RELIABILITY ANALYSIS OF A STANDBY REDUNDANT COMPLEX SYSTEM WITH CHANGING ENVIRONMENT UNDER PREEMPTIVE-REPEAT REPAIR DISCIPLINE

In this repair discipline, priority is given to the repair of subsystem A over the repair of partial failure in subsystem B, and after completion of repair of subsystem A, the repair of partial failure restarts and is considered as a fresh failure i.e., the repair already carried out on the partial failure goes waste.

In addition to the notations stated in the beginning of this chapter, the following notation is also defined to formulate the mathematical model:

\[ P_{A2}(x,t) = \text{Probability that at time } t, \text{ the complex system is in failed state and is under repair of subsystem A, } x \text{ time has elapsed in the repair of subsystem A and the partial failure, which has been in repair is awaiting repair a fresh.} \]

The state transition diagram showing the various probability states of the complex system under this repair discipline is given in Fig 1.3.1
FIG 1.3.1: STATE TRANSITION DIAGRAM
FORMULATION OF THE MATHEMATICAL MODEL

By elementary probability and continuity arguments, the difference-differential equations governing the stochastic behaviour of the complex system are.

\[
\left[ \frac{d}{dt} + \lambda_1 + \mu_1 + \mu_2 + \eta \right] P_{0,0}(t) = \int_0^\infty P_{A2,0}(x,t) \gamma_A(x) dx + \int_0^\infty P_{0,1}(y,t) \gamma_1(y) dy + \int_0^\infty \left[ P_{0,2}(z,t) + P_{A1,2}(z,t) \right] \gamma_2(z) dz + \delta P_{\text{atm}}(t) \tag{1.3.1}
\]

\[
\left[ \frac{d}{dt} + \lambda_2 + \mu_2 + \mu_1 + \eta \right] P_{A1,0}(t) = \lambda_1 P_{0,0}(t) + \int_0^\infty P_{A1,1}(y,t) \gamma_1(y) dy \tag{1.3.2}
\]

\[
\left[ \frac{\partial}{\partial t} + \lambda_1 + \mu_1 + \mu_2 + \eta \right] P_{A2,0}(x,t) = 0 \tag{1.3.3}
\]

\[
\left[ \frac{\partial}{\partial t} + \gamma_1(y) + \lambda_1 + \mu_2 + \eta \right] P_{0,1}(y,t) = 0 \tag{1.3.4}
\]

\[
\left[ \frac{\partial}{\partial t} + \gamma_1(y) + \lambda_2 + \mu_2 + \eta \right] P_{A1,1}(y,t) = \lambda_1 P_{0,1}(y,t) \tag{1.3.5}
\]

\[
\left[ \frac{\partial}{\partial t} + \gamma_A(x) \right] P_{A2,1}(x,t) = 0 \tag{1.3.6}
\]

\[
\left[ \frac{\partial}{\partial t} + \gamma_2(z) \right] P_{a2,2}(z,t) = 0 \tag{1.3.7}
\] (\(a=0, A1\))

\[
\left[ \frac{d}{dt} + \delta \right] P_{\text{atm}}(t) = \eta \left[ P_{A1,0}(t) + P_{0,0}(t) + P_{A1,1}(t) + P_{0,1}(t) \right] \tag{1.3.8}
\]

BOUNDARY CONDITIONS

\[
P_{A2,0}(0,t) = \lambda_2 P_{A1,0}(t) \tag{1.3.9}
\]

\[
P_{0,1}(0,t) = \mu_1 P_{0,0}(t) + \int_0^\infty P_{A2,1}(x,t) \gamma_A(x) dx \tag{1.3.10}
\]
\[ P_{A1,1}(0,t) = \mu_{i}P_{A1,0}(t) \] ..(1.3.11)
\[ P_{A2,1}(0,t) = \lambda_{2}P_{A1,1}(t) \] ..(1.3.12)
\[ P_{A2,2}(0,t) = \mu_{2}[P_{0,0}(t) + P_{0,1}(t)] \] ..(1.3.13)
\[ P_{A1,2}(0,t) = \mu_{2}[P_{A1,0}(t) + P_{A1,1}(t)] \] ..(1.3.14)

**INITIAL CONDITIONS**

\[ P_{i}(0) = \begin{cases} 1, & \text{when } i = 0 \\ 0, & \text{otherwise} \end{cases} \] ..(1.3.15)

