ACKNOWLEDGEMENT

First and foremost, I wish to pay my most humble obeisance to the almighty God for giving me the inspiration and wisdom to take up this task. I have been his guiding hand that has brought this task to a successful completion.

I feel great pleasure in expressing my deep and profound sense of gratitude to my esteemed teacher, mentor and research guide Dr. H. D. Doctor, Professor and Head, Department of Mathematics, Veer Narmad South Gujarat University, Surat. It could hardly have become possible for me to venture in the particular field of research, without his continuous guidance, encouragement, motivative attitude, punctuality and parental care. I feel honored and consider myself very lucky to work under his guidance. He has not only guided me but also acted as co-traveler too, throughout my research work and ensured that I reach destination. He has been a guiding light to me during my research work will always remains so.

I feel great pleasure to my deep and sincere to Dr. D. C. Joshi, Professor Department of Mathematics, Veer Narmad South Gujarat University, Surat for his perceptive guidance, moral support, constant encouragement, enduring passion and help out of the way led me to overcome all the difficulties at every stage in my research work.

I am thankful to Dr. M.G. Timol, Dr. K. B. Patel, and Dr. P.V. Tandcl, of Mathematics Department, Veer Narmad South Gujarat University, Surat for that kindly support valuable help and continuous encouragement they have rendered to me.
I am thankful to the Principal **Shri Ashokbhai Patel** of Jagruti Vidhyalay Rumla and Dakshin Gujarat Kolcha Kotvalia Aadivasi Seva Samaj, Khergam Trust for give me necessary permission and extending their co-operation during my research work.

Who in this world can entirely and adequately thank the parents who have given us everything that we possess in this life. During my research work there was continuous support from my wife **Hetal**.

And last but not the least me heartfelt thanks to my son **Het** and daughter **Jisha** for supporting and encouraging me to pursue this degree.

At the end, I am taking opportunity to thank sincerely each and everybody who have directly and indirectly helped me in preparation of present work.

**Place:** (UMESH S. PATEL)

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SYNOPSIS

The present dissertation entitled “A STUDY OF NON-NEWTONIAN FLUID FLOW USING SPLINE COLLOCATION TECHNIQUE” submitted for the award of Ph. D. degree is an outcome of studies of non-linear mathematical models related to Non-Newtonian flow arising in various branches of Fluid Dynamics with Quartic spline method.

One of the most important thing engineers and scientists do is to model physical phenomena. Virtually every phenomenon in nature, whether aerospace, biological, chemical, geological, or mechanical can be described with the aid of the laws of physics or other fields in terms of algebraic, differential, and/or integral equations relating various quantities of interest. Determining the stress distribution in a pressure vessel with oddly shaped holes and numerous stiffeners and subjected to mechanical, thermal, and/or aerodynamic loads, finding the concentration of pollutants in lakes and sea water or in the atmosphere, and simulating weather in an attempt to understand and predict the formation of thunderstorms and tornadoes are example of many practical problems that Engineers deal with.

Analytical descriptions of physical phenomena and processes are called Mathematical models. Mathematical models of a process are developed using assumptions concerning how the process work and using appropriate or laws governing the process, and they are often characterized by very complex differential and/or integral equations posed on geometrically complicated domains. Consequently the processes to be studies, until the advent of electronic computation, were drastically simplified so that the governing equations can be solved analytically. Over the last few decades, however, computers have made it possible, with the help of suitable mathematical models and numerical methods, to solve many practical problems of engineering. The use of a numerical method and a computer to evaluate the mathematical model of a process and estimate its characteristics is called Numerical simulation. There now exists a new and growing body of knowledge connected with the
development of a mathematical models and use of Numerical simulation of physical system and it is known as computational mechanics.

This dissertation emphasizes the use of Quartic spline collocation technique for analyzing problems related to Non-Newtonian MHD boundary layer flow.

The dissertation consists of six chapters. The first chapter contains a general discussion about the numerical method over finite difference and Method of Weighted Residual (MWR). The rapid development of high speed digital computer and the increasing development of recent technology lead to demand of numerical methods. As the majority of differential equations in science and Engineering cannot be integrated analytically, it is necessary to apply some methods of approximation. Also sometimes it happens that such type of physical conditions arise which need a very close solution of the system, essentially the approximate methods are useful. When analytic solution is not available we have to consider approximate methods, and we have to be careful about selection of method. Normally a method is selected which requires a minimum number of steps, consumes the shortest computing time and yet that does not produce any excessive errors.

