Chapter-3

NON-LINEAR ANISOTROPIC FILTERING IN PCA DOMAIN

3.1 INTRODUCTION

Fusion is the process of merging information obtained from two or more images into a single image which would be more informative than the any of the input images applied. This chapter proposes a novel image fusion approach for medical images that utilizes Non-Linear Anisotropic Filtering (NLAF) in Principal Component Analysis (PCA) domain, which preserves the texture information of fused images most effectively. The base and detail layers are obtained with NLAF by decomposing the source medical images then the outcome of these layers is calculated utilizing the PCA. At last, utilizing the linear combination of these two layers a fused image is acquired. Qualitative and Quantitative functioning of NLAF-PCA algorithm is evaluated with the help of image quality metrics like Peak Signal-to-Noise Ratio (PSNR), Correlation Coefficient (CC), Entropy (E), Root Mean Square Error (RMSE) and Structural Similarity Index Measurement (SSIM).

3.2 NON-LINEAR ANISOTROPIC FILTERING (NLAF)

The NLAF process will smooth a given image at homogeneous regions while preserving the nonhomogeneous regions (edges) using Partial Differential Equations (PDE). It overcomes the drawbacks of non-linear isotropic filtering in which there is a loss of information at the edges due to the smoothing in inter-region. In counterpoint, production of images with coarser resolution is done by smoothing intraregional
space. At every edge the coarser resolutions are sharp and substantive. Flux operation utilized by NLAF for the diffusion control of an input image $X$ is given by,

$$X_t = F(m,n,t)\Delta X + \nabla F \cdot \nabla X$$  \hspace{1cm} (3.2.1)

where $F(m,n,t)$ denotes flux function, $\Delta$ is the Laplacian operator, $\nabla$ is the gradient operator and $t$ is time or scaling constant.

The equation (3.2.1) also referred as heat equation, can be resolved by the approach called Forward-Time-Central Space (FTCS). The solution for this given by the following Partial Differentiation Equation (PDE) is

$$X_{ij}^{t+1} = X_{ij}^t + \beta \left[ F_N \cdot \nabla_N X_{ij}^t + F_S \cdot \nabla_S X_{ij}^t + F_E \cdot \nabla_E X_{ij}^t + F_W \cdot \nabla_W X_{ij}^t \right]$$  \hspace{1cm} (3.2.2)

where, the image with coarser resolution at $t + 1$ scale is denoted as $X_{ij}^{t+1}$, which relies on the past image with coarser scale $X_{ij}^t$. Constant of stability is denoting with $\beta$ which must be in the range $0 \leq \beta \leq 1/4$. Differences of nearest neighbourhood in the directions of North, South, East and West are denoted as $\nabla_N$, $\nabla_S$, $\nabla_E$, $\nabla_W$ respectively and are defined as

$$\nabla_N X_{ij} = X_{i-1,j} - X_{ij}$$  \hspace{1cm} (3.2.3)

$$\nabla_S X_{ij} = X_{i+1,j} - X_{ij}$$  \hspace{1cm} (3.2.4)

$$\nabla_E X_{ij} = X_{ij+1} - X_{ij}$$  \hspace{1cm} (3.2.5)

$$\nabla_W X_{ij} = X_{ij-1} - X_{ij}$$  \hspace{1cm} (3.2.6)

Similarly, the flux functions are denoted as $F_N$, $F_S$, $F_E$ and $F_W$ respectively and defined by
In equations (3.2.7) to (3.2.10), \( g(\cdot) \) is a function which monotonically decreases with \( g(0) = 1 \). Various operations can be considered for \( g (\cdot) \). But couple of operations are advised by [1] and are given below:

\[
\begin{align*}
F^t_{N_{ij}} &= g\left( \| (\nabla X)_{i-1/2,j} \| \right) = g(\| \nabla N_{ij} \|) \\
F^t_{S_{ij}} &= g\left( \| (\nabla X)_{i+1/2,j} \| \right) = g(\| \nabla S_{ij} \|) \\
F^t_{E_{ij}} &= g\left( \| (\nabla X)_{i,j+1/2} \| \right) = g(\| \nabla E_{ij} \|) \\
F^t_{W_{ij}} &= g\left( \| (\nabla X)_{ij-1/2} \| \right) = g(\| \nabla W_{ij} \|)
\end{align*}
\]

A trade-off between preservation of texture and smoothing is provided by the two functions given in equations (3.2.11) and (3.2.12) where the equation (3.2.11) is utilized if there are the edges in an image with high-contrast over the low-contrast and the equation (3.2.12) is utilized if the image contains the broad regions over the smaller regions. A parameter \( k \), presented in both the equations is utilized to settle the region limit validity based on the strength of its edge.