**SOLUTION OF THE MODEL**

Taking Laplace transforms of equation (1.3.1) through (1.3.14) and using equation (1.3.15), one may obtain

\[ s + \lambda_{1} + \mu_{i} + \mu_{2} + \eta\overline{P}_{0,0}(s) = 1 + \int_{0}^{\infty} \overline{P}_{A2,0}(x,s)\gamma_{A}(x)dx + \int_{0}^{\infty} \overline{P}_{0,1}(y,s)\gamma_{1}(y)dy \] ..(1.3.16)

\[ + \int_{0}^{\infty} [\overline{P}_{0,2}(z,s) + \overline{P}_{A1,2}(z,s)]\gamma_{2}(z)dz + \delta\overline{P}_{a,t}(s) \]

\[ s + \lambda_{2} + \mu_{2} + \mu_{1} + \eta\overline{P}_{A1,0}(s) = \lambda_{1}\overline{P}_{0,0}(s) + \int_{0}^{\infty} \overline{P}_{A1,1}(y,s)\gamma_{1}(y)dy \] ..(1.3.17)

\[ s + \frac{\partial}{\partial x} + \gamma_{A}(x)\overline{P}_{A2,0}(x,s) = 0 \] ..(1.3.18)

\[ s + \frac{\partial}{\partial y} + \gamma_{1}(y) + \lambda_{1} + \mu_{2} + \eta\overline{P}_{0,1}(y,s) = 0 \] ..(1.3.19)

\[ s + \frac{\partial}{\partial y} + \gamma_{1}(y) + \lambda_{2} + \mu_{2} + \eta\overline{P}_{A1,1}(y,s) = \lambda_{1}\overline{P}_{0,1}(y,s) \] ..(1.3.20)

\[ s + \frac{\partial}{\partial x} + \gamma_{A}(x)\overline{P}_{A2,1}(x,s) = 0 \] ..(1.3.21)

\[ s + \frac{\partial}{\partial z} + \gamma_{2}(z)\overline{P}_{a,2}(z,s) = 0 \] ..(1.3.22)

(a=0,A1)
\[ s + \delta \mathcal{P}_{\alpha} = \eta \left( \mathcal{P}_{\alpha,0} + \mathcal{P}_{\alpha,1} + \mathcal{P}_{\alpha,2} \right) \]  
\[ \mathcal{P}_{\alpha,0} = \lambda_{\alpha} \mathcal{P}_{\alpha,0} \]  
\[ \mathcal{P}_{\alpha,1} = \mu_{\alpha} \mathcal{P}_{\alpha,0} + \int_{0}^{\infty} \mathcal{P}_{\alpha,1}(x) \gamma_{\alpha}(x) dx \]  
\[ \mathcal{P}_{\alpha,2} = \lambda_{\alpha} \mathcal{P}_{\alpha,1} \]  
\[ \mathcal{P}_{\alpha,2} = \mu_{\alpha} \left( \mathcal{P}_{\alpha,0} + \mathcal{P}_{\alpha,1} \right) \]  
\[ \mathcal{P}_{\alpha,2} = \mu_{\alpha} \left( \mathcal{P}_{\alpha,0} + \mathcal{P}_{\alpha,1} \right) \]  

Integrating equations (1.3.18), (1.3.19), (1.3.21) and (1.3.22), one gets,

\[ \mathcal{P}_{\alpha,0}(x) = \mathcal{P}_{\alpha,0}(0) \exp \left[ -sx - \int_{0}^{x} \gamma_{\alpha}(y) dy \right] \]  
\[ \mathcal{P}_{\alpha,1}(y) = \mathcal{P}_{\alpha,1}(0) \exp \left[ -(s + \lambda_{\alpha} + \eta + \mu_{\alpha})y - \int_{0}^{y} \gamma_{\alpha}(y) dy \right] \]  
\[ \mathcal{P}_{\alpha,2}(x) = \mathcal{P}_{\alpha,2}(0) \exp \left[ -sx - \int_{0}^{x} \gamma_{\alpha}(y) dy \right] \]  
\[ \mathcal{P}_{\alpha,2}(z) = \mathcal{P}_{\alpha,2}(0) \exp \left[ -sz - \int_{0}^{z} \gamma_{\alpha}(y) dy \right] \]  
\[ \text{Use of relation (1.3.24), in equation (1.3.30), one may get,} \]  
\[ \mathcal{P}_{\alpha,0}(x) = \lambda_{\alpha} \mathcal{P}_{\alpha,0}(s) \exp \left[ -sx - \int_{0}^{x} \gamma_{\alpha}(y) dy \right] \]  

Integrating equation (1.3.34) w.r.t x from 0 to \( \infty \), one obtain

\[ \mathcal{P}_{\alpha,0}(s) = \lambda_{\alpha} \mathcal{J}_{\alpha,0}(s,0) \mathcal{P}_{\alpha,0}(s) \]
where,
\[ J_i(s, \alpha) = \left[ 1 - S_i(s + \alpha) \right] (s + \alpha)^{-1} \text{ where } i = A, 1 \text{ or } 2 \]

Multiplying equation (1.3.32) by \( \gamma_A(x) \) and then integrating from 0 to \( \infty \), one gives

\[
\int_0^\infty P_{A1,1}(x, s) \gamma_A(x) dx = \lambda_2 S_A(s) P_{A1,1}(s) \tag{1.3.36}
\]

