Comprehensive Analysis of BER and SNR in OFDM Systems

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ABSTRACT: The proliferation of mobile platforms has put lot of stress in terms of power limitations and Quality of Service being offered by OFDM systems. BER and SNR analysis plays an important and imperative role in understanding and improving the design of OFDM systems. In this paper we have presented a comprehensive analysis about BER and SNR for various scenarios which include different channels, different modulation techniques and carrier frequency offsets.

Keywords: OFDM, BER, SNR, Offsets, Quality of Service

1. LITERATURE REVIEW ON OFDM CONCEPTS

The OFDM transmission scheme seems to be a promising candidate for future broadband radio systems. That transmission scheme is currently deployed in the well known standard IEEE 802.16a/d [1]. The crucial part which determines the performance is thereby the combination of dynamic sub-carrier allocation, transmission power allotment, and adaptive modulation. Many communication systems require knowledge of the signal-to-noise ratio (SNR), with efficient signal detection and link adaptation as most prominent examples. Signal-to-Noise ratio (SNR) is an indicator commonly used to evaluate the quality of a communication link. More specifically, SNR knowledge enables wireless systems to improve propagation channel estimation and is a key decision parameter in adaptive processes such as dynamic reconfiguration of cognitive radios, adaptive modulation and coding (AMC) or adaptive power allocation. Frequency division multiplexing (FDM) extends the concept of single carrier modulation by using multiple sub carriers within the same single channel. The total data rate to be sent in the channel is divided between the various sub carriers [3]. An OFDM baseband signal is the sum of a number of orthogonal sub-carriers, each sub-carrier being independently modulated (for instance using QAM or PSK) by its own data. The orthogonality allows simultaneous transmission on a lot of sub-carriers in a tight frequency space without interference from each other. Thus, they are able to overlap without interfering. As a result, OFDM systems are able to maximize spectral efficiency without causing adjacent channel interference. BPSK is the simplest form of phase shift keying (PSK) A digital signal alternating between +1 and -1 (or 1 and 0) will create phase reversals, i.e. 180 degree phase shifts as the data shifts state. QPSK Higher order modulation schemes, such as QPSK, are often used in preference to BPSK when improved spectral efficiency is required [6]. QPSK uses four points on the constellation diagram. With four phases, QPSK can encode two bits per symbol. Quadrature Amplitude Modulation refers to QPSK with Amplitude Modulation. Basically, it is a mix of phase modulation and amplitude modulation. QAM phase modulates the carrier and also modulates the amplitude of the carrier [5] [8]

Additive White Gaussian Noise (AWGN) channel is a Channel Model used for analyzing modulation schemes used for transmission of radio OFDM Signal. In this Model the channel adds a white Gaussian noise to the OFDM signal which is passing through it. By this the signal gets two properties like the amplitude frequency response is flat which means that can signal pass through channel without any amplitude loss and having infinity bandwidth. Phase frequency response is linear, so that no phase distortion of frequency component occurs. Rayleigh fading channel is used when there is no direct path between transmitter and receiver [4]. If there is no line of site then the constructive and destructive nature of Multipath Signal in flat fading can be approximated by Rayleigh Distribution [7].

II. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is an efficient bandwidth signaling scheme for the wideband digital communications. One important difference between OFDM and frequency division multiplexing (FDM) is that,
the individual carriers mutually overlaps in the OFDM spectrum. OFDM carriers exhibit orthogonality property based on a symbol interval if their spacing in frequency is exactly at the reciprocal of the symbol interval, which can be accomplished by utilizing the Discrete Fourier Transform (DFT) [1]. OFDM is a technique based on multicarrier communication. The basic approach of multicarrier communications is to divide the total available signal bandwidth into a number of subcarriers, and information is transmitted on each of the subcarriers. Unlike the conventional multicarrier communication scheme, in which spectrum of each subcarrier is non-overlapping and band pass filtering is used to extract the frequency of interest. The spectra of subcarriers overlap but individual subcarrier can be extracted by baseband processing. This overlapping property makes OFDM technique more spectral efficient than the conventional multicarrier communication scheme. With the development of modern digital signal processing technology, OFDM technique has become practical to implement and has been proposed as an efficient digital modulation scheme for applications ranging from modems, digital audio broadcast, to next-generation high-speed wireless data communications.

Orthogonal Frequency Division Multiplexing (OFDM) and Multiple-Input Multiple-Output (MIMO) are cutting edge physical layer technologies to be employed in fourth generation (4G) wireless cellular standards such as 3GPP Long Term Evolution (LTE/LTE-A), Worldwide Interoperability for Microwave Access (WiMAX) and high speed Wireless LAN standards. Such 4G cellular standards are expected to support data rates exceeding 100 Mbps through OFDMA, MIMO, dynamic carrier aggregation and thus enable a diverse number of applications in the wireless system such as broadcast/multicast video, on demand HDTV, high speed access to internet, interactive gaming amongst others. [10] At the same time there is an impressive effort towards fixed mobile convergence to enable smooth and effortless mobility across future Wireless LAN and cellular networks. Such factors are driving the wireless telecommunication system designers and operators to invest hugely in the development of OFDM compatible technologies and applications with the aim of tapping into the potentially vast revenue opportunity in futuristic 4G cellular network systems. High capacity and variable bit rate transmission information with high bandwidth efficiency are some of the requirements that the modern transceivers have to meet a variety of new high value added quality services to be delivered to the customers. A worldwide convergence has occurred for the use of Orthogonal Frequency Division Multiplexing (OFDM) as an emerging technology for high data rates. In particular, many wireless standards have adopted the OFDM technology as a mean to increase dramatically future wireless communications. The main reason behind OFDM's increased popularity is the desire for high speed wireless technologies and the increased demand for multimedia applications, which require higher data rates. Because in the wireless environment signals are usually impaired by fading and multipath delay spread phenomenon, traditional single carrier mobile communication systems do not perform well. In such channels, extreme fading of the signal amplitude occurs and Inter Symbol Interference (ISI) due to the frequency selectivity of the channel appears at the receiver side [2]. This leads to a high probability of errors and the system's overall performance becomes very poor. Techniques like channel coding and adaptive equalization have been widely used as a solution to these problems. However, due to the inherent delay in the coding and equalization process and high cost of the hardware, it is quite difficult to use these techniques in systems operating at high bit rates.

