INTRODUCTION

The basic concept of reliability is that of failure free operation, which means the ability of an equipment to function properly without failure throughout a specified period of time under specified conditions. In the modern age of science, every equipment/system depends more or less on reliability. In defences, industries and space research programs, a high degree of reliability is most essential. Both government and commercial sectors are emphasizing the designers, engineers and manufacturers for the reliable performance of their systems or equipments.

In various reliability models of practical utility, we often come across with the situation of maximizing the profit. The profit earned out of an operable system, depends upon the cost incurred by the repairman needed to repair the failure stage of the system. Therefore, in the study of repairable complex systems, the main interest lies in predicting and estimating the cost involved to run a system. The idea of estimating the cost involved to run a system has deep sense of systems behaviour rather than what is generally obtained as availability/reliability of the system.

In recent past various researchers, viz., Gupta, P.P. (1973); Gopalan, M.N. (1981); Goel, L.R and Gupta, R. (1985); Pandey, D and
Jacob, M. (1995); Wang, K.H., Kuo, C.C. (2000); Parashar, B., Taneja, G. (2007); Sharma, S., Pandey, S.B. and Singh, S.B. (2009); Goyal, Taneja and Singh (2010); have considered cost analysis of several 2-state and 3-state repairable systems but surprisingly very little work seems to have been done in the direction of forecasting, evaluating and optimizing the cost involved to run repairable systems under waiting time distribution and environmental failure.

The study of this chapter is broadly classified into the following two sections:

**SECTION-1**

Cost analysis of Solar Thermal Electric Power Plant

**SECTION-2**

Reliability Characteristic of Cold-Standby Redundant System
SECTION-1

COST ANALYSIS OF SOLAR THERMAL ELECTRIC POWER PLANT

INTRODUCTION

An unreliable system has higher probability of failure. When any item of the associated equipment is damaged or destroyed due to some reason or the other, the unreliability of system may cause not only the wastage of time and cost, but the national security may also be endangered. Thus, a high degree of reliability is very essential. One of the purposes of system reliability analysis is to identify the weakness of a system and to quantify the impact of component failures.

Reliability analysis techniques have been gradually accepted as standard tools for the planning and operation of automatic and complex thermal power plants. In the present paper, the author has considered a solar thermal power plant, which is the power conversion system that is used to convert the heat into electricity. Solar thermal electric power plants generally use concentrated sun light to produce high-temperature heat. The heat energy is then transferred to a high temperature tank, which is used in a typical power plant cycle to convert the heat energy to
mechanical energy and then electricity. The two major parts of a solar thermal electric power plant are:

(i) The component that collects the solar energy and convert it to heat, and

(ii) The component that then converts the heat energy into electricity.

The Solar Thermal Electric Power Plant under consideration consists of three subsystems A, B and C, arranged in series. Subsystem A is the collector-storage loop having minor and major failure. Minor failure reduces the efficiency of the system causing degraded state while major failure results into a non-operable state of the system. It is used to collect the heat and store in a heat exchanger after heating to a high temperature. Then hot gases circulate in subsystem B, a Boiler, with an identical unit in standby redundancy with a perfect switching. Subsystem C contains two units in series and these units are turbine and generator. Failure of any unit causes complete failure of the system. Subsystem C generates electric power on rotating of turbine. This system can also fail due to environmental failure. The Laplace transform of the time dependent probabilities of the system being in various states have been obtained by employing the supplementary variable technique. Various reliability parameters have been computed and some tabular and graphical illustrations are also given in the end.

The state transition diagram of the system is shown in Fig 3.1.1
ASSUMPTIONS

The following assumptions have been associated with this model:

1. Initially the system works with full efficiency.
2. The system has 3-states: good, degraded and failed.
3. The system has only one repair facility which is always available to repair every type of failure.
4. After repair the system is as good as new.
5. The failure and repair times follow exponential and general time distributions respectively.
6. The whole system can also fail due to environmental failure.

NOTATIONS

\( \lambda_{mi}, \lambda_{ma} \) : Minor and Major failure rates of Subsystem A

\( \mu_{mi}(z), \mu_{ma}(z) \) : Repair rates of minor and major failures of subsystem A

\( \lambda_i, \mu_i(x) \) : Failure and repair rates of the principal and standby units of Subsystem B, where \( i=\text{B1,B2} \)

\( \lambda_i, \mu_i(y) \) : Failure and repair rates of generator and turbine of subsystem C, where \( i=\text{G,T} \)

\( x, y, z \) : Elapsed repair times
\( \lambda, \delta \) : Constant failure and repair rate of the system due to environmental failure.

\( P_0(t) \) : Probability that the system is in good state at time \( t \).

\( P_i(j,t) \Delta \) : The probability that at time ‘\( t \)’, the system is in degraded state due to failure of \( i^{th} \) unit and under repair, elapsed repair time \( j; t \), where \( i = B1, m_i, j=x, z \) respectively.