Using equation (1.3.31), equation (1.3.20) on integration, one gets

\[
P_{A1,1}(y, s) = \left[ \lambda_i (\lambda_2 - \lambda_1)^{-1} \left\{ e^{(\lambda_2 - \lambda_1)y} - 1 \right\} P_{01}(0, s) + P_{A1,1}(0, s) \right] \\
\exp \left[ - (s + \lambda_2 + \mu_2 + \eta) y - \int_0^y \gamma_1(y) dy \right] \tag{1.3.37}
\]

Using equations (1.3.25), (1.3.26) (1.3.36), equation (1.3.37) on integration w.r.t \( y \) from 0 to \( \infty \), one yields

\[
P_{A1,1}(s) = L(s) P_{00}(s) + N(s) P_{A1,0}(s) \tag{1.3.38}
\]

where,

\[
L(s) = \lambda_1 \mu_e E_i(s, \lambda_1 + \mu_2 + \eta, \lambda_2 + \mu_2 + \eta) \left[ 1 - K(s) \right]^{-1}
\]

\[
N(s) = \mu_e J_i(s, \lambda_1 + \mu_2 + \eta) \left[ 1 - K(s) \right]^{-1}
\]

\[
K(s) = \lambda_1 \lambda_2 \bar{S}_A(s) E_i(s, \lambda_1 + \mu_2 + \eta, \lambda_2 + \mu_2 + \eta)
\]

\[
E_i(s, \alpha, \beta) = \left[ J_i(s, \alpha) - J_i(s, \beta) \right] (\beta - \alpha)^{-1}
\]

Utilizing equation (1.3.36) and (1.3.38), equation (1.3.25) becomes

\[
P_{01}(0, s) = \left[ \mu_e + \lambda_2 \bar{S}_A(s) L(s) \right] P_{00}(s) + \lambda_2 \bar{S}_A(s) N(s) P_{A1,0}(s) \tag{1.3.39}
\]
Using equation (1.3.39), equation (1.3.31) on integration w.r.t $y$ from 0 to $\infty$, one may get
\[
\overline{P}_{0,1}(s) = J_1(s, \lambda_1 + \mu_2 + \eta) [\mu_1 + \lambda_2 S_A(s)L(s)]\overline{P}_{0,0}(s) + \lambda_2 S_A(s)N(s)\overline{P}_{A1,0}(s) \quad \text{(1.3.40)}
\]

Utilizing equations (1.3.27) and (1.3.38), equation (1.3.32) on integration w.r.t $x$ from 0 to $\infty$, one gives
\[
\overline{P}_{A2,1}(s) = \lambda_2 J_2(s,0) [L(s)\overline{P}_{0,0}(s) + N(s)\overline{P}_{A1,0}(s)] \quad \text{(1.3.41)}
\]

Using equations (1.3.28) and (1.3.40), equation (1.3.33) for $a=0$ on integration w.r.t $z$ from 0 to $\infty$, one yields
\[
\overline{P}_{0,2}(s) = \mu_2 J_2(s,0) [1 + \mu_1 J_1(s, \lambda_1 + \mu_2 + \eta) + \lambda_2 S_A(s)L(s)J_1(s, \lambda_1 + \mu_2 + \eta)] \quad \text{(1.3.42)}
\]
\[
\overline{P}_{0,2}(s) = \lambda_2 S_A(s)N(s)J_1(s, \lambda_1 + \mu_2 + \eta)\overline{P}_{A1,0}(s) \quad \text{(1.3.42)}
\]

Using equations (1.3.29) and (1.3.38), equation (1.3.33) for $a=A1$ on integration w.r.t $z$ from 0 to $\infty$, one obtain
\[
\overline{P}_{A1,2}(s) = \mu_2 J_2(s,0) [L(s)\overline{P}_{0,0}(s) + 1 + N(s)\overline{P}_{A1,0}(s)] \quad \text{(1.3.43)}
\]

Employing (1.3.38) and (1.3.40) in equation (1.3.23), one may get
\[
\overline{P}_{ana}(s) = \frac{\eta}{s+\delta} [1 + L(s) + \mu_1 J_1(s, \lambda_1 + \mu_2 + \eta) + \lambda_2 S_A(s)L(s)J_1(s, \lambda_1 + \mu_2 + \eta)]\overline{P}_{0,0}(s)
\]
\[
+ [1 + N(s) + \lambda_2 S_A(s)N(s)J_1(s, \lambda_1 + \mu_2 + \eta)\overline{P}_{A1,0}(s)] \quad \text{(1.3.44)}
\]

