CHAPTER I

INTRODUCTION

1.1. Introduction.

In many disciplines of the social and natural sciences dynamic systems are encountered that are made up of a large number of separate but interacting units. Due to complexity, inherent random effects or incompleteness of information about the dynamic structure, a stochastic model is appropriate for many of these systems.

This thesis is devoted to the study of some stochastic models in inventories. An inventory system is a facility at which items of materials are stocked. In order to promote smooth and efficient running of business, and to provide adequate service to the customers, an inventory of materials is essential for any enterprise. When uncertainty is present, inventories are used as a protection against risk of stock out. It is advantageous to procure the item before it is needed at a lower marginal cost. Again, by bulk purchasing, the advantage of price discounts can be availed. All these contribute to the formation of inventory.

Maintaining inventories is a major expenditure for any organization. For each inventory, the fundamental question is how much new stock should be ordered and when should the orders be placed. If large quantities are ordered, the organization has to pay excessive storage cost. On the other hand, very small order quantities result in very high procurement cost. Hence, a trade off between the two is called for. Management of any such inventory involves monitoring the input and withdrawals of inventoried items, as well as making decisions as to the best means of replenishing the inventory.
In the present study, we have considered several models for single and two commodity stochastic inventory problems. By model building, we mean providing a model that will provide a good fit to a set of data and that will give good estimates of parameters and good prediction of future values for given values of the independent variables.

1.2. Historical Background.

The first quantitative analysis in inventory studies started with the work of Harris in 1915. He formulated mathematically a simple inventory situation and obtained its solution. Wilson rediscovered the same formula in 1918. After the second world war, several researchers like Pierre Masse (1946), Arrow, Harris and Marschack (1951) Dvoretsky, Kiefer and Wolfowitz (1952) and Whitin (1953) have discussed the stochastic nature of inventory problems.

A systematic analysis of \((s, S)\) inventory model based on renewal theory is first provided by Arrow Karlin and Scarf (1958). The book by Hadley and Whitin (1963), provides an excellent account of applications. A computational approach for finding optimal \((s, S)\) inventory policies is given by Veinott and Wagner (1965). An excellent review by Veinott (1966), summarizes the status of mathematical theory of inventory until the early sixties. He focuses his attention on the determination of optimal policies of multi-item and/or for multi echelon inventory systems with certain and uncertain demands. The cost analysis of different inventory systems along with several other characteristics is given in Naddor (1966). Gross and Harris (1971) develop continuous review \((s, S)\) inventory models with state dependent lead times. Sivazlian (1974) considered a continuous review \((S, s)\) inventory system with arbitrary inter arrival time distribution between demands, where each arrival demands exactly one unit. He obtains the transient and steady state distribution for the position inventory and shows that the limiting distribution of the position inventory is uniform and is independent of the inter
arrival time distribution under many sharp assumptions. The same result for the case with arbitrarily distributed demand quantity has been obtained by Richards (1975). An indepth study of (s, S) inventory policy with arbitrarily distributed lead time is available in Srinivasan (1979). Here he assumes the demand process as a renewal process where as Sahin (1979) considers an inventory problem with the item being continuously measured; inter arrival times form a renewal process. However, she assumes the lead time to be a degenerate random variable. This was further extended by Manoharan, Krishnamoorthy and Madhusoodhanan (1987) to the case of non-identically distributed inter arrival demand times and random lead times, which however is restricted to demand quantity being exactly equal to one unit.

An (s, S) inventory system with demand for items dependent on an external environment is studied by Feldmann (1975). Ramaswami (1981) obtains algorithms for an (s, S) inventory model where the demand is according to a versatile Markovian point process. The binomial moments of the time dependent and limiting distributions of the deficit in the case of a continuous review (s, S) policy with random lead time and demand process following a compound renewal process have been obtained by Sahin (1983).

Thangaraj and Ramanarayanan (1983) discuss an inventory system with two reordering levels and random lead time. Ramanarayanan and Jacob (1986) analyze the same problem with relaxation that the lead time is random and several reordering levels. Krishnamoorthy and Manoharan (1991) discuss the same problem in which they have obtained the time dependent probability distribution of the inventory level and the correlation between the number of demands during a lead time and the length of the next inventory dry period. Krishnamoorthy and Manoharan (1990) consider an (s,S) inventory problem with state dependent demand quantities. They obtain the system state probabilities.

The review by Nahmias (1982) provides the state of art on perishable inventory models until the beginning of the eighties. Kalpakom and Arivarignan introduce
perishability of exhibiting item(s) and provide several characterization of the underlying inventory process. They (1985a) consider the case of an inventory system with arbitrary inter arrival time between demands in which one item is put into operation as an exhibiting item whose lifetime has the exponential distribution. Non exhibited items do not deteriorate. The transient and steady state distributions for position inventory are derived under assumption that quantity demanded at a demand epoch depends the time elapsed since the previous arrival. Again the same system having one exhibiting item subject to random failures with failure times following exponential distribution and unit demand is dealt with by the same authors (1985b) and the expression for the limiting distribution of the position inventory is derived by applying the techniques of semi-regenerative process. Manoharan and Krishnamoorthy (1989) consider an inventory problem with all items subject to decay and derive the limiting probability distribution. They assume that quantities demanded by arrivals are independently and identically distributed random variables and inter arrival times follow an arbitrary distribution. Kalpakom and Arivarignan (1989) analyze a perishable inventory model in which the inventoried items have life times with negative exponential distribution with demands forming a Poisson process which is extended by Krishnamoorthy and Varghese (1995) to one, subject to disasters.