### 3.3 RESEARCH METHODOLOGY

This section describes the brief explanation of our proposed fusion framework. Fused output image is obtained by implementation of NALF-PCA process. NLAF is used to obtain the approximate and detail layers with PCA fusion rule. Proposed NLAF-PCA fusion methodology shown in Figure 3.1.
Fig. 3.1: Proposed NLAF-PCA fusion process flow

3.3.1 EXTRACTION OF APPROXIMATED AND DETAIL LAYERS UTILIZING NLAF

Let the source MR and CT images are denoted as $I(r,m,n)$, $J(r,m,n)$ respectively. Both the images are assumed to be of size $p \times q$. The source images are the co-registered images obtained from the registration block. As shown in Figure 3.1, both the source images are first passed through the NLAF block to obtain the approximate layers.
A_{Ir}(m,n) = nlaf(I_r(m,n)) \quad (3.3.1)

A_{Jr}(m,n) = nlaf(J_r(m,n)) \quad (3.3.2)

Where \( A_{Ir}(m,n) \) and \( A_{Jr}(m,n) \) are \( r \)th approximate layers and nlaf is a sub function that processes the source image. Now, the detail layers are obtained by subtracting the output of NLAF from the source images \( I_r(m,n) \), \( J_r(m,n) \) by utilizing equations (3.3.3) and (3.3.4).

\[ D_{Ir}(m,n) = I_r(m,n) - A_{Ir}(m,n) \quad (3.3.3) \]

\[ D_{Jr}(m,n) = J_r(m,n) - A_{Jr}(m,n) \quad (3.3.4) \]

Figure 3.2 demonstrates the output layers obtained from NLAF process i.e., approximation and detail layers of MR and CT images.

**Algorithm: NLAF-PCA based fusion process**

**Step 1:** Select and read MR and CT source images from the MATLAB current directory (data set2 shown in Figure 1.2).

**Step 2:** Convert the source images into gray scale if the source images are Red Green Blue (RGB) images.

**Step 3:** Apply NLAF process to obtain approximate layers of MR and CT images as described in section 3.3.1.

**Step 4:** Subtract the source images from the obtained approximate layers to get the detail layers of MR and CT images.

**Step 5:** Compute the covariance of detail layers obtained from step 4.

**Step 6:** Calculate the Eigen vectors for step 5 output.

**Step 7:** Now, apply PCA fusion rule to obtain final fused output of MR and CT images.
**3.3.2 PRINCIPAL COMPONENT ANALYSIS (PCA)**

One of the basic underlying complication in the statistics of multivariate is the issue if data visualization which has several changing parameters. In practice, several altered parameters within the datasets frequently move with each other. The reason behind this is that, there might be more than one variable which is measuring the same driving principle governing the behaviour of the system. There are only few such driving forces in many systems.

Because of the availability of abundance instrumentation, dozens of such system variables can be measured. This is possible when the redundant information is taken into consideration. The problem can be simplified by replacing a group of variables with a new single variable.

**Figure 3.2:** (a)-(b) Approximate Layer of both MR and CT images (c)-(d) Detail Layer of both MR and CT images
3.3.2.1 DEFINITION

To attain the modification discussed above, an extensive approach is required which is often cited as PCA that produces a subset of novel changing parameters which are named as principal components and every principal component is a linear combination of the original variables. In addition to this, all the principal components are orthogonal to each other hence there is no existence of information redundancy.

3.3.2.2 PRINCIPAL COMPONENTS

In general, the first principal component of PCA is a single axis in space that generates a novel changing parameter by projecting every observation on to this axis. The variance of this changing parameter is the highest amongst all the possible options of the first axis. The second principal component of PCA is one more axis in space which is exactly in the perpendicular direction to the initial one. Projecting the observations on this axis generates another new variable. The variance of this variable is the maximum among all possible choices of this second axis. The full set of principal components is as large as the original set of variables. But it is commonplace for the sum of the variances of the first few principal components to exceed 80% of the total variance of the original data.