Employing (1.3.37), (1.3.26) and (1.3.39) in eq. (1.1.17) and then simplifying, one gets,
\[
H(s)\overline{P}_{A1,0}(s) - Q(s)\overline{P}_{0,0}(s) = 0 \quad \text{(1.3.45)}
\]
where,

\[ H(s) = s + \lambda_2 + \mu_1 + \mu_2 + \eta - \mu_1 S_1(s + \lambda_2 + \mu_2 + \eta) \]
\[ - \lambda_2 S_A(s)N(s)D_i(s, \lambda_1 + \mu_2 + \eta, \lambda_2 + \mu_2 + \eta) \]

\[ Q(s) = \lambda_1 \left[ 1 + \mu_1 D_i(s, \lambda_1 + \mu_2 + \eta, \lambda_2 + \mu_2 + \eta) \right] + \lambda_2 S_A(s)N(s)D_i(s, \lambda_1 + \eta, \lambda_2 + \mu_2 + \eta) \]

\[ D_i(s, \alpha, \beta) = \frac{S_i(s + \alpha) - S_i(s + \beta)}{\beta - \alpha} \]

Similarly, utilization of equations (1.3.30), (1.3.31), (1.3.33) and other relevant relationship in (1.3.16) and then simplifying, one gives

\[ A(s)\bar{P}_{0,0}(s) - B(s)\bar{P}_{A1,0}(s) = 0 \quad \text{(1.3.46)} \]

where,

\[ A(s) = s + \lambda_1 + \mu_1 + \mu_2 + \eta - \left[ \mu_1 + \lambda_2 S_A(s)L(s)\bar{S}_1(s + \lambda_1 + \mu_2 + \eta) \right] \]
\[ - \left\{ \mu_2 S_2(s) + \frac{\eta \delta}{s + \delta} \right\} \left[ 1 + L(s) + \mu_1 J_1(s, \lambda_1 + \mu_2 + \eta) + \lambda_2 S_A(s)L(s)J_1(s, \lambda_1 + \mu_2 + \eta) \right] \]

\[ B(s) = \lambda_2 S_A(s) + \lambda_2 N(s)S_A(s)\bar{S}_1(s + \lambda_1 + \mu_2 + \eta) \]
\[ + \left\{ \mu_2 S_2(s) + \frac{\eta \delta}{s + \delta} \right\} \left[ 1 + N(s) + \lambda_2 S_A(s)N(s)J_1(s, \lambda_1 + \mu_2 + \eta) \right] \]

Solving equations (1.3.45) and (1.3.46), one gets:

\[ \bar{P}_{0,0}(s) = \frac{H(s)}{T(s)} \quad \text{(1.3.47)} \]

\[ \bar{P}_{A1,0}(s) = \frac{Q(s)}{T(s)} \quad \text{(1.3.48)} \]

where,

\[ T(s) = H(s)A(s) - Q(s)B(s) \]
Now, one may get

\[ \overline{P}_{0,0}(s) = \frac{H(s)}{T(s)} \]  \quad \text{..(1.3.49)}

\[ \overline{P}_{a1,0}(s) = \frac{Q(s)}{T(s)} \]  \quad \text{..(1.1.50)}

\[ \overline{P}_{a2,0}(s) = \lambda_2 J_A(s,0) \frac{Q(s)}{T(s)} \]  \quad \text{..(1.3.51)}

\[ \overline{P}_{0,1}(s) = J_1(s, \lambda_1 + \mu_2 + \eta) \left[ \left( \mu_1 + \lambda_2 \overline{S}_A(s) L(s) \right) \frac{H(s)}{T(s)} + \lambda_2 \overline{S}_A(s) N(s) \frac{Q(s)}{T(s)} \right] \]  \quad \text{..(1.3.52)}

\[ \overline{P}_{a1,1}(s) = L(s) \frac{H(s)}{T(s)} + N(s) \frac{Q(s)}{T(s)} \]  \quad \text{..(1.3.53)}

\[ \overline{P}_{a2,1}(s) = \lambda_2 J_A(s,0) \left[ L(s) \frac{H(s)}{T(s)} + N(s) \frac{Q(s)}{T(s)} \right] \]  \quad \text{..(1.3.54)}

\[ \overline{P}_{0,2}(s) = \mu_2 J_2(s,0) \left[ 1 + \mu_1 J_1(s, \lambda_1 + \mu_2 + \eta) + \lambda_2 \overline{S}_A(s) L(s) J_1(s, \lambda_1 + \mu_2 + \eta) \right] \]  \quad \text{..(1.3.55)}

\[ \frac{H(s)}{T(s)} + \lambda_2 \overline{S}_A(s) N(s) J_1(s, \lambda_1 + \mu_2 + \eta) \frac{Q(s)}{T(s)} \]  \quad \text{..(1.3.56)}

\[ \overline{P}_{a1,2}(s) = \mu_2 J_2(s,0) \left[ L(s) \frac{H(s)}{T(s)} + \left( 1 + N(s) \right) \frac{Q(s)}{T(s)} \right] \]  \quad \text{..(1.3.56)}

\[ \overline{P}_{\alpha \alpha}(s) = \frac{\eta}{s + \delta} \left[ 1 + L(s) + \mu_1 J_1(s, \lambda_1 + \mu_2 + \eta) + \lambda_2 \overline{S}_A(s) L(s) J_1(s, \lambda_1 + \mu_2 + \eta) \right] \]  \quad \text{..(1.3.57)}
UP AND DOWN STATE PROBABILITIES