In this proposed work we have presented an extensive analysis about the significance of BER (Bit Error Rate) and SNR (Signal to Noise Ratio) under various scenarios which include different channels and different modulation techniques. Carrier frequency offset (CFO) in orthogonal frequency-division multiplexing (OFDM) systems, which can induce the orthogonality loss among subcarriers resulting in significant performance degradation, is critical and to be estimated and compensated for. In this paper we have also studied the effect of CFO on AWGN channel and its subsequent impact on BER[9]. The advent of MIMO-OFDM systems the complexity in design and analysis of OFDM systems have reached another plane, the paper also presents the performance analysis of MIMO OFDM systems. We have presented a comprehensive analysis of performance of OFDM system considering different factors in regard to the BER and SNR of OFDM systems.

III. SIMULINK MODEL

To validate the research objectives a simulink model is developed for MIMO-OFDM system. The system has been developed using communication tool box available in MATLAB version 7.14 R2012.
The above model offers flexibility in selecting number of channels, channel bandwidth, type of channel etc. Apart from the above model text coding in MATLAB is also employed to validate the objectives of the proposed work in having a comprehensive study of performance of OFDM in regard to its BER and SNR performance measures.

IV. RESULTS

The below figure depicts an OFDM signal generated for the proposed work.
High power Amplification (HPA) induces distortion and subsequently affects BER and SNR. The below figure depicts an OFDM signal simulated for distortion after HPA.

![Fig.3. An OFDM Signal after HPA](image)

The kind of modulation has a direct bearing on the performance and hence the variation between SNR and BER. The below figure depicts the variation for QAM type.

![Fig.4. BER Compared with SNR for different levels of QAM](image)

The type of channel also influences the performance of OFDM. The below figure presents a comparative plot between Rayleigh and AWGN channels for BPSK type modulation.

![Fig.5. BER compared with SNR Plot for BPSK over AWGN and Rayleigh](image)
The relationship between SNR and BER is inversely proportional. An enhanced SNR indicates a reduce error rate and an improved performance. The below figures validates the above point in terms of different channels.

![Fig. 6. OFDM BER Compared with SNR in Selective Rayleigh Fading Channel](image)

OFDM is extremely sensitive to synchronization errors, especially the carrier frequency offset (CFO), which is induced by oscillator discrepancies between the transmitter and receiver and/or Doppler shifts. As a result, CFO estimation for OFDM is an active area of research. The below figure depicts the BER sensitivity for CFO in an AWGN channel.

![Fig. 7. BER sensitivity for CFO in an AWGN channel.](image)
It can be observed that there is a clear difference in comparison between two channels in terms of BER and SNR comparison for the same modulation technique. With the advent of mobile platforms and other devices, MIMO systems are becoming an integral part of OFDM environments; it is essential to analyze their performance as well. The below figures depict the signal power in the context of MIMO systems.

![Capacity plot in comparison with SNR in a Rayleigh Fast Fading MIMO System](image_url)

**Fig. 8.** Capacity plot in comparison with SNR in a Rayleigh Fast Fading MIMO System

![Relative Achievable capacity of different receiver structures in Rayleigh Fading MIMO](image_url)

**Fig. 9.** Relative Achievable capacity of different receiver structures in Rayleigh Fading MIMO

From the above discussions, we can understand that the BER and SNR estimation pose a complex challenge due to the multitude of reasons and complex interactions between different components of an OFDM system. The proliferation of MIMO systems adds to the complexity of the environment. Understanding the performance measures of BER and SNR is very significant in the better design of OFDM systems.

V. CONCLUSION

The portable and battery-operated devices are getting integrated into most of the day to day applications. OFDM being the chief player in these devices has to be updated and evolved continuously. This offers researchers ample opportunities and challenges alike in improving the performance of the OFDM systems. BER and SNR play a very crucial role in performance analysis. This paper brings out the complexity in BER and SNR analysis and their importance in the development of better OFDM systems.
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BIOGRAPHY

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A Critical Analysis of Pilot and Blind channel Estimation Techniques for OFDM systems

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Abstract: The channel estimation techniques for OFDM systems based on pilot arrangement and blind estimation are investigated in the proposed work. Various channel estimation techniques are employed in order to judge the physical effects of the medium present. In this proposed work, we have analyzed and implemented various estimation techniques for MIMO OFDM Systems such as Least Squares (LS), Minimum Mean Square Error (MMSE), Constant Modulus Algorithm (CMA) and linear Pre-coding. These techniques are therefore compared to effectively estimate the channel in MIMO OFDM Systems. The objective of the proposed work is to further aid in the development of NDA based channel estimation methods by serving as an analytical tool for comparison between the existing methods and new methods being developed.

Keywords: OFDM, CMA, linear precoding, minimum mean square error

I. Introduction

The increasing require for high-bit-rate digital mobile communications has incited the appearance of Orthogonal Frequency-Division Multiplexing (OFDM) for achieving good performance in high rate data transmission [1] [2]. It is also an effective technique that produces a high spectral efficiency and a good scheme to combat frequency-selective fading channels in wireless communication systems without forgetting the major property that is subcarrier orthogonality. Hence, the symbol duration must be significantly larger than the channel delay spread. In orthogonal frequency division multiplexing (OFDM), the entire channel is divided into many narrow sub channels. Splitting the high-rate serial data stream into many low-rate parallel streams, each parallel stream modulates orthogonal subcarriers by means of the inverse fast Fourier transform (IFFT). If the bandwidth of each subcarrier is much less than the channel coherence bandwidth, a frequency flat channel model can be assumed for each subcarrier. Moreover, inserting a cyclic prefix (or guard interval) results in an inter-symbol interference (ISI) free channel assuming that the length of the guard interval is greater than the delay spread of the channel. Therefore, the effect of the multipath channel on each subcarrier can be represented by a single complex multiplier, affecting the amplitude and phase of each subcarrier. Hence, the equalizer at the receiver can be implemented by a set of complex multipliers, one for each subcarrier. Under multi path spread situation, a dynamic estimation of channel is necessary before the demodulation of OFDM signals to ensure a coherent detection and since the radio channel is frequency selective and time-varying for wideband mobile communication systems [2].