\( P_i(j,t) \Delta \) : The probability that at time ‘\( t \)’, the system is in failed state due to failure of \( i^{th} \) unit under repair, elapsed repair time \( j; t \), where \( i = B2, G, T, C, ma, j=x, y, y, z \) respectively.

\( M_i \) : \(-s'_i(0)\)
**FIG 3.1.1: STATE TRANSITION DIAGRAM**

- Circle: Good state
- Diamond: Degraded state
- Square: Failed state
FORMULATION OF THE MATHEMATICAL MODEL

By elementary probability and continuity arguments, the difference-differential equations governing the stochastic behaviour of the complex system are:

\[
\begin{align*}
\left[ \frac{d}{dt} + \lambda_{B1} + \lambda_T + \lambda_G + \lambda_{ma} + \lambda_{mi} + \lambda_E \right] P_0(t) &= \int_0^\infty P_{B1}(x,t) \mu_{B1}(x) dx \\
&+ \int_0^\infty P_{B2}(x,t) \mu_{B2}(x) dx + \int_0^\infty P_T(y,t) \mu_T(y) dy \\
&+ \int_0^\infty P_G(y,t) \mu_G(y) dy + \int_0^\infty P_{ma}(z,t) \mu_{ma}(z) dz \\
&+ \int_0^\infty P_{mi}(z,t) \mu_{mi}(z) dz + \delta P_E(t) \\
&= (3.1.1)
\end{align*}
\]

\[
\begin{align*}
\left[ \frac{\partial}{\partial t} + \mu_{ma}(z) + \lambda_G + \lambda_{B1} + \lambda_{B2} + \lambda_T + \lambda_{ma} + \lambda_E \right] P_{mi}(z,t) &= 0 \\
&= (3.1.2)
\end{align*}
\]

\[
\begin{align*}
\left[ \frac{\partial}{\partial t} + \mu_{ma}(z) \right] P_{ma}(z,t) &= 0 \\
&= (3.1.3)
\end{align*}
\]

\[
\begin{align*}
\left[ \frac{\partial}{\partial t} + \mu_G(y) \right] P_G(y,t) &= 0 \\
&= (3.1.4)
\end{align*}
\]

\[
\begin{align*}
\left[ \frac{\partial}{\partial t} + \mu_T(y) \right] P_T(y,t) &= 0 \\
&= (3.1.5)
\end{align*}
\]

\[
\begin{align*}
\left[ \frac{\partial}{\partial t} + \mu_{B1}(x) + \lambda_{B2} + \lambda_G + \lambda_{ma} + \lambda_T + \lambda_E \right] P_{B1}(x,t) &= 0 \\
&= (3.1.6)
\end{align*}
\]

\[
\begin{align*}
\left[ \frac{\partial}{\partial t} + \mu_{B2}(x) \right] P_{B2}(x,t) &= 0 \\
&= (3.1.7)
\end{align*}
\]

\[
\begin{align*}
\frac{d}{dt} P_E(t) &= \lambda_E \left[ P_0(t) + P_{ma}(t) + P_{B1}(t) \right] \\
&= (3.1.8)
\end{align*}
\]
BOUNDARY CONDITIONS

\[ P_{mi}(0,t) = \lambda_{mi} [P_0(t) + P_{B1}(t)] \]  \hspace{1cm} \text{(3.1.9)}

\[ P_{ma}(0,t) = \lambda_{ma} [P_0(t) + P_{mi}(t) + P_{B1}(t)] \]  \hspace{1cm} \text{(3.1.10)}

\[ P_G(0,t) = \lambda_G [P_0(t) + P_{mi}(t) + P_{B1}(t)] \]  \hspace{1cm} \text{(3.1.11)}

\[ P_T(0,t) = \lambda_T [P_0(t) + P_{mi}(t) + P_{B1}(t)] \]  \hspace{1cm} \text{(3.1.12)}

\[ P_{B1}(0,t) = \lambda_{B1} [P_0(t) + P_{mi}(t)] \]  \hspace{1cm} \text{(3.1.13)}

\[ P_{B2}(0,t) = \lambda_{B2} [P_{mi}(t) + P_{B1}(t)] \]  \hspace{1cm} \text{(3.1.14)}

INITIAL CONDITIONS

\[ p_i(0) = \begin{cases} 1, & \text{when } i = 0 \\ 0, & \text{otherwise} \end{cases} \]  \hspace{1cm} \text{(3.1.15)}