Laplace transforms of operational availability and non-availability of the system are:

\[ \bar{P}_{up}(s) = \bar{P}_{0,0}(s) + \bar{P}_{A1,0}(s) + \bar{P}_{0,1}(s) + \bar{P}_{A1,1}(s) \]

\[ = \left[ 1 + L(s) + \left\{ \mu_1 + \lambda_2 \bar{S}_A(s) \right\} J_1(s, \lambda_1 + \mu_2 + \eta) \right] \left[ H(s) / T(s) \right] \]

\[ + \left[ 1 + N(s) + \lambda_2 \bar{S}_A(s) N(s) J_1(s, \lambda_1 + \mu_2 + \eta) \right] \left[ Q(s) / T(s) \right] \]

..(1.3.58)

\[ \bar{P}_{down}(s) = \bar{P}_{A2,0}(s) + \bar{P}_{A2,1}(s) + \bar{P}_{0,2}(s) + \bar{P}_{A2,2}(s) + \bar{P}_{am}(s) \]

\[ = \left[ \lambda_2 L(s) J_2(s,0) + \left\{ \mu_2 J_2(s,0) + \frac{\eta}{(s + \delta)} \right\} \right] \left[ 1 + \mu_1 J_1(s, \lambda_1 + \mu_2 + \eta) \right] \]

\[ + L(s) + \lambda_2 \bar{S}_A(s) L(s) J_1(s, \lambda_1 + \mu_2 + \eta) \right] \left[ H(s) / T(s) \right] \]

\[ + \left[ \lambda_2 \left\{ J_A(s,0) + N(s) J_2(s,0) \right\} + \left\{ \mu_2 J_2(s,0) + \frac{\eta}{(s + \delta)} \right\} \right] \]

\[ \left[ 1 + N(s) + \lambda_2 \bar{S}_A(s) N(s) J_1(s, \lambda_1 + \mu_2 + \eta) \right] \left[ Q(s) / T(s) \right] \]

..(1.3.59)

STEADY STATE BEHAVIOUR OF THE SYSTEM

Using Abel’s Lemma in Laplace Transforms, viz.,

\[ \lim_{s \to 0} \left[ s \bar{F}(s) \right] = \lim_{t \to \infty} F(t) = F_1 \text{ (say)} \]

provided the limit on the right hand side exists, the following time independent probabilities are:
\[ P_{0,0} = \frac{H(0)}{T'(0)} \] ..(1.3.60)

\[ P_{A1,0} = \frac{Q(0)}{T'(0)} \] ..(1.3.61)

\[ P_{A2,0} = \lambda_2 M_A P_{A1,0} \] ..(1.3.62)

\[ P_{0,1} = J_1(0, \lambda_1 + \mu_2 + \eta)[\mu_1 + \lambda_2 L(0)]P_{0,0} + \lambda_2 N(0)P_{A1,0} \] ..(1.3.63)

\[ P_{A1,1} = L(0)P_{0,0} + N(0)P_{A1,0} \] ..(1.3.64)

\[ P_{A2,1} = \lambda_2 M_A [L(0)P_{0,0} + N(0)P_{A1,0}] \] ..(1.3.65)

\[ P_{0,2} = \mu_2 M_2 \left[ (1 + \mu_1 J_1(0, \lambda_1 + \mu_2 + \eta) + \lambda_2 L(0)J_1(0, \lambda_1 + \mu_2 + \eta) \right] P_{0,0} \]
\[ + \{ \lambda_2 N(0)J_1(0, \lambda_1 + \mu_2 + \eta) \} P_{A1,0} \] ..(1.3.66)

\[ P_{A1,2} = \mu_2 M_2 [L(0)P_{0,0} + \{ 1 + N(0) \} P_{A1,0}] \] ..(1.3.67)

\[ P_{alm} = \frac{\eta}{\delta} \left[ (1 + L(0) + \mu_1 J_1(0, \lambda_1 + \mu_2 + \eta) + \lambda_2 L(0)J_1(0, \lambda_1 + \mu_2 + \eta) \right] \]
\[ P_{0,0} + \{ 1 + N(0) + \lambda_2 N(0)J_1(0, \lambda_1 + \mu_2 + \eta) \} P_{A1,0} \] ..(1.3.68)