In the literature, many channel estimation schemes are found and depends on if the channel is constant, slowly or fast time varying. Traditionally, channel estimation is achieved by sending training sequences through the channel. However, when the channel is varying, even slowly, the training sequence needs to be sent periodically in order to update the channel estimates. Hence, the transmission efficiency is reduced [2]. The increasing demand for high-bit-rate digital mobile communications makes blind channel identification and equalization very attractive, since they do not require the transmission of a training sequence.

This paper investigates and compares both pilot based and blind channel based estimators for OFDM systems. The primary objective of the proposed work is to aid in further development of blind channel estimation techniques by providing a critical review of the existing systems. Such analysis will be of great help in comparing and analyzing the performance of new techniques in related to the techniques existing in the literature.

II. System Model

The baseband OFDM system is practically the same for all the schemes of channel estimations and differs just from the block of the channel but some schemes can add another block used especially for interpolation or for equalization.
Figure 1: A Base Band OFDM System.

III. OFDM System for Channel Estimation based on Pilot Arrangement

The OFDM system model used for training sequence (pilot signal) consists of mainly of a mapper block forwarded by a S/P conversion, then there is an insertion pilot block followed by an IDFT calculation of the information data [4]. After that we find the guard insertion block and a P/S conversion before reaching the channel which is affected by an AWGN noise. The data stream will be converted on a parallel stream, and then the guard interval is removed and will sail towards the frequency domain. Channel estimation is afterward performed before carrying out a P/S conversion and attainment of the demapper block to restore back the data stream. After crossing the S/P block, the pilot used here will be inserted in all subcarriers of one OFDM symbol with a specific period or uniformly between data sequence for a block pilot type estimator or in some specific subcarriers for the comb pilot type. Then, the data sequence will pass up the IDFT block for the transformation to time domain and the expression of \( x(n) \) (\( N \) being the DFT length) is given as follows:

\[
x(n) = IDFT(x(k)) \quad n = 0, 1, 2, \ldots, N - 1
\]

After that, the cyclic prefix will be inserted to preserve orthogonality of the subcarriers on the one hand and to evade inter symbol interference between adjacent OFDM symbols on the other hand. The guard time which contains this cyclic prefix having a length \( N_g \) is chosen to be greater than the delay spread [5]. Then the resulting symbol is:

\[
x_f(n) = \begin{cases} 
  x(n + n), & n = -N_g, -N_g + 1, \ldots, -1 \\
  x(n), & n = 0, 1, \ldots, N - 1 
\end{cases}
\]

After a P/S conversion, the OFDM symbol will cross the channel expected to be frequency selective and time varying fading channel with an additive noise and will be given by

\[
y_f(n) = x_f(n) \otimes h(n) + w(n)
\]

\( h(n) \) is the channel impulse response represented by

\[
h(n) = \sum_{i=0}^{r-1} h_i \delta \left( \frac{2\pi}{N} \delta(n - \tau_i) \right), \quad 0 \leq n \leq N - 1
\]
Where \( r \) is the total number of propagation paths, \( h_i \) is the complex impulse response of the \( i \)th path, \( f_i \) is the \( \text{Di} \)th path Doppler frequency shift is the delay spread index, \( T \) is the sample period and \( \Delta_i \) is the \( i \)th path delay normalized by the sampling time.

\[
y_f(n) \text{ for } -N_g \leq n \leq N - 1
\]

\[
y(n) = y_f(n + N_g) \quad n = 0,1,2, ..., N - 1
\]  

(5)

The frequency form of this resulting signal will be expressed as follow:

\[
Y(k) = \text{DFT}(y(n)) \quad k = 0,1,2, ..., N - 1
\]

\[
= \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j \frac{2\pi nk}{N}}
\]  

(6)

By supposing a transmission without an inter symbol interference ISI, the relation between the frequency components is

\[
Y(k) = X(k)H(k) + I(k) + W(k)
\]  

(7)

Where \( H(k) = \text{DFT}(h(n)) \), \( I(k) \) is the inter-carrier interference because of the Doppler frequency and \( W(k) = \text{DFT}(w(n)) \) The pilot signals are then extracted and cross the channel estimation block, after that the estimated channel \( H_e(k) \) for the data sub-channel is obtained and the transmitted data is estimated. At last, the binary data sequence is recovered by the signal demapper block.

IV. OFDM System for Blind Estimation

In OFDM systems, the serial data are converted into \( M \) parallel streams. Each parallel data stream modulates a different carrier [3][6]. The frequency separation between the adjacent carriers is \( 1/T \), where \( T \) is the symbol duration for the parallel data that is \( M \) times of the symbol duration for the serial data. Let us consider an OFDM signal in the interval \((nT, (n+1)T)\) as

\[
s(t) = \sum_{m=0}^{M-1} a_m(n)e^{jw_mt}
\]  

(8)

Where \( a_m(n) \) are symbols resulting from a modulation constellation like QAM. \( w_m \) is the frequency of \( m \)th carrier

\[
s(nM + i) = \sum_{m=0}^{M-1} a_m(n)e^{j \frac{2\pi}{M} mi}
\]  

(9)

From this equation, the \( M \) samples can be seen as the inverse discrete Fourier transform (IDFT) of a block for \( M \) input symbols. Theoretically speaking, when the number of carriers is large enough, symbol duration \( T \) is much larger than the duration of FIR channel; IS1 is negligible. However, for the high-bit-rate communications, it is impractical to choose very large \( M \) to make ISI negligible. Therefore, a cyclic prefix of length \( P \) is added into each block of IDFT output at the transmitter. The length of the prefix is chosen to be longer than the length of the channel impulse response in order to avoid inter-block interference (IBI). That results with total cancellation of IS1 and inter carrier interference (ICI). The input data will be as follow

\[
s(n(M + P) + i) = \sum_{m=0}^{M-1} a_m(n)e^{j \frac{2\pi}{M} (i-P)}
\]

\[
i = 0,1, ..., M + P - 1
\]  

(10)

\[
y_i(n) = a_i(n)H\left( \frac{2\pi i}{M} \right) + v_i(n)
\]  

(11)
Where $H(\cdot)$ is the frequency response of the channel. It is evident from the equation above that the ISI is completely cancelled and the effect of the channel at the receiver is simply a complex gain and AWGN.