SOLUTION OF THE MODEL

Taking Laplace transforms of equation (3.1.1) through (3.1.14) and using equation (3.1.15), one may obtain

\[
\left[ s + \lambda_{B1} + \lambda_{T} + \lambda_{G} + \lambda_{ma} + \lambda_{mi} + \lambda_{E} \right] \overline{\bar{P}}_{0}(s) = \int_{0}^{\infty} \overline{\bar{P}}_{B1}(x,s) \mu_{B1}(x) \, dx \\
+ \int_{0}^{\infty} \overline{\bar{P}}_{B2}(x,s) \mu_{B2}(x) \, dx + \int_{0}^{\infty} \overline{\bar{P}}_{T}(y,s) \mu_{T}(y) \, dy \\
+ \int_{0}^{\infty} \overline{\bar{P}}_{G}(y,s) \mu_{G}(y) \, dy + \int_{0}^{\infty} \overline{\bar{P}}_{ma}(z,s) \mu_{ma}(z) \, dz \\
+ \int_{0}^{\infty} \overline{\bar{P}}_{mi}(z,s) \mu_{mi}(z) \, dz + \delta \overline{\bar{P}}_{E}(s)
\]

\[
\left[ s + \frac{\partial}{\partial z} + \mu_{mi}(z) + \lambda_{G} + \lambda_{B1} + \lambda_{B2} + \lambda_{T} + \lambda_{ma} + \lambda_{E} \right] \overline{\bar{P}}_{mi}(z,s) = 0
\]  \hspace{1cm} \text{(3.1.16)}

139
\[
\left[ s + \frac{\partial}{\partial z} + \mu_{ma}(z) \right] P_{ma}(z, s) = 0 \quad \text{(3.1.18)}
\]
\[
\left[ s + \frac{\partial}{\partial y} + \mu_G(y) \right] P_G(y, s) = 0 \quad \text{(3.1.19)}
\]
\[
\left[ s + \frac{\partial}{\partial y} + \mu_T(y) \right] P_T(y, s) = 0 \quad \text{(3.1.20)}
\]
\[
\left[ s + \frac{\partial}{\partial x} + \mu_{B1}(x) + \lambda_{B2} + \lambda_G + \lambda_{mi} + \lambda_{ma} + \lambda_T + \lambda_E \right] P_{B1}(x, s) = 0 \quad \text{(3.1.21)}
\]
\[
\left[ s + \frac{\partial}{\partial x} + \mu_{B2}(x) \right] P_{B2}(x, s) = 0 \quad \text{(3.1.22)}
\]
\[
\left[ s + \delta \right] P_E(s) = \lambda_E \left[ P_0(s) + P_{mi}(s) + P_{B1}(s) \right] \quad \text{(3.1.23)}
\]
\[
P_{mi}(0, s) = \lambda_{mi} \left[ P_0(s) + P_{B1}(s) \right] \quad \text{(3.1.24)}
\]
\[
P_{mi}(0, s) = \lambda_{mi} \left[ P_0(s) + P_{mi}(s) + P_{B1}(s) \right] \quad \text{(3.1.25)}
\]
\[
P_G(0, s) = \lambda_G \left[ P_0(s) + P_{mi}(s) + P_{B1}(s) \right] \quad \text{(3.1.26)}
\]
\[
P_T(0, s) = \lambda_T \left[ P_0(s) + P_{mi}(s) + P_{B1}(s) \right] \quad \text{(3.1.27)}
\]
\[
P_{B1}(0, s) = \lambda_{B1} \left[ P_0(s) + P_{mi}(s) \right] \quad \text{(3.1.28)}
\]
\[
P_{B2}(0, s) = \lambda_{B2} \left[ P_{mi}(s) + P_{B1}(s) \right] \quad \text{(3.1.29)}
\]

Integrating equation (3.1.17) through (3.1.22), one gets,

\[
\bar{P}_{ma}(z, s) = \bar{P}_{mi}(0, s) \exp \left[ - \left( s + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_{B1} + \lambda_{B2} + \lambda_E \right) z - \int_0^z \mu_{ma}(z) dz \right] \quad \text{(3.1.30)}
\]
\[
\bar{P}_{ma}(z, s) = \bar{P}_{ma}(0, s) \exp \left[ - sz - \int_0^z \mu_{ma}(z) dz \right] \quad \text{(3.1.31)}
\]
\begin{align*}
\overline{P}_G(y,s) &= \overline{P}_G(0,s) \exp \left[ -sy - \int_0^y \mu_G(y)dy \right] \tag{3.1.32} \\
\overline{P}_T(y,s) &= \overline{P}_T(0,s) \exp \left[ -sy - \int_0^y \mu_T(y)dy \right] \tag{3.1.33} \\
\overline{P}_{B_1}(x,s) &= \overline{P}_{B_1}(0,s) \exp \left[ -(s + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_{B_2} + \lambda_E)x - \int_0^x \mu_{B_1}(x)dx \right] \tag{3.1.34} \\
\overline{P}_{B_2}(x,s) &= \overline{P}_{B_2}(0,s) \exp \left[ -sx - \int_0^x \mu_{B_2}(x)dx \right] \tag{3.1.35} \\
\end{align*}