Ramanarayanan and Jacob (1987) analyze an inventory system with random lead time and bulk demands. They use the matrix of transition time densities and its convolutions to arrive at the expression for the probability distribution of the inventory level. Inventory systems with random lead times and server vacations when the inventory becomes dry is introduced by Daniel and Ramanarayanan (1987, 1988).

Sivazlian and Stanfel (1975) discuss a two commodity single period inventory problem. Krishnamoorthy, Basha and Lakshmi (1994) consider a two commodity inventory system with demand quantities exactly one unit of either or both type at each demand epoch. They investigate the stationary distribution of the system state. Some optimization problems associated with this model are also examined. Also
Krishnamoorthy, Lakshmi and Basha (1997) generalize the above set up by analyzing a two commodity inventory problem with Markov shift in demand of either type of commodity, and derive the stationary distribution of the system state. They provide a characterization for the system state distribution to be uniform.


N Policy is introduced into inventory problem by Krishnamoorthy and Raju (1998a, b) wherein local purchase is resorted to when the backlog reaches a threshold N. Three types of local purchases are discussed by them-local purchase to bring the level to S cancelling outstanding order, local purchase to bring the level to s and the local purchase to meet the backlog alone without cancelling the outstanding orders. They examine the N value that minimizes the total expected cost.

1.3. An Outline of the Present Work :

The thesis is divided into six chapters, including this introductory chapter. Chapters two and three are about single commodity inventory problems and the last three derived on two commodity problems. We have analyzed the models to get the inventory level probabilities at any instant of time and determined the cost functions. Most of the models are illustrated with numerical examples.

Chapter two deals with single commodity, continuous review, (s, S) inventory system with disasters. In most of the analysis of inventory systems the decay and disaster factors are ignored. But in several practical situations, these factors play an important role in decision making. Examples are electronic equipment stored and exhibited on a sales counter where there is possibility of damage to the equipment due to lightning, crops subject to natural calamity etc.
We have examined two models. In Model I, inventory level depletes due to both disasters and demands. Shortages are not allowed and lead time is zero. The interarrival times of disasters have arbitrary distribution $G(.)$ and the quantity destructed depends on the time elapsed between disasters. Demands form a Compound Poisson process. The assumptions of Model II are similar to Model I except that the time elapsed between two consecutive demand points are independently and identically distributed with common distribution function $G(.)$ and demand magnitude depends only on the time elapsed since the previous demand points. The probability distribution of stock level at arbitrary time points and also the steady state inventory level distribution are obtained for both the models. Cost functions associated with the models are also studied.

In chapter III, we have introduced correlation in $(s, S)$ inventory problems in two different ways. Model I discusses analysis of correlated order quantity. Model II studies correlation between order quantity and replenishment quantity. The inventory level at arbitrary time point and its limiting distribution are computed. Some optimization problems are also examined for both the models.

Chapter IV deals with linearly correlated bulk demand two commodity inventory problem, where each arrival demands a random number of items of each commodity $C_1$ and $C_2$, the maximum quantity demanded being $a (< s_1)$ and $b (< s_2)$ respectively. The particular case of linearly correlated demand is also discussed. Numerical illustrations are also provided.

Chapter V deals with two models. First model describes a bulk demand two commodity inventory problem. We follow $(s_k, S_k)$ policy for the commodity $C_k$ ($k = 1, 2$). The probability that a demand occurs for commodity $C_k$ alone is $p_k$ and a demand for both $C_1$ and $C_2$ together is assumed not to occur. Thus $p_1 + p_2 = 1$. Lead time is assumed to be zero.
In Model II, all assumptions are similar to Model I except that the probability for a demand of both commodities together is allowed. Lead time is exponentially distributed for first commodity and sales of $C_1$ restricted to those customers, that demand second commodity $C_2$ also until $C_1$ is replenished. The limiting probabilities and optimization problems are examined for both models. Some numerical illustrations are also provided.

In the last chapter, we analyze a two commodity inventory problem with lead time under N policy. Local purchase by shopkeepers are very common. Situations of this sort arise in practice in shops when certain goods run out of stock and on reaching a threshold (negative level), the owner goes for local purchase. Though this results in higher cost to the system, it ensures goodwill of customers.

In this model, all assumptions are similar to Model II described in Chapter V except that we introduce the N policy for local purchase of the first commodity. Three variants of the problem are investigated. The limiting probabilities of the system size are derived. An optimization problem is examined. Numerical illustrations are also provided.

The notations used in this thesis are explained in each chapter. The thesis ends with a list of references.