V. Channel Estimation Techniques

Channel Estimation is the process of characterizing the effect of the physical medium on the input sequence. It is an important and necessary function for wireless systems [3]. Even with a limited knowledge of the wireless channel properties, a receiver can gain insight into the data sent over by the transmitter. The main goal of Channel Estimation is to measure the effects of the channel on known or partially known set of transmissions [3]. Orthogonal Frequency division multiplexing (OFDM) Systems are especially suited for channel estimation. The sub carriers are closely spaced. While the system is generally used in high speed applications that are capable of computing channel estimates with minimum delay. There are primarily two major classification of channel estimation techniques into Pilot based channel Estimation and Blind Channel estimation [3] [8].

Pilot based Channel estimation is based on the training sequence which is known to both transmitters and receiver. The receiver can utilize the known training bits and the corresponding received samples for estimating the Channel. Some of the major approaches in this technique include Least Squares (LS) and Minimum Mean Squares (MMSE) among others. In the Least Squares Error (LSE) estimation method can be used to estimate the system $h[m]$ by minimizing the squared error between estimation and detection. In Minimum Mean Square Error (MMSE) the estimator minimizes the mean-square error [7]. Although ISI can be avoided, via the use of cyclic prefix in OFDM modulation, the phase and gain of each sub channel is needed for coherent symbol detection. An estimate of these parameters can be obtained with pilot/training symbols, at the expense of bandwidth. Blind channel estimation methods avoid the use of pilot symbols, which makes them good candidates for achieving high spectral-efficiency[3][8]. Existing blind channel estimation methods for OFDM systems can be classified as: 1. Statistical. 2. Deterministic.

The statistical methods explore the cyclo-stationarity that the cyclic prefix induces to the transmitted signal [9]. They recover the channel using cyclic statistics of the received signal, or subspace decomposition of the correlation matrix of the pre-DFT received blocks. The deterministic methods process the post DFT received blocks, and exploit the finite alphabet property of the information bearing symbols. Maximum likelihood and iterative Bayesian methods are two examples taking into account, specific properties of M-PSK or QAM signals, while utilizing an exhaustive search. In comparison to the statistical methods, the deterministic ones converge much faster; however, they involve high complexity, which becomes even higher as the constellation order increases [3] [9].

Equalization technique employed is from the deterministic class of blind channel estimation. It involves the use of equalizers. An equalizer removes the channel effects on a transmitted signal and reduces the Intersymbol Interference (ISI). The type of equalization, capable of tracking a slowly time-varying channel response is known as adaptive equalization. It can be implemented to perform tap-weight adjustments periodically or continually. Periodic adjustments are accomplished by periodically transmitting a preamble or short training sequence of digital data that is known to the receiver in advance. The receiver also uses the preamble to detect start of transmission, to set the automatic gain control (AGC) level. Continual adjustments are accomplished by replacing the known training sequence with a sequence of data. These algorithms adjust filter co-efficients in response to sample statistics rather than in response to sample decisions [9].

![Figure 2: Adaptive equalization.](image-url)
namely, Bussgang algorithm, Stato algorithm, Constant Modulus algorithm and Godard algorithm among others [9]. In the proposed work we have implemented and analyzed one deterministic type method namely the Constant Modulus algorithm and one Statistical type method in the form of Linear Precoding.

VI. Simulation, Results, and Discussion

The proposed system was coded in MATLAB environment. We have coded the system for both MIMO and SISO based OFDM systems. In order to illustrate the effect of Carrier Frequency Offset (CFO) and Symbol Timing Offset in OFDM in regard to ISI we have also coded their effects in OFDM signal. The following figures and discussion summarizes the simulation results of the proposed work. The following figure demonstrates the effect of CFO and STO in OFDM. It is demonstrated for QAM type Modulation.

![Figure 3: A 16 QAM Constellation under the effect of CFO.](image)

![Figure 4: A 16 QAM Constellation under the effect of STO.](image)

From the above figures it can be inferred that the pilots are not static but are rotating around the centre. During such a situation the phase estimation of the signal can not be perfect because of the presence of a time varying phase which is not constant over a symbol period. Higher the offset more the phase changes over one OFDM symbol and estimation becomes much more difficult.

The following figures demonstrate the results of simulation of channel estimation using the pilot based approach of LSE and MMSE.

![Figure 5: The Plot of Actual Channel.](image)
Figure 6: The Plot of Channel Estimated Using LS.

Figure 7: The Plot of Channel Estimated Using MMSE.

Figure 8: The Plot of SNR and Mean Squared Error for LS and MMSE based Estimator.

Figure (6) and Figure (7) depict the estimation using LS and MMSE methods in comparison to the original signal as depicted in Figure (5). The above pilot based approaches are effective as long as the training sequence is available to the receiver. Figure (8) demonstrates the mean square error of channel estimation at different SNR in dB as SNR increases mean square error decreases for both LSE and MMSE. It can also be observed that for a given SNR, MMSE estimator shows better performance than LSE estimator. The complexity of MMSE estimators will be larger than LSE estimators but gives a better performance in comparison to LSE.

The below mentioned figures demonstrate the implementation of estimation using blind channel estimation techniques.
Figure 9: Blind Channel Estimation Based on CMA QAM based frequency response.

Figure 10: Blind Channel Estimation Based on Precoding for QAM based frequency response.

Figure 11: Symbol or Bit error For Blind Channel Estimation.
Figure 12: MSE Estimated using CMA over 8000 Data Points.

Figure (9) and Figure (10) blind channel approaches exhibits better performance as compared to the training-based one for the case of fast varying channels. The BER result is shown in Figure (11) and by using the MSE estimate in Figure (12) one can see that these methods achieves much better performance that the training based one. But these methods are too computationally intensive to be used with higher N and constellation orders.