Integrating equations (3.1.30) through (3.1.35) from 0 to \( \infty \), one obtain

\begin{align*}
\overline{P}_{ma}(s) &= \overline{P}_{ma}(0,s) J_{ma}(s, \lambda_{ma} + \lambda_G + \lambda_T + \lambda_{B_1} + \lambda_{B_2} + \lambda_E) \tag{3.1.36} \\
\overline{P}_{ma}(s) &= \overline{P}_{ma}(0,s) J_{ma}(s,0) \tag{3.1.37} \\
\overline{P}_G(s) &= \overline{P}_G(0,s) J_{G}(s,0) \tag{3.1.38} \\
\overline{P}_T(s) &= \overline{P}_T(0,s) J_{T}(s,0) \tag{3.1.39} \\
\overline{P}_{B_1}(s) &= \overline{P}_{B_1}(0,s) J_{B_1}(s, \lambda_{ma} + \lambda_G + \lambda_T + \lambda_{B_2} + \lambda_E) \tag{3.1.40} \\
\overline{P}_{B_2}(s) &= \overline{P}_{B_2}(0,s) J_{B_2}(s,0) \tag{3.1.41} \\
\end{align*}

where,

\[ J_i(s, \alpha) = \left[ 1 - \overline{S}_i(s + \alpha) \right] (s + \alpha)^{-1} \]

Utilization of equations (3.1.28), (3.1.36) and (3.1.24), equation (3.1.40) yields

\[ \overline{P}_{B_1}(s) = K(s) \overline{P}_0(s) \tag{3.1.42} \]
where,

\[
K(s) = \lambda_{b_1}J_{b_1}(s, \lambda_{m_a} + \lambda_{m_b} + \lambda_G + \lambda_T + \lambda_{b_2} + \lambda_E) \left[1 + \lambda_{m_i}J_{m_i}(s, \lambda_{m_a} + \lambda_G + \lambda_T + \lambda_{b_1} + \lambda_{b_2} + \lambda_E)\right]
\]

\[
\left[1 - \lambda_{b_1}\lambda_{m_i}J_{m_i}(s, \lambda_{m_a} + \lambda_G + \lambda_T + \lambda_{b_1} + \lambda_{b_2} + \lambda_E)J_{b_1}(s, \lambda_{m_a} + \lambda_{m_i} + \lambda_G + \lambda_T + \lambda_{b_2} + \lambda_E)\right]^{-1}
\]

Using equation (3.1.24) and (3.1.42) in equation (3.1.36), one may get

\[
\overline{P}_m(s) = \lambda_{m_i}J_{m_i}(s, \lambda_{b_1} + \lambda_{b_2} + \lambda_{m_a} + \lambda_G + \lambda_T + \lambda_E) \left[1 + K(s)\right]P_0(s)
\]  \hspace{1cm} (3.1.43)

Using equations (3.1.25), (3.1.26), (3.1.27), (3.1.42) and (3.1.43), equations (3.1.37), (3.1.38) and (3.1.39), becomes

\[
\overline{P}_m(s) = \lambda_{m_a}J_{m_a}(s, 0) \left[1 + \lambda_{m_i}J_{m_i}(s, \lambda_{b_1} + \lambda_{b_2} + \lambda_{m_a} + \lambda_G + \lambda_T + \lambda_E)\right] \left[1 + K(s)\right]P_0(s)
\]  \hspace{1cm} (3.1.44)

\[
\overline{P}_G(s) = \lambda_GJ_{G}(s, 0) \left[1 + \lambda_{m_i}J_{m_i}(s, \lambda_{b_1} + \lambda_{b_2} + \lambda_{m_a} + \lambda_G + \lambda_T + \lambda_E)\right] \left[1 + K(s)\right]P_0(s)
\]  \hspace{1cm} (3.1.45)

\[
\overline{P}_T(s) = \lambda_TJ_{T}(s, 0) \left[1 + \lambda_{m_i}J_{m_i}(s, \lambda_{b_1} + \lambda_{b_2} + \lambda_{m_a} + \lambda_G + \lambda_T + \lambda_E)\right] \left[1 + K(s)\right]P_0(s)
\]  \hspace{1cm} (3.1.46)

Utilizing equations (3.1.29), (3.1.42) and (3.1.43), equation (3.1.41) gives

\[
\overline{P}_{b_2}(s) = \lambda_{b_2}J_{b_2}(s, 0) \left[K(s) + \lambda_{m_i}\left[1 + K(s)\right]J_{m_i}(s, \lambda_{b_1} + \lambda_{b_2} + \lambda_{m_a} + \lambda_G + \lambda_T + \lambda_E)\right]P_0(s)
\]  \hspace{1cm} (3.1.47)