3.3.3 FUSION RULE

After obtaining the approximate and detail layers from the source MR and CT images PCA is applied to find out principal components (as described in section 3.3.2) for getting better analysis over conventional fusion algorithms presented in the literature. Now, to get a fused output image a rule must be initiated to obtain optimum output from the proposed NLAF-PCA fusion process. First the approximate layers of
MR and CT images are combined. Then, the detail layers are multiplied with the principal components denoted as \( p \) which are obtained by PCA algorithm and the products are added. Finally, integrating these two process outputs produces a fused image.

\[
\mathcal{F}(m, n) = A(m, n) + D(m, n) \tag{3.3.5}
\]

where, \( A(m, n) = A_{Ir}(m, n) + A_{Jr}(m, n) \)

\[
D(m, n) = p(1) \ast D_{Ir}(m, n) + p(2) \ast D_{Jr}(m, n) \tag{3.3.6}
\]

3.4 RESULTS AND DISCUSSION

All the experiments are done using MATLAB software. Figure 3.3 displays the datasets of source medical images that are utilized for executing the test outcome with NLAF-PCA and other conventional fusion algorithms. These datasets are cited as dataset 1 and dataset 2 which comprises of CT and MR images respectively.

**Figure 3.3**: Source medical images (a) CT i.e., dataset 1 (b) MR i.e., dataset 2
As mentioned in earlier sections, incorporation of the required info into one image from one or more images is the basic theory of fusion concept. The outcome of fusion procedure cannot be evaluated entirely by observing the fused image or by quantifying the metrics of fusion. It should be evaluated both qualitatively by observing the visual expose and quantitatively by employing the metrics of fusion.

This subsection presented both qualitative and quantitative assessment of obtained fused medical images by employing several fusion approaches like, Discrete Wavelet Transform (DWT) [2], Stationary Wavelet Transform (SWT) [3], PCA [4], DWT with PCA [5], SWT with PCA [6] and Fast Discrete Curvelet Transform (FDCT) [7] with comparison to the NLAF-PCA methodology. For disclosing the efficacy of NLAF-PCA, the quantitative assessment is done by computing several metrics of image quality like PSNR, SSIM, RMSE, CC and entropy. All the metrics should have an optimum value for the best fusion output images since the motive of any fusion approach is to attain a well qualitative outcome. Therefore, to visualize it easily, bold letter is employed for the best value of computed quality metric. Figure 3.4 depicts the visual assessment of conventional fusion algorithms mentioned earlier and the NLAF-PCA for dataset 1 from Figure 3.3. Similarly, Figure 3.5 demonstrates the fused outcome of likely algorithms discussed above for dataset 2 of Figure 3.3. It can be clearly observed that, the perceptual quality of fused output using PCA, shown in Figure 3.4(a) appears as low resolute image and the gray levels are not up to the mark. Other transformation methods like the methodology proposed in [2], SWT and the algorithm demonstrated in [7] shown in Figure 3.4(b-d) respectively, which performed superior to the PCA method in terms of visual perception, however these methods suffer from lack of contrast and edge preservation.
Figure 3.4: Visualization of fused output images with Data set 1 (a) PCA (b) DWT (c) SWT (d) FDCT (e) SWT-PCA and (f) NLAF-PCA

Figure 3.4 (e) shows that the fused output of the method presented in [6], which was far better than the PCA, DWT, SWT, FDCT and SWT-PCA algorithms. However, all the existing fusion methods outputs poor visual perception, poor contrast at the edges and fails to preserve the texture information. The output of the NLAF-PCA method which is presented in Figure 3.4(f), which looks more quality in visualization with good contrast and proper edge information and has excellent texture preservation as the value of entropy is much higher.
Figure 3.5: Visualization of fused output images with Data set 2 (a) PCA (b) DWT (c) SWT (d) FDCT (e) SWT-PCA and (f) NLAF-PCA

Figure 3.5 (a-f) demonstrates fused outputs obtained using PCA, DWT, SWT, FDCT, SWT+PCA and the proposed NLAF-PCA method with data set 2. The same analysis which we have discussed above is applicable for this also.