VII. Conclusions

A MATLAB based schema for performance analysis of pilot based channel estimation techniques and blind channel estimation techniques is implemented. In CMA type Blind channel Estimation BPSK type modulation was considered over 8000 data Points. The results for different techniques are compared and observed. The obtained results show that blind channel estimation can be used in future wireless communications especially, when the spectrum efficiency, low complexity and low level of received signal powers are considered. In an embedded transceiver design, the blind techniques can easily be employed with bearable performance degradation. The performance of LSE with MMSE estimator is also investigated. It is observed that MMSE estimation is better that LSE estimator in low SNRs; whereas at high SNRs, performance of LSE estimator is comparable to that of the MMSE estimator.

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Cram´er- Rao Lower Bounds and Bayesian Cram´er- Rao Lower Bound Estimators for Blind Channel Estimation of SNR and Phase Noise

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ABSTRACT: Channel Estimation is the process of characterizing the effect of the physical medium on the input sequence and Orthogonal Frequency division multiplexing (OFDM) Systems are especially suited for channel estimation. In this work we have designed a Cram´er- Rao Lower Bound estimation for blind channel estimator of SNR in OFDM systems. Phase Noise is a time varying process which changes the symbol from time to time and has a high deteriorating effect on higher order OFDM system. We have derived a Bayesian Cram´er- Rao Lower Bound Estimator for estimating phase noise.

KEY WORDS: SNR, Phase Noise, Cram´er- Rao Lower Bound, Bayesian Cram´er- Rao Lower Bound

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technology can be utilized to enhance the throughput and reliability of wireless communication links by introducing multiplexing and diversity gains to the overall system [1], [2]. As a result, MIMO systems are an effective means to meet the stringent requirements on today’s wireless communication systems that demand higher spectral efficiencies and throughputs [3]. On the other hand, phase noise severely deteriorates the performance of MIMO systems [4]. Accurate SNR estimate is required for measuring the channel quality for adaptive modulation schemes as well as for soft decoding procedures as shown in [5], [6] and [7]. In addition to low-complexity requirement, it is essential to assess the truthfulness of SNR estimators in term of their statistical variances. For this purpose, the well-known CRLB is a prominent benchmark to evaluate the statistical variance performance of unbiased estimators.

Actually both data aided (DA) and non-data aided (NDA) trends are considered for either performance bounds derivation or estimation algorithms. Data aided approach, which relies on the transmission of known data streams such as training sequences and also pilot symbols, should expedite and ease the estimation process. Unfortunately, this approach limits the system through-put in the sense that adding known pilot symbols to the data stream should drop down the spectral efficiency of the communication system. Hence NDA SNR estimation approach receives substantial attention in recent literature. CRLB for NDA SNR estimation is derived in [8] from both BPSK and QPSK modulated signals with AWGN channel. Derived bounds are compared to those obtained for DA estimation. In [9], a straightforward approximation of the CRLB for NDA SNR estimation from BPSK modulated signals over AWGN channel is presented in efficient form that avoids tedious numerical integration. Authors, in [10], derive a lower bound for SNR estimation from general one/two dimensional modulation signals with axis/half plane symmetry over AWGN channel. Exact analytical CRLB of unbiased NDA SNR estimation from square QAM signals using I/Q received signal model is addressed in [11], where a generalization of the elegant CRLB expressions presented in [8] is also introduced.

Phase noise is a time varying process that changes from symbol to symbol [12]–[14]. Moreover, the deteriorating effect of phase noise may be more severe in MIMO systems employing higher order modulations, given that in MIMO systems, independent oscillators may be used at each transmit and receive antenna resulting in multiple phase noise processes that need to be jointly estimated at the MIMO receiver[13]. The use of independent oscillators at each transmit and each receive antenna is well motivated in applications where antennas need to be placed far apart from one another.
another, e.g., in the case of line-of-sight (LoS) MIMO systems. As a result, even though Cram’er- Rao lower bounds (CRLBs) and algorithms for estimation of phase noise in single-input single-output (SISO) systems have been extensively and thoroughly studied in [12], these results cannot be applied to the case of MIMO systems. Similarly, phase locked loops, that can be used in SISO systems for phase noise tracking, cannot be applied in LoS-MIMO systems where multiple phase noise parameters need to be tracked simultaneously at the receiver [10].

The proposed work discusses a CRLB for NDA SNR estimation. Different types of time domain channel estimators are also studied. The performance of the proposed estimator is compared with the performance of ML estimator and Subspace blind channel estimator. Similarly, the Bayesian Cram’er-Rao lower bounds (BCRLBs) for online, i.e., filtering, and offline, i.e., smoothing, estimation of phase noise over the length of a frame is also derived.

II. OFDM SYSTEM FOR BLIND ESTIMATION

In OFDM systems, the serial data are converted into M parallel streams. Each parallel data stream modulates a different carrier. The frequency separation between the adjacent carriers is 1/T, where T is the symbol duration for the parallel data that is M times of the symbol duration for the serial data. Let us consider an OFDM signal in the interval (nT,(n+1)T) as [6]:

\[ s(t) = \sum_{m=0}^{M-1} a_{m} e^{j2\pi f_{m}t} \]  

(1)

Where \( a_{m}(n) \) are symbols resulting from a modulation constellation like 16 QAM, \( f_{m} \) is the frequency of \( m \)th carrier that is \( \frac{2\pi f_{m}}{T} \), and the \( M \) samples that are sampled at \( t = nT + \left( i \times \frac{1}{T} \right) \) with \( i = 0,1,2...,M-1 \) as follows:

\[ s(nM + i) = \sum_{m=0}^{M-1} a_{m} e^{j2\pi f_{m}i} \]  

(2)

From this equation, the M samples can be seen as the inverse discrete Fourier transform (IDFT) of a block for M input symbols. Theoretically speaking, when the number of carriers is large enough, symbol duration \( T \) is much larger than the duration of FIR channel; IS1 is negligible. However, for the high-bit-rate communications, it is impractical to choose very large \( M \) to make ISI negligible. Therefore, a cyclic prefix of length \( P \) is added into each block of IDFT output at the transmitter. The length of the prefix is chosen to be longer than the length of the channel impulse response in order to avoid inter-block interference (IBI). That results with total cancellation of IS1 and inter carrier interference (ICI). The input data will be as follow:

\[ s(n(M + P) + i) = \sum_{m=0}^{M-1} a_{m} e^{j2\pi f_{m}(i-P)} \]  

(3)

Where \( S(n(M + P) + i) \) denotes the cyclic prefix.