Second chapter contains a discussion and preliminaries of the problems considered. In Engineering and Applied Sciences many of the problems gives rise to mathematical models that produce non-linear Boundary value problems (BVPs) and the second end conditions may be specified at infinity. We have been able to tackle such type of problems very effectively. Particularly in simulating models associated to Falkner-Skan (F. S.) problems, non-Newtonian laminar boundary layer flow with magnetic field, M.H.D. flow due to shrinking / extensible surface and problems of heat transfer form a continuous surface are attempted here.

Third chapter gives a survey of spline functions. It is commonly accepted that the first mathematical reference to spline was give by Schoenberg in1946, which is probably the first place that word “spline” is used in the aircraft and shipbuilding
industries. General spline functions with their minimum norm properties were discussed by Lynch et al (1964, 66). Ahlberg et al (1964a, 1965b) extended the integral relation to spline in more than one dimensions. The period of 1960 to 1972 was significant in the field of spline theory, as a remarkable research on existence, uniqueness, minimum norm property and best approximation property was done. In spline function theory, various types of splines are found like Generalized splines, Cardinal splines, Lg-splines, Natural splines, B-splines, Polynomial splines, Parabolic splines, Trigonometric splines. These functions were found as well as studied during the years 1964 to 1969. Albasiny et al (1969) tried to use spline functions as interpolants of the solution to a linear differential equations. Callender (1971) presented the procedure for obtaining low order, high accuracy spline approximations of solution to initial value problem in ordinary differential equations. The use of B-spline was made to obtain the solution of the equation by Sincovec (1972) arising from collocation.

In 1968 Brickley brought forward an useful aspect of spline functions in light that can be employed to solve a linear two point boundary value problem approximately. The spline presented by him is expressed in terms of infinite series or in truncated powers. This work was supported by Fyfe (1969). In fact a substantial work regarding the error estimates in cubic spline approximations was presented here. Again Fyfe (1970), extended the use of same cubic spline function to the solution of forth order linear two point boundary value problem and this was analyzed by Hoskins (1973), with a view point of error.

Here we have developed an algorithm to solve various BVPs using the Bickley’s definition of cubic spline and Quartic spline. We discuss the combination of Quasilinearization technique and Quartic spline collocation method when the governing equation is nonlinear.

Chapter four discusses numerical study of the mathematical models associated to an unsteady two-dimensional laminar hydrodynamic boundary layer flow past a wedge. The governing boundary layer equations are solved numerically using Quartic
spline collocation technique. The results in the form of velocity profiles and skin frictions for various parameters are obtained and analyzed the effect of these parameters in detail. Also some the results are compared with available results. The analysis of boundary layer flow induced by a moving rigid surface due to Sakiadis is well known. As an extension to this study, the flow over a stretching sheet so far, recently, Crane [1970] the analysis to flow induced by a stretching sheet. We considered here the boundary layer flow of a viscous fluid over a nonlinear axisymmetric stretching sheet. Results are obtained and presented by Graph.

Chapter five gives an insight into numerical study of the boundary layer flow of a power-law non-Newtonian fluid over a continuously moving surface in the presence of magnetic field applied perpendicular to the surface. The effect of the Stewart number (N) and the power law-index on the velocity profiles and skin friction are studied. Also this chapter concerned with the boundary layer flow of a power law non-Newtonian fluid in the presence of a magnetic field applied perpendicular to the surface and an electric field perpendicular to magnetic field. The combined effects of the magnetic forces and the flow index on the velocity profile, the sheer stress on the surface, the displacement thickness and the momentum thickness are studied. It is found that the velocity of the fluid increases with increase of either magnetic forces or the flow index individually, with the other kept constant.

Chapter six contains an analysis for the hydro magnetic flow and heat transfer adjacent to a stretching vertical sheet [Crane (1970)]. An analysis is done for the flow and heat transfer of a non-Newtonian fluid known as Casson fluid over a permeable stretching surface through a porous. The transformed boundary layer equations are solved numerically using Quartic spline collocation method for the involved parameters. Also we study the problem of MHD Non Newtonian Power law fluid and Heat Transfer past a non-linear stretching surface with thermal radiation and viscous dissipation. A systematic study is carried out to illustrate the effects of various parameters on the fluid velocity and the temperature distribution in the boundary layer through graphs. The
results for the local skin-friction coefficient and the local Nusselt number are tabulated and discussed and found to be in good agreement with available results.

The present work justifies the numerical simulation of mathematical models related to laminar boundary layer MDH flow with suction / injection over varying surface using Quartic spline collocation technique and establishes the reliability of the method in computational fluid dynamics and suggests a wide range of application to various physical phenomena.
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6.2.11 Skin friction different values of $m$

6.2.12 Velocity Profile different values of $m$

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