Utilization of equations (3.1.42) and (3.1.43), equation (3.1.23) yields

\[
\overline{P}_E(s) = \lambda_E(s + \delta)^{-1} \left[1 + K(s)\right] \left[1 + \lambda_{m_i}J_{m_i}(s, \lambda_{b_1} + \lambda_{b_2} + \lambda_{m_a} + \lambda_G + \lambda_T + \lambda_E)\right]P_0(s)
\]  \hspace{1cm} (3.1.48)

Similarly, utilization of equations (3.1.30) through (3.1.35) and other relevant relationship in (3.1.16) and then simplifying, one obtain

\[
\overline{P}_0(s) = \frac{1}{A(s)}
\]  \hspace{1cm} (3.1.49)

where,
\[
A(s) = s + \lambda_G + \lambda_B + \lambda_T + \lambda_{ma} + \lambda_{mi} + \lambda_E \\
- \{1 + K(s)\} \{1 + \lambda_{mi} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \\
\{ \lambda_{ma} \bar{S}_{ma}(s) + \lambda_G \bar{S}_{G}(s) + \lambda_T \bar{S}_{T}(s) + \delta \lambda_E (s + \delta)^{-1} \} \\
- \lambda_{mi} \{1 + K(s)\} \{1 + K(s)J_{ma}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \\
- \lambda_{mi} \{1 + K(s)\} \{1 + \lambda_{mi} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \\
- \lambda_{mi} \{1 + K(s)\} \{1 + \lambda_{mi} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \\
- \lambda_{mi} \{1 + K(s)\} \{1 + \lambda_{mi} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \tag{3.1.50}
\]

Now, one may get
\[
\bar{P}_0(s) = \frac{1}{A(s)} \tag{3.1.51}
\]
\[
\bar{P}_{mi}(s) = \lambda_{mi} \{1 + K(s)\} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E) \bar{P}_0(s) \tag{3.1.52}
\]
\[
\bar{P}_{ma}(s) = \lambda_{ma} \{1 + K(s)\} J_{ma}(s, 0) \{1 + \lambda_{mi} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \bar{P}_0(s) \tag{3.1.53}
\]
\[
\bar{P}_G(s) = \lambda_G \{1 + K(s)\} J_{G}(s, 0) \{1 + \lambda_{mi} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \bar{P}_0(s) \tag{3.1.54}
\]
\[
\bar{P}_T(s) = \lambda_T \{1 + K(s)\} J_{T}(s, 0) \{1 + \lambda_{mi} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \bar{P}_0(s) \tag{3.1.55}
\]
\[
\bar{P}_{B1}(s) = K(s) \bar{P}_0(s) \tag{3.1.56}
\]
\[
\bar{P}_{B2}(s) = \lambda_{B2} \bar{J}_{B2}(s, 0) \{K(s) + \lambda_{mi} \{1 + K(s)\} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \tag{3.1.57}
\]
\[
\bar{P}_E(s) = \lambda_E (s + \delta)^{-1} \{1 + K(s)\} \{1 + \lambda_{mi} J_{mi}(s, \lambda_B + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\} \bar{P}_0(s) \tag{3.1.58}
\]

**UP AND DOWN STATE PROBABILITIES**
Laplace transforms of operational availability and non-availability of the system are:

\[
\tilde{P}_{up}(s) = \tilde{P}_0(s) + \tilde{P}_m(s) + \tilde{P}_{B1}(s) = \{1 + K(s)\}[1 + \lambda_{mi} J_{mi}(s, \lambda_{B1} + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)] \tilde{P}_0(s) \quad \text{..(3.1.59)}
\]

\[
\tilde{P}_{down}(s) = \tilde{P}_{ma}(s) + \tilde{P}_G(s) + \tilde{P}_T(s) + \tilde{P}_{B2}(s) + \tilde{P}_E(s) = \left\{\left[1 + K(s)\right]\left[1 + \lambda_{mi} J_{mi}(s, \lambda_{B1} + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)\right]\right\} \\
\left\{\lambda_{ma} J_{ma}(s,0) + \lambda_G J_{G}(s,0) + \lambda_T J_{T}(s,0) + \lambda_E(s + \delta)^{-1}\right\} + \lambda_{B2} J_{B2}(s,0) \{K(s) + \lambda_{ma} (1 + K(s))\} \\
J_{ma}(s, \lambda_{B1} + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E) \} \tilde{P}_0(s) \quad \text{..(3.1.60)}
\]

It is interesting to note that \( \tilde{P}_{up}(s) + \tilde{P}_{down}(s) = \frac{1}{s} \)