III. CRAM’ER- RAO LOWER BOUNDS AND BAYESIAN CRAM’ER- RAO LOWER BOUND ESTIMATORS

The model adopted in this case is very near to the classical system model that is consists of \( N \) sub-carriers and a cyclic prefix of length \( N_{p} \). Supposing that \( T_{s} \) denotes the sampling time and \( v = N + N_{p} \), the duration of OFDM symbol will be \( T = vT_{s} \). In the case of a transmission over a multipath Rayleigh channel and for a \( n \)th transmitted OFDM symbol given as

\[ x_{(n)} = \begin{bmatrix} x_{(n)}[\left[ \frac{-N}{2} \right]], x_{(n)}[\left[ \frac{-N}{2} + 1 \right]], \ldots, x_{(n)}[\left[ \frac{N}{2} - 1 \right]] \end{bmatrix}^{T} \]  

(4)

Where (\( x_{(n)}[\left[ b \right]\) are modulated 4-QAM or 16 QAM) and the \( n \)th received OFDM symbol
The system used for estimating correlated with respect to each other. They are wide sense sed as a ratio between the input data sequence and the output. The LMS estimator is used mainly for the tracking of the channel and is usually clustered with an equalizer or a decision and the following values are calculated based on the previous estimation LMS estimator uses one tap LMS adaptive filter at each pilot frequency. The first value is found directly through LS channel estimation, the LS estimator will be expres

\[ y_{(n)} = H_{(n)} x_{(n)} + w_{(n)} \]

Where \( W(n) \) \( N \times 1 \) zero-mean complex Gaussian noise vector with covariance matrix \( \sigma^2 I_N \) and \( H_{(n)} \) is a \( N \times N \) diagonal matrix with diagonal elements given by:

\[ \frac{\sum_{i=1}^{L} \alpha_i(n) x_i e^{-j2\pi \left( \frac{k-1}{N} - \frac{1}{2} \right) \pi t}}{\sum_{i=1}^{L} \alpha_i^2} \]

L is the total number of propagation paths; \( \alpha_i(n) \) is the i\text{th} complex gain of variance with \( \sum_{i=1}^{L} \alpha_i^2 = 1 \), \( \sigma_{\alpha_i}^2 = 1 \). The L individual elements are uncorrelated with respect to each other. They are wide sense stationary narrowband complex Gaussian processes, with the so-called Jakes’ power spectrum with Doppler frequency \( f_d \). It means that \( \alpha_i(n) \) are correlated complex Gaussian variables with zero means and correlation coefficients given by:

\[ R_{\alpha_i} = E \left[ \alpha_i(n) \alpha_i(n+m) \right] = \sigma_{\alpha_i}^2 \delta_{mn} \]

Then, the observation model for the nth OFDM symbol can be re-written as

\[ y_{(n)} = \text{diag} \{ x_{(n)} \} F a_{(n)} + \omega_{(n)} \]

Where \( a_{(n)} = [a_{1}^{(n)}, \ldots, a_{L}^{(n)}]^{T} \) is \( L \times 1 \) vector and \( F \) is the \( N \times L \) Fourier matrix. In order to find benchmarks for the performance of phase noise estimators over the length of a frame, we have derived a Bayesian Cramer–Rao Lower Bounds (BCRlb) for offline and online estimation of time varying phase noise in an MIMO system. The BCRLB assumes no knowledge of the transmitted symbols, and can be derived by taking into account the a priori distribution of the transmitted symbols. Thus, to evaluate the BCRLB, the likelihood function \( p(y|\theta) \) is achieved by averaging out \( p(y|\theta, s) \) over a priori distribution of s. Accordingly, assuming all elements of \( A \) are equiprobable, the log likelihood function is given by

\[ \log p(y(k)|\theta) = \log \lambda \sum_{i=0}^{L-1} \frac{1}{y(\omega)} e^{\exp \left[ -\frac{||y(k) - e^{[2j\pi \omega]k} H \theta(k) q(k)||^2}{2\sigma^2} \right]} \]

The estimates are calculated using both analytical and numerical methods. A 2 x 2 MIMO system used for estimating the phase noise variation.

IV. RESULTS AND DISCUSSIONS

The LS estimator is shown to be the basic algorithm and gives regular results used with practically all the schemes of channel estimation, the LS estimator will be expressed as a ratio between the input data sequence and the output. The LMS estimator uses one tap LMS adaptive filter at each pilot frequency. The first value is found directly through LS and the following values are calculated based on the previous estimation and the current channel output. The LMS estimator is used mainly for the tracking of the channel and is usually clustered with an equalizer or a decision feedback equalizer. The LMMSE estimator is of considerable complexity, since a matrix inversion is needed every time
the training data in exchanges. The Fig 1 gives the plot of different channel estimators typically used in OFDM systems.

![Graph showing different channel estimation techniques](image)

**Fig. 1 Simulation of Different Channel Estimation Techniques**

In Fig 1 the legend TD LMMSE refer to a time domain channel estimation technique, TDD LMMSE refer to the technique in which the channel covariance is ignored in the estimation and TD Qabs LMMSE refers to estimation in which the smoothing matrix is involved. LMMSE techniques are complex and computationally intensive TDD LMMSE and TD Qabs LMMSE reduces the complexity and computational time.

Maximum Likelihood estimation (MLE) is an important tool in determining the actual probabilities of the assumed model of communication. Maximum likelihood estimation is a method to determine these unknown parameters associated with the corresponding chosen models of the communication channel. The Cramer-Rao Lower Bound is widely used in statistical signal processing as a benchmark to evaluate unbiased estimators. In this work we have derived a CRLB for NDA estimation of SNR. Fig 2 gives the MSE performance of SNR for ML estimator and the designed CRLB estimator.