Also

\[ P_{up} = \left[ (1 + L(0) + \mu_1 + \lambda_2 L(0)J_1(0, \lambda_1 + \mu_2 + \eta)) \right] \]
\[ \left[ H(0) / T'(0) \right] \]
\[ + \left[ (1 + N(0) + \lambda_2 N(0)J_1(0, \lambda_1 + \mu_2 + \eta)) \right] \]
\[ \left[ Q(0) / T'(0) \right] \] ..(1.3.69)

\[ P_{down} = \left[ \lambda_2 L(0)M_2 + \left\{ \mu_2 M_2 + \frac{\eta}{\delta} \right\} \right] \]
\[ \{ 1 + L(0) + \mu_1 J_1(0, \lambda_1 + \mu_2 + \eta) \} \]
\[ + \lambda_2 L(0)J_1(0, \lambda_1 + \mu_2 + \eta) \]
\[ \left[ H(0) / T'(0) \right] + \left[ \lambda_2 \{ M_A + N(0)M_2 \} \right] \]
\[ + \left\{ \mu_2 M_2 + \frac{\eta}{\delta} \right\} \left[ (1 + N(0) + \lambda_2 N(0)J_1(0, \lambda_1 + \mu_2 + \eta)) \right] \]
\[ \left[ Q(0) / T'(0) \right] \] ..(1.3.70)
where,

\[ T'(0) = \left[ \frac{d}{dt} T(s) \right]_{at \ s=0} \]

**PARTICULAR CASE**

When repair follows exponential time distribution

Setting \( \bar{S}_A(s) = \frac{\phi_A}{s + \phi_A} \), \( \bar{S}_1(s) = \frac{\phi_1}{s + \phi_1} \) and \( \bar{S}_2(s) = \frac{\phi_2}{s + \phi_2} \)

in equations (1.3.49) through (1.3.59), the Laplace transforms of various state probabilities are obtained as follows:

\[ \bar{P}_{0,0}(s) = \frac{H_i(s)}{T_i(s)} \quad \text{..(1.3.71)} \]

\[ \bar{P}_{A1,0}(s) = \frac{Q_i(s)}{T_i(s)} \quad \text{..(1.3.72)} \]

\[ \bar{P}_{A2,0}(s) = \lambda_2 (s + \phi_A)^{-1} \bar{P}_{A1,0}(s) \quad \text{..(1.3.73)} \]

\[ \bar{P}_{0,1}(s) = (s + \lambda_1 + \mu_2 + \eta + \phi_1)^{-1} \left[ \mu_1 + \lambda_2 \phi_A(s + \phi_A)^{-1} L_4(s) \right] \bar{P}_{0,0}(s) \]

\[ + \lambda_2 \phi_A(s + \phi_A)^{-1} N_1(s) \bar{P}_{A1,0}(s) \quad \text{..(1.3.74)} \]

\[ \bar{P}_{A1,1}(s) = L_1(s) \bar{P}_{0,0}(s) + N_1(s) \bar{P}_{A1,0}(s) \quad \text{..(1.3.75)} \]

\[ \bar{P}_{A2,1}(s) = \lambda_2 (s + \phi_2)^{-1} \left[ L_4(s) \bar{P}_{0,0}(s) + N_1(s) \bar{P}_{A1,0}(s) \right] \quad \text{..(1.3.76)} \]

\[ \bar{P}_{0,2}(s) = \mu_2 (s + \phi_2)^{-1} \left[ 1 + \mu_1 (s + \lambda_1 + \mu_2 + \eta + \phi_1)^{-1} \right]\]

\[ + \lambda_2 \phi_A L_4(s)(s + \phi_A)^{-1}(s + \lambda_1 + \mu_2 + \eta + \phi_1)^{-1} \bar{P}_{0,0}(s) \]

\[ + \lambda_2 \phi_A N_1(s)(s + \phi_A)^{-1}(s + \lambda_1 + \mu_2 + \eta + \phi_1)^{-1} \bar{P}_{A1,0}(s) \quad \text{..(1.3.77)} \]
\[
\bar{P}_{Al,2}(s) = \mu_2(s + \phi_2)^{-1}[L_2(s)\bar{P}_{0,0}(s) + [1 + N_1(s)]\bar{P}_{A1,0}(s)]\] ..(1.3.78)

\[
\bar{P}_{ana}(s) = \dfrac{\eta}{s + \delta} \left[ [1 + L_1(s) + \mu_1(s + \lambda_1 + \mu_2 + \eta + \phi_1)^{-1}
+ \lambda_2 \phi_A L_1(s)(s + \phi_A)^{-1}(s + \lambda_1 + \mu_2 + \eta + \phi_1)^{-1}\bar{P}_{0,0}(s)
+ [1 + N(s) + \lambda_2 \phi_A N_1(s)(s + \phi_A)^{-1}(s + \lambda_1 + \mu_2 + \eta + \phi_1)^{-1}\bar{P}_{A1,0}(s)] \right] \] ..(1.3.79)