**STEADY STATE BEHAVIOUR OF THE SYSTEM**

Using Abel’s Lemma in Laplace Transforms, viz.,

\[
\lim_{s \to 0} \left[s \tilde{F}(s)\right] = \lim_{t \to \infty} F(t) = F_1 \text{ (say)}
\]

provided the limit on the right hand side exists, the following time independent up and down state probabilities are:

\[
P_{up} = P_0 + P_{mi} + P_{B1} = \{1 + K(0)\}[1 + \lambda_{mi} J_{mi}(0, \lambda_{B1} + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E)]P_0 \quad \text{..(3.1.61)}
\]
\[ P_{\text{down}} = P_{ma} + P_G + P_T + P_{B2} + P_E \]
\[ = \left[ (1 + K(0)) \right] \left[ 1 + \lambda_{ma} J_m (0, \lambda_{B1} + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E) \right] \]
\[ \{ \lambda_{ma} M_{ma} + \lambda_G M_G + \lambda_T M_T + \lambda_E \delta^{-1} \} \]
\[ + \lambda_{B2} M_{B2} \{ K(0) + \lambda_{ma} (1 + K(0)) J_m (0, \lambda_{B1} + \lambda_{B2} + \lambda_{ma} + \lambda_G + \lambda_T + \lambda_E) \} ] P_0 \]
\[ \text{(3.1.62)} \]

where
\[ P_0 = \frac{1}{A'(0)} \]
\[ K(0) = \left[ K(s) \right]_{s=0} \]
\[ A'(0) = \left[ \frac{d}{dt} A(s) \right]_{s=0} \]

**PARTICULAR CASE**

When repair follows exponential time distribution

Setting \[ \bar{S}_i(s) = \frac{\phi_i}{s + \phi_i} \] in equation (3.1.51) through (3.1.60), the Laplace transforms of various state probabilities are obtained as follows:

\[ \bar{P}_0(s) = \frac{1}{A(s)} \]
\[ \text{..(3.1.63)} \]
\[ \bar{P}_{mi}(s) = \frac{\lambda_{mi} K_2 (s)}{A(s)} \]
\[ \text{..(3.1.64)} \]
\[ \bar{P}_{ma}(s) = \frac{\lambda_{ma} K_3 (s)}{(s + \phi_{ma}) A(s)} \]
\[ \text{..(3.1.65)} \]
\[ \overline{P}_G(s) = \frac{\lambda_G K_1(s)}{(s + \phi_G)A(s)} \]  ..(3.1.66)

\[ \overline{P}_T(s) = \frac{\lambda_T K_1(s)}{(s + \phi_T)A(s)} \]  ..(3.1.67)

\[ \overline{P}_{B1}(s) = \frac{K_1(s)}{A(s)} \]  ..(3.1.68)

\[ \overline{P}_{B2}(s) = \frac{\lambda_{B2} \left[ K_1(s) + \lambda_{mi} K_2(s) \right]}{(s + \phi_{B2})A(s)} \]  ..(3.1.69)

\[ \overline{P}_E(s) = \frac{\lambda_E K_3(s)}{(s + \delta)A(s)} \]  ..(3.1.70)

\[ \overline{P}_{up}(s) = \frac{K_3(s)}{A(s)} \]  ..(3.1.71)

Also,

\[ \overline{P}_{up}(s) = \frac{K_3(s)}{A(s)} \]  ..(3.1.72)

\[ \overline{P}_{down}(s) = \frac{1}{A(s)} \left[ K_3(s) \left\{ \frac{\lambda_G}{s + \phi_G} + \frac{\lambda_{ma}}{s + \phi_{ma}} + \frac{\lambda_T}{s + \phi_T} + \frac{\lambda_E}{s + \delta} \right\} \right. \]  ..(3.1.73)

\[ + \frac{\lambda_{B2}}{s + \phi_{B2}} \{ K_1(s) + \lambda_{mi} K_2(s) \} \]  ..(3.1.74)

where,

\[ K_1(s) = \lambda_{B1} \left[ \frac{(s + \lambda_G + \lambda_{B1} + \lambda_{B2} + \lambda_T + \lambda_{ma} + \lambda_{mi} + \lambda_E + \phi_{mi})}{(s + \lambda_G + \lambda_{B1} + \lambda_{B2} + \lambda_T + \lambda_{ma} + \lambda_{mi} + \lambda_E + \phi_{B1})} - \lambda_{B1} \lambda_{mi} \right] \]
\[ K_2(s) = \frac{1 + K_1(s)}{s + \lambda_{B_1} + \lambda_{B_2} + \lambda_G + \lambda_T + \lambda_{ma} + \lambda_E + \phi_{mi}} \]

\[ K_3(s) = K_2(s)\left[ s + \lambda_{B_1} + \lambda_{B_2} + \lambda_G + \lambda_T + \lambda_{ma} + \lambda_{mi} + \lambda_E + \phi_{mi} \right] \]

\[ A(s) = s + \lambda_G + \lambda_{B_1} + \lambda_T + \lambda_{ma} + \lambda_{mi} + \lambda_E \]

\[-K_3(s) \left[ \frac{\lambda_G \phi_G}{s + \phi_G} + \frac{\lambda_{ma} \phi_{ma}}{s + \phi_{ma}} + \frac{\lambda_T \phi_T}{s + \phi_T} + \frac{\lambda_E \phi_E}{s + \phi_E} \right] - K_2(s)\lambda_{mi} \phi_{mi} \]

\[-\frac{\lambda_{B_1} \phi_{B_1} \left[ 1 + \lambda_{mi} K_2(s) \right]}{(s + \lambda_{B_2} + \lambda_G + \lambda_T + \lambda_{ma} + \lambda_{mi} + \lambda_E + \phi_{B_1})} - \frac{\lambda_{B_2} \phi_{B_2} \left[ K_1(s) + \lambda_{mi} K_2(s) \right]}{s + \phi_{B_2}} \]