Quantitative analysis of the proposed NLAF-PCA method with the existing methods for dataset 1 and dataset 2 are shown in Table 3.1 and Table 3.2 respectively. Table 3.1 consists of various fusion metric parameters such as PSNR, RMSE, CC, SSIM and Entropy.
### Table 3.1: Obtained quality metrics of fused outcome for Dataset 1

<table>
<thead>
<tr>
<th>Methodology</th>
<th>PSNR (in dB)</th>
<th>RMSE</th>
<th>CC</th>
<th>SSIM</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCA [4]</td>
<td>60.595</td>
<td>0.16</td>
<td>0.888</td>
<td>0.9989</td>
<td>6.09</td>
</tr>
<tr>
<td>SWT [3]</td>
<td>62.253</td>
<td>0.1967</td>
<td>0.7928</td>
<td>0.986</td>
<td>6.11</td>
</tr>
<tr>
<td>DWT [2]</td>
<td>62.257</td>
<td>0.1966</td>
<td>0.7935</td>
<td>0.986</td>
<td>6.099</td>
</tr>
<tr>
<td>FDCT [7]</td>
<td>65.156</td>
<td>0.146</td>
<td>0.9</td>
<td>0.9983</td>
<td>5.963</td>
</tr>
<tr>
<td>DWT-PCA [5]</td>
<td>63.159</td>
<td>0.165</td>
<td>0.895</td>
<td>0.9987</td>
<td>6.12</td>
</tr>
<tr>
<td>SWT-PCA [6]</td>
<td>64.96</td>
<td>0.143</td>
<td>0.9</td>
<td>0.997</td>
<td>6.19</td>
</tr>
<tr>
<td>NLAF-PCA</td>
<td><strong>65.06</strong></td>
<td><strong>0.142</strong></td>
<td><strong>0.913</strong></td>
<td><strong>0.997</strong></td>
<td><strong>6.24</strong></td>
</tr>
</tbody>
</table>

### Table 3.2: Obtained quality metrics of fused outcome for Dataset 2

<table>
<thead>
<tr>
<th>Methodology</th>
<th>PSNR (in dB)</th>
<th>RMSE</th>
<th>CC</th>
<th>SSIM</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCA [4]</td>
<td>69.937</td>
<td>0.056</td>
<td>0.87</td>
<td>0.999</td>
<td>4.637</td>
</tr>
<tr>
<td>SWT [3]</td>
<td>68.95</td>
<td>0.0909</td>
<td>0.933</td>
<td>0.988</td>
<td>0.9684</td>
</tr>
<tr>
<td>DWT [2]</td>
<td>68.98</td>
<td>0.0906</td>
<td>0.934</td>
<td>0.988</td>
<td>0.9683</td>
</tr>
<tr>
<td>FDCT [7]</td>
<td>70.215</td>
<td>0.140</td>
<td>0.9206</td>
<td>0.9983</td>
<td>5.05</td>
</tr>
<tr>
<td>DWT+PCA [5]</td>
<td>71.159</td>
<td>0.0486</td>
<td>0.9</td>
<td>0.999</td>
<td>5.09</td>
</tr>
<tr>
<td>SWT+PCA [6]</td>
<td>72.932</td>
<td>0.047</td>
<td>0.96</td>
<td>0.999</td>
<td>4.9</td>
</tr>
<tr>
<td>NLAF-PCA</td>
<td><strong>74.18</strong></td>
<td><strong>0.049</strong></td>
<td><strong>0.973</strong></td>
<td><strong>0.999</strong></td>
<td><strong>5.16</strong></td>
</tr>
</tbody>
</table>

The proposed method has produced high PSNR for both the data sets. The RMSE calculated with respect to dataset 1 and dataset 2 is less for the proposed NLAF-PCA method but [6] have also given promising results. The NLAF-PCA method has outperformed all the listed methods in terms of Entropy. Higher Entropy values are obtained for the proposed method. The best values are highlighted in bold letters.
proposed method obtained far better values over all the existing fusion methods discussed in the literature.

3.5 CONCLUSION

This chapter described a new texture preserving fusion approach for MR and CT images by utilizing NLAF-PCA methodology. NLAF is utilized to extract the approximate and detail layers from the MR and CT source images. Then the principal components are computed according to the PCA algorithm. Finally, fusion is applied to obtain a fused image with texture preservation. Performance of NLAF-PCA fusion process also assessed with several medical image fusion methodologies presented in the literature. Further, comparative analysis is done according to the image quality metrics and shown that the NLAF-PCA fusion process performed superior to the conventional medical fusion algorithms. Although this method has given promising results by preserving texture, its PSNR is less and there is a scope in the improvement of the fusion output. Better fusion performance is expected if Active Slope Meagerness (ASM) characteristic of the image is taken into consideration. An algorithm which enhances ASM property with Statistics Based filtration is proposed in chapter-4.
REFERENCES