Also

\[
\bar{P}_{up}(s) = \left[ 1 + L_1(s) + \dfrac{1}{(s + \lambda_1 + \mu_2 + \eta + \phi_1)} \left\{ \mu_1 + \dfrac{\lambda_2 \phi_A L_1(s)}{(s + \phi_A)} \right\} \left[ H_1(s)/T_1(s) \right] \right] 
+ \left[ 1 + N_1(s) + \dfrac{\lambda_2 \phi_A N_1(s)}{(s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1)} \right] \left[ Q_1(s)/T_1(s) \right] \] ..(1.3.80)

\[
\bar{P}_{down}(s) = \left[ \dfrac{\lambda_2 L_1(S)}{(s + \phi_2)} + \left\{ \dfrac{\mu_2}{(s + \phi_2)} + \dfrac{\eta}{(s + \delta)} \right\} \left[ 1 + L_1(S) + \dfrac{\mu_1}{(s + \lambda_1 + \mu_2 + \eta + \phi_1)} \right] \right] \bar{P}_{0,0}(s) + \left[ \dfrac{\lambda_2 \phi_A L_1(S)}{(s + \phi_A)} \left[ 1 + N_1(S) + \dfrac{\lambda_2 \phi_A N_1(S)}{(s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1)} \right] \right] \bar{P}_{A1,0}(s) \] ..(1.3.81)

where,

\[
H_1(s) = s + \lambda_2 + \mu_1 + \mu_2 + \eta - \mu_1 \phi_1 (s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \]^{-1}

\[
\{ (s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1)(s + \lambda_2 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \}^{-1}
\]

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\[ Q_1(s) = \lambda_1 \left[ 1 + \mu_1 \phi_A(s + \lambda_A) \left\{ (s + \lambda_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1)(s + \lambda_2 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \right\} \right]^{-1} \]

\[ A_1(s) = s + \lambda_1 + \mu_1 + \mu_2 + \eta - \mu_1 \phi_A(s + \lambda_1 + \mu_2 + \eta + \phi_1) \]

\[ \left\{ (s + \lambda_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1)(s + \lambda_2 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \right\}^{-1} \]

\[ - \left\{ \mu_2 \phi_2(s + \phi_2)^{-1} + \eta \delta(s + \delta)^{-1} \right\} \left[ 1 + \mu_1 \left\{ (s + \lambda_1 + \lambda_2 + \mu_2 + \eta + \phi_1) \right\} \right] \]

\[ \left\{ (s + \lambda_2 + \mu_2 + \eta + \phi_1)(s + \lambda_2 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \right\}^{-1} \]

\[ B_1(s) = \lambda_2 \phi_A(s + \phi_A)^{-1} + \lambda_2 \mu_1 \phi_A \left\{ (s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1) \right\} \]

\[ (s + \lambda_2 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \right\}^{-1} \]

\[ \frac{\mu_2 \phi_2}{(s + \lambda_2 + \mu_2 + \eta + \phi_1)} \left\{ (s + \lambda_1 + \mu_2 + \eta + \phi_1) \right\} \]

\[ \left\{ (s + \phi_A)(s + \lambda_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \right\}^{-1} \]

\[ T_1(s) = H_1(s).A_1(s) - Q_1(s).B_1(s) \]

\[ K_1(s) = \lambda_1 \lambda_2 \phi_A \left\{ (s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \right\}^{-1} \]

\[ L_1(s) = \lambda_1 \mu_1(s + \phi_A) \left\{ (s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \right\}^{-1} \]

\[ N_1(s) = \mu_1(s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1) \]

\[ \left\{ (s + \phi_A)(s + \lambda_1 + \mu_2 + \eta + \phi_1) - \lambda_1 \lambda_2 \phi_A \right\}^{-1} \]
EFFECT OF MINOR FAILURE ON THE LONG RUN AVAILABILITY
OF THE SYSTEM IN STEADY STATE

Setting

\[ \lambda_1 = 0.01, \lambda_2 = 0.02, \mu_2 = 0.1, \eta = 0.03, \phi_A = 0.40, \phi_1 = 0.5, \phi_2 = 1, \text{ and } \delta = 0.7 \]

in equations (1.1.60), (1.1.61), (1.1.63), (1.1.64) and (1.1.69) for exponential repair rates, one may obtain the following table:

Table 1.3.1

<table>
<thead>
<tr>
<th>( \mu_1 )</th>
<th>Pr. of normal availability ( P_{0,0} + P_{A1,0} )</th>
<th>Pr. of availability with reduced efficiency ( P_{0,1} + P_{A1,1} )</th>
<th>operational availability Pup</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.8589794</td>
<td>0.013635</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.02</td>
<td>0.8457643</td>
<td>0.02685</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.03</td>
<td>0.8329497</td>
<td>0.039664</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.04</td>
<td>0.8205176</td>
<td>0.052096</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.05</td>
<td>0.8084512</td>
<td>0.064163</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.06</td>
<td>0.7967345</td>
<td>0.075879</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.07</td>
<td>0.7853526</td>
<td>0.087261</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.08</td>
<td>0.7742913</td>
<td>0.098323</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.09</td>
<td>0.7635372</td>
<td>0.109077</td>
<td>0.872614</td>
</tr>
<tr>
<td>0.1</td>
<td>0.7530778</td>
<td>0.119536</td>
<td>0.872614</td>
</tr>
</tbody>
</table>
COMPUTATION OF THE TIME DEPENDENT AVAILABILITY OF THE COMPLEX SYSTEM