**RELIABILITY**

Taking all repair rates equal to zero and by inversion process, one may obtain the reliability of the system is given as follows:

\[ R(t) = \frac{\lambda_{B_2}}{\lambda_{B_2} - \lambda_{B_1} - \lambda_{mi}} e^{-\alpha_1 t} + \frac{\lambda_{B_1} + \lambda_{mi}}{\lambda_{B_1} - \lambda_{B_2} + \lambda_{mi}} e^{-\alpha_2 t} \quad ..(3.1.74) \]

where,

\[ \alpha_1 = \lambda_G + \lambda_{B_1} + \lambda_T + \lambda_{ma} + \lambda_E \quad ..(3.1.75) \]

\[ \alpha_2 = \lambda_G + \lambda_{B_2} + \lambda_T + \lambda_{ma} + \lambda_E \quad ..(3.1.76) \]

**M.T.T.F**

The mean time to failure of the system is given by

\[ \text{M.T.T.F} = \int_0^\infty R(t) dt = \frac{\lambda_{B_2}}{(\lambda_{B_2} - \lambda_{B_1} - \lambda_{mi})\alpha_1} + \frac{\lambda_{B_1} + \lambda_{mi}}{(\lambda_{B_1} - \lambda_{B_2} + \lambda_{mi})\alpha_2} \quad ..(3.1.77) \]

where \( \alpha_1 \) and \( \alpha_2 \) have been mentioned earlier.
NUMERICAL COMPUTATIONS

1. Availability and Cost Analysis

Setting
\[ \lambda_{mi} = .001, \lambda_{ma} = .002, \lambda_{f} = .002, \lambda_{G} = .001, \lambda_{B1} = .01, \lambda_{B2} = .02, \lambda_{E} = .03, \phi = 0.8, \phi_i = 1 \]

in equation (3.1.72) and taking the Inverse Laplace transform, the availability of the system is obtained as follows:

\[
P_{up}(t) = 0.9590 + 0.0336e^{-0.8290t} + 0.0239e^{-1.0130t} - 0.0284e^{-1.0340t} + 0.0085e^{-1.0833t}
+ 2e^{(-1.0644t)}\left[0.0017 \cos(0.0216t) + 0.0112 \sin(0.0216t)\right] \tag{3.1.78}
\]

The value of \( P_{up}(t) \) for different values of \( t \) is shown in Table 3.1.1 and the corresponding graph is shown in the Figure 3.1.2. For repairable system, the cost function \( H(t) \) in the interval \((0, t)\) is given by

\[
H(t) = C_1 \int_0^t P_{up}(t)dt - C_2 t - C_3
= C_1 \left[0.9590t - 0.0405(e^{-0.8290t} - 1) - 0.0236(e^{-1.0130t} - 1) + 0.0275(e^{-1.0340t} - 1)\right.
\]
\[
-0.0789(e^{-1.0833t} - 1) + \left(-0.0018 + 0.0105i\right)e^{(-1.0644 + 0.0216i)t} - 1\right]
\]
\[
\left.+(-0.0018 - 0.0105i)e^{(-1.0644 - 0.0216i)t} - 1\right] - C_2 t - C_3 \tag{3.1.79}
\]

where \( C_1, C_2 \) and \( C_3 \) are revenue per unit time, service cost per unit time, and system establishment cost respectively.

The value of \( H(t) \) for different values of \( C_2 \) is shown in Table 3.1.2 and the corresponding graph is shown in the Figure 3.1.3.
2. Reliability Analysis

Setting \( \lambda_{mi} = .001, \lambda_{ma} = .002, \lambda_T = .002, \lambda_G = .001, \lambda_{B1} = .01, \lambda_{B2} = .02, \lambda_E = .03 \) in equation (3.1.74) and taking the Inverse Laplace transform, the reliability of the system is obtained as follows:

\[
R(t) = 2.2222e^{-0.044t} - 1.2222e^{-0.053t} \quad (3.1.80)
\]

The value of \( R(t) \) for different values of \( t \) is shown in Table 3.1.3 and the corresponding graph is shown in the Figure 3.1.4.