![Graph showing MSE Performance versus SNR for ML and CRLB Estimators](image)

**Fig. 2 MSE Performance versus SNR for ML and CRLB Estimators**
It can be observed from Fig 2 that the performance of both the estimators almost converge at high SNR but at low SNR the mse performance of CRLB is much better than ML estimator. This makes it possible the deployment of CRLB estimators for highly sensitive MIMO OFDM systems. Fig 3 gives the MSE performance of SNR estimation by subspace blind channel estimation and the proposed CRLB based estimator.

Bayesian Cramer-Rao Lower Bounds (CRLB) is derived for analyzing and estimating phase noise. The offline and online BCRLBs are evaluated analytically using the closed-form expressions in [12] and [14]. The online (filtering) BCRLB is plotted for frame length K=20 and offline (smoothening) BCRLB is plotted for frame lengths K=[3; 5; 10; 15; 20]. Similarly the offline and online BCRLBs are evaluated by numerically evaluating the expectation of the elements of the Hessian matrix, or 1000 Monte-Carlo simulations. The online BCRLB is plotted for frame length K=20 and offline BCRLB is plotted for frame lengths K=[3; 5; 10; 15; 20]
Fig. 5 Online and offline BCRLB versus the number of observations for 2 * 2 MIMO system with SNR = 5 dB using Analytical Approach

Fig 4 and Fig 5 show the Plot of the offline and online BCRLBs for frame lengths of K = [3; 5; 10; 15; 20] and a 2 x 2 MIMO system with an SNR of 1=2x2 w = 5 dB. The results in Fig 4 and Fig 5 reveal that the minimum and maximum values of the offline BCRLB are achieved at the middle and end-points of a frame, respectively, for any frame length, K. This implies that the best phase noise estimate can be achieved for the middle symbol within the frame, whereas the estimates get poorer as one move to the boundary points. This behaviour is expected since the phase noise for the symbol in the middle of the frame is followed by the largest number of past and future symbols with highly correlated phase noise values. Thus, by exploiting the observed symbols and correlations, the phase noise values corresponding to the middle symbol can be estimated with the highest accuracy. This can be observed from the time dependency of the BCRLB. Moreover Fig 4 and Fig 5 shows that the online BCRLB decreases with increasing observation length, K, since the longer the length of the observation sequence the more information is available for estimation of the kth symbol’s phase noise

V. CONCLUSIONS

Different types of channel estimators are simulated and studied for performance. It can be observed that the Cram’er-Rao Lower Bound (CRLB) estimator has superior performance in comparison to ML estimator especially in low SNR scenario. Even though the performance of ML estimator is comparable to that of Cram er-Rao Lower Bound (CRLB) it can be concluded that in a blind channel estimation scenario CRLB gives us the best possible estimation of SNR. Similarly it can also be observed that CRLB maintains its superior performance over another blind channel estimation technique namely subspace coding across the entire spectrum of SNR from low to high. Also in this Bayesian Cram’er-Rao Lower Bound estimator for estimating phase noise which is very crucial in avoiding signal deterioration and improving signal quality is derived. The performance of BCRLB for both online and offline for 2* 2 MIMO System using both analytical and numerical approaches is evaluated. It can be concluded that a high degree of estimate can be achieved for phase noise of middle symbol within a frame and the estimates get poorer as moved towards the boundary.

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A Robust Hybrid NDA Estimation Technique for SNR Estimation in OFDM System

Sivanagaraju V., Siddaiah P.

Abstract – Channel Estimation techniques play a very crucial role in characterizing the effect of the physical medium on the input sequence. Orthogonal Frequency division multiplexing (OFDM) Systems are especially suited for channel estimation. In this paper we have proposed a novel robust blind channel estimation technique. This technique is developed on the basis of the Envelope Based Estimator. A hybrid approach based on EM and simpler moments-based techniques is adopted. Modified moments based estimator is designed for this proposed work. In the adopted hybrid approach the SNR is estimated by the moments based estimators up to a certain threshold point, ρ0. EM estimator is used to estimate SNR greater than ρ0. Since the Phase Noise is a time varying process which changes the symbol from time to time and has a high deteriorating effect on higher order OFDM system. We have derived a Bayesian Cramér-Rao Lower Bound Estimator for estimating phase noise. Copyright © 2015 Praise Worthy Prize S.r.l. - All rights reserved.

Keywords: Channel Estimation, OFDM, Phase Noise, SNR

I. Introduction

Futuristic and advanced communication systems and standards like 3GPP Long Term Evolution (LTE/LTE-A), Worldwide Interoperability for Microwave Access (WiMAX) and high speed Wireless Local Area Network (WLAN) standards have cutting edge physical layer technologies in the form of Orthogonal Frequency Division Multiplexing (OFDM) and Multiple-Input Multiple-Output (MIMO).

Data rates in excess of 100 Mbps can be supported by these 4G systems through MIMO and dynamic carrier aggregation bringing to for a multitude of applications in the wireless environment. Providing seamless mobility across future WLAN and cellular networks with emphasis towards fixed mobile convergence have driven the development and deployment of OFDM compatible technologies and applications. Through multiplexing and diversity gains of the overall system the Multiple-input multiple-output (MIMO) technology can be used to improve the throughput and reliability of the wireless communication links [1][2]. The stringent requirement of maintaining high spectral efficiencies and throughput makes these MIMO systems effective and suitable means for achieving them [3]. In spite of all these things the phase noises severely impair the performance of the MIMO systems [4]. Accurate Signal to Noise Ratio (SNR) estimate is required for measuring the channel quality for adaptive modulation schemes as well as for soft decoding procedures as shown in [5]-[7]. In addition to low-complexity requirement, it is essential to assess the truthfulness of SNR estimators in term of their statistical variances.

For this purpose, the well-known Cramér-Rao Lower Bounds (CRLB) is a prominent benchmark to evaluate the statistical variance performance of unbiased estimators. Several OFDM SNR estimators have already been suggested in the literature but most of them are based on the pilot symbol knowledge.

This kind of estimation method is commonly classified as Data-Aided (DA). DA algorithms proved their efficiency but cannot be applied in every context.