Setting $\lambda_1 = \lambda_2 = 0.5$, $\mu_1 = \mu_2 = 0.2$, $\eta = 0.25$, $\phi_1 = \phi_2 = 1$, and $\delta = \frac{1}{3}$ in equation (1.1.80) and then taking inverse L.T. one may get

$$P_{up}(t) = 0.4712 + 0.1346e^{-0.4688t} + 0.0018e^{-0.8090t} - 0.0017e^{-0.8508t}$$

$$+ 2e^{-1.3830t} \left[ 0.1987 \cos(0.5289t) + 0.1507 \sin(0.5289t) \right]$$

$$+ 2e^{-2.0330t} \left[ 0.0143 \cos(0.4550t) - 0.0145 \sin(0.4550t) \right]$$

$$+ 2e^{-2.2367t} \left[ -0.0160 \cos(0.4411t) + 0.0035 \sin(0.4411t) \right]$$

This further yields the following table:

**Table 1.3.2**

<table>
<thead>
<tr>
<th>S.NO.</th>
<th>TIME</th>
<th>$P_{up}(t)$</th>
<th>$P_{down}(t)$</th>
<th>$P_{up}(\infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.9999</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.6787</td>
<td>0.3213</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.5526</td>
<td>0.4474</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.5089</td>
<td>0.4911</td>
<td>0.4711</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.4921</td>
<td>0.5079</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>0.4840</td>
<td>0.5160</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.4792</td>
<td>0.5208</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.4763</td>
<td>0.5237</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.4744</td>
<td>0.5256</td>
<td></td>
</tr>
</tbody>
</table>
Fig 1.3.2: Normal Availability V/S Minor Failure
Fig 1.3.3: Availability with Reduced Efficiency V/S Minor Failure.
Fig 1.3.4: Operational Availability V/S Minor Failure
Fig 1.3.5: Operational Availability V/S Time
INTERPRETATION OF THE RESULTS

1. An inspection of Table 1.3.1 and Fig 1.3.2, 1.33, 1.3.4 shows that the effect of minor failure for such a complex system under given parameters is the same as in the case of Preemptive-Resume repair discipline.

2. A critical examination of Table 1.3.2 and the Graph “Operational Availability V/S Time” (Fig 1.3.5) indicate that the availability of the complex system too is the same as in the case of Preemptive-Resume repair discipline.

COMPARISON OF THE EFFECT OF THREE REPAIR DISCIPLINE ON THE OPERATIONAL AVAILABILITY OF THE COMPLEX SYSTEM

I. EFFECT ON THE OPERATIONAL AVAILABILITY IN STEADY STATE

A minute inspection of tables 1.1.1, 1.2.1, 1.3.1 and graph “Operational Availability V/S minor failure” (Fig 1.3.6) indicates that the availability of the complex system after a sufficient long interval of time decreases with the increase of minor failure rate in case of Head-of-Line repair discipline, whereas it remains throughout constant in both Preemptive-Resume and Preemptive-Repeat repair disciplines. Besides, the availability for such a complex system under given parameters is always much higher in the case of Preemptive-Resume and Preemptive-Repeat repair disciplines.
II. EFFECT ON THE TIME DEPENDENT OPERATIONAL AVAILABILITY

The effect of the three repair disciplines on the availability of the complex system at any time t has been studied in the graph “Operational Availability V/S Time” (Fig 1.3.7). A critical examination of Tables 1.1.2, 1.2.2, 1.3.2 and Fig 1.3.7 reveals that, the availability in both Preemptive-Resume and Preemptive-Repeat repair disciplines is always greater than that in Head-of-Line repair discipline. Moreover, the availability in all three repair disciplines decreases with respect to time and attains the stationary values 0.4660, 0.4711, and 0.4711 in the steady state. This shows that the availability of such complex system lies in the range as indicated below:

<table>
<thead>
<tr>
<th>OPERATIONAL AVAILABILITY</th>
<th>REPAIR DISCIPLINES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.4660 \leq P_{wp}(t) \leq 1$</td>
<td>Head - of - Line</td>
</tr>
<tr>
<td>$0.4711 \leq P_{wp}(t) \leq 1$</td>
<td>Preemptive - Resume</td>
</tr>
<tr>
<td>$0.4711 \leq P_{wp}(t) \leq 1$</td>
<td>Preemptive - Repeat</td>
</tr>
</tbody>
</table>

CONCLUSION

Hence in practical situations for such type of models, either preemptive-Resume or Preemptive-Repeat repair disciplines should be adopted for their purposes to obtain optimum returns.
Fig 1.3.6: Operational Availability V/S Partial Failure Rate in Three Repair Discipline
Fig 1.3.7: Operational Availability V/S time in Three Repair Disciplines