3. M.T.T.F Analysis

Setting \( \lambda_T = .002, \lambda_G = .001, \lambda_{mi} = .001, \lambda_{B1} = .01, \lambda_{B2} = .02, \lambda_E = .03 \) in equation (3.1.77), one may obtain

\[
MTTF = \frac{2.2222}{0.044 + \lambda_{ma}} - \frac{1.2222}{0.053 + \lambda_{ma}} \quad (3.1.81)
\]

The value of MTTF for different values of \( \lambda_{ma} \) is shown in Table 3.1.4 and the corresponding graph is shown in the Figure 3.1.5.
### Table 3.1.1

<table>
<thead>
<tr>
<th>Time</th>
<th>$P_{up}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
</tr>
<tr>
<td>1</td>
<td>0.9765</td>
</tr>
<tr>
<td>2</td>
<td>0.9665</td>
</tr>
<tr>
<td>3</td>
<td>0.9622</td>
</tr>
<tr>
<td>4</td>
<td>0.9604</td>
</tr>
<tr>
<td>5</td>
<td>0.9596</td>
</tr>
<tr>
<td>6</td>
<td>0.9593</td>
</tr>
<tr>
<td>7</td>
<td>0.9591</td>
</tr>
<tr>
<td>8</td>
<td>0.9590</td>
</tr>
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</table>

**Fig 3.1.2: Availability V/S Time**
Table 3.1.2

<table>
<thead>
<tr>
<th>Time</th>
<th>Expected Profit $H(t)$, $C_1=50$</th>
<th>$C_2=10.0$</th>
<th>$C_2=12.5$</th>
<th>$C_2=15.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>36.6764</td>
<td>34.1764</td>
<td>31.6764</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>76.0087</td>
<td>71.0087</td>
<td>66.0087</td>
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<td>3</td>
<td></td>
<td>114.4788</td>
<td>106.9788</td>
<td>99.4788</td>
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<tr>
<td>4</td>
<td></td>
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<td>142.6274</td>
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<tr>
<td>5</td>
<td></td>
<td>190.6543</td>
<td>178.1543</td>
<td>165.6543</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>228.6346</td>
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<td></td>
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<td>231.5967</td>
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<tr>
<td>8</td>
<td></td>
<td>304.5516</td>
<td>284.5516</td>
<td>264.5516</td>
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<tr>
<td>9</td>
<td></td>
<td>342.5036</td>
<td>320.0036</td>
<td>297.5036</td>
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<tr>
<td>10</td>
<td></td>
<td>380.4544</td>
<td>355.4544</td>
<td>330.4544</td>
</tr>
</tbody>
</table>

Fig 3.1.3: Expected Profit V/S Time
Table 3.1.3

<table>
<thead>
<tr>
<th>t</th>
<th>R(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>1</td>
<td>0.9655</td>
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<tr>
<td>2</td>
<td>0.931992</td>
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<td>0.89946</td>
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<td>0.867889</td>
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<td>5</td>
<td>0.837264</td>
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<td>6</td>
<td>0.807566</td>
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<tr>
<td>7</td>
<td>0.778778</td>
</tr>
<tr>
<td>8</td>
<td>0.750881</td>
</tr>
<tr>
<td>9</td>
<td>0.723859</td>
</tr>
<tr>
<td>10</td>
<td>0.69769</td>
</tr>
</tbody>
</table>

Fig 3.1.4: Reliability V/S Time
Table 3.1.4

<table>
<thead>
<tr>
<th>$\lambda_{\text{mut}}$</th>
<th>MTTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>26.74896296</td>
</tr>
<tr>
<td>0.02</td>
<td>26.08694862</td>
</tr>
<tr>
<td>0.03</td>
<td>25.45591945</td>
</tr>
<tr>
<td>0.04</td>
<td>24.85379386</td>
</tr>
<tr>
<td>0.05</td>
<td>24.27866995</td>
</tr>
<tr>
<td>0.06</td>
<td>23.72880678</td>
</tr>
<tr>
<td>0.07</td>
<td>23.20260784</td>
</tr>
<tr>
<td>0.08</td>
<td>22.69860656</td>
</tr>
<tr>
<td>0.09</td>
<td>22.21545344</td>
</tr>
<tr>
<td>0.10</td>
<td>21.75190476</td>
</tr>
</tbody>
</table>

Fig 3.1.5: MTTF V/S Failure Rate
INTERPRETATION OF THE RESULTS

[1] The inspection of Table 3.1.1 and Fig 3.1.2 reveals that, as time increases, the availability of the system decreases catastrophically in the beginning but thereafter it decreases approximately in constant manner.

[2] Table 3.1.2 and Fig 3.1.3 show that as service cost $C_2$ increases, expected profit decreases with the passage of time.

[3] Table 3.1.3 and Fig 3.1.4 make it clear that reliability of the system decreases with respect to time.

[4] Further, the graph corresponding to Table 3.1.4 and Fig 3.1.5 decrease the fact that the M.T.T.F. decreases rapidly in the beginning, but as the time passes, it decreases approximately at a uniform rate.