For instance, in some cognitive radio applications, terminals need to sense the link quality with all the surrounding networks to decide which one is the most appropriate to communicate with.

A cognitive network cannot accept any delay in having frames synchronized with each network to make a decision that which network is most suited to its needs. Non Data Aided (NDA) SNR estimator for OFDM systems were introduced to overcome the limitation of the Data Aided (DA) estimation methods.

The Algorithms for SNR estimation can be classified according to their operational conditions: a) data-aided (DA) vs. Non data-aided (NDA), b) coherent vs. envelope-based (EVB), c) over sampled vs. baud-rate sampled. The NDA methods like moments based schemes are applied when the pilot symbols are not available. They fall under the category of Envelope based schemes which are highly robust towards carrier induced phase uncertainties. NDA EVB methods are of particular interest because they need no training symbols and carrier phase reference. Actually both data aided (DA) and non-data aided (NDA) trends are considered for either performance bounds derivation or estimation algorithms.

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Data aided approach, which relies on the transmission of known data streams such as training sequences and also pilot symbols, should expedite and ease the estimation process. Unfortunately, this approach limits the system through-put in the sense that adding known pilot symbols to the data stream should drop down the spectral efficiency of the communication system.

Hence NDA SNR estimation approach receives substantial attention in recent literature. CRLB for NDA SNR estimation is derived in [8] from both Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK) modulated signals with Additive white Gaussian noise (AWGN) channel. Derived bounds are compared to those obtained for DA estimation. In [9], a straightforward approximation of the CRLB for NDA SNR estimation from BPSK modulated signals over AWGN channel is presented in efficient form that avoids tedious numerical integration.

Authors in [10], derive a lower bound for SNR estimation from general one/two dimensional modulation signals with axis/half plane symmetry over AWGN channel. Exact analytical CRLB of unbiased NDA SNR estimation from square QAM signals using I/Q received signal model is addressed in [11], where a generalization of the elegant CRLB expressions presented in is also introduced.

The primary contribution of this paper is a new hybrid method which Expected Maximum (EM) and simpler moments based method. This method is proposed to be non data aided making it suitable for blind channel estimation. The main goal is to bring the performance of the proposed method to a close proximity with that of the CRLB based estimators.

The method exploits the performance characteristics of moment based estimators which provide a good performance in the high SNR region and the EM based estimators which provide a better performance in the low SNR region. Having a hybrid method which utilizes both the above methods have resulted in having good estimation performance at both high and low SNR values in comparison to the CRLB based estimation method.

The robustness of the proposed approach is compared with that of subspace and CRLB based methods by performing a BER (Bit Error Rate) analysis.

II. Problem Statement

Survey of literature points to the fact that although solutions based on Continuous-time or over sampled signals can be found in literature most SNR estimators in the literature assume the symbol timing to be established.

The significance of the envelope-based (EVB) estimators can be appreciated from the fact that they only make use of the received signal magnitude and thus can be applied even if the carrier phase has not been completely acquired. This is significant in applications like SNR estimation where it has to be estimated even when the value is low that it precludes accurate synchronization and decoding.

The EVB maximum-likelihood (ML) SNR estimator requires the numerical solution of a set of nonlinear equations derived from the related likelihood function; the computational complexity motivates the development of the EVB expectation-maximization (EM) algorithm.

In this paper we have proposed a new a NDA SNR estimation method, which is developed on the basis of the Envelope Based Estimator. We have adopted a hybrid approach based on EM and simpler moments-based techniques capable of performing CRLB (Cramér-Rao lower bound) over a wide SNR range. Similarly we have derived a Bayesian Cramér-Rao Lower Bound Estimator for estimating phase noise. In OFDM systems, the serial data are converted into M parallel streams and each parallel data stream modulates a different carrier.

The frequency separation between the adjacent carriers is 1/T, where T is the symbol duration for the parallel data that is M times of the symbol duration for the serial data. Let us consider an OFDM signal in the interval (aT, (n+1)T) as [6]:

$$s(t) = \sum_{m=0}^{M-1} a_m(n) e^{j\omega_m t}$$

where \(a_m(n)\) are symbols resulting from a modulation constellation like 16 QAM. \(\omega_m\) is the frequency of \(m^{th}\) carrier that is \(\frac{2\pi}{T}\), and the \(M\) samples that are sampled at \(t = nT + (i* 1/T)\) with \(i = 0, 1, 2, \ldots, M-1\) as follows:

$$s(nM + i) = \sum_{m=0}^{M-1} a_m(n) e^{j\frac{2\pi}{T} mi}$$

A cyclic prefix of length \(P\) is added into each block of IDFT output at the transmitter. The length of the prefix is chosen to be longer than the length of the channel impulse response in order to avoid inter-block interference (IBI).

That results with total cancellation of IS1 and inter carrier interference (ICI). The input data will be as follow:

$$s(n(M + P) + i) = \sum_{m=0}^{M-1} a_m(n) e^{j\frac{2\pi}{T} (i-P)}$$

where \(s(n(M+P) + i)\) denotes the cyclic prefix.

III. Cramér-Rao Lower Bound Estimator for NDA Estimation

The model adopted in this case is very near to the classical system model that consists of N sub-carriers and a cyclic prefix of length \(N_c\). Supposing that \(T_c\) denotes the sampling time and \(v = N + N_c\), the duration of OFDM symbol will be \(T = v T_c\). In the case of a transmission over a multipath Rayleigh channel and for an \(n^{th}\) transmitted OFDM symbol given as:
where \( \{ x(n)[b] \} \) are modulated 4-QAM or 16 QAM) and the \( n \)th received OFDM symbol:

\[
y_n = [y_n \left( \frac{N}{2} \right), y_n \left( \frac{N}{2} + 1 \right), \ldots, y_n \left( 1 \right)]^T
\]

(5)

\[
y_{(n)} = H(n)x_{(n)} + w_{(n)}
\]

where \( W(n) \) \( N \times 1 \) zero-mean complex Gaussian noise vector with covariance matrix \( \sigma^2 I_N \) and \( H(n) \) is a \( N \times N \) diagonal matrix with diagonal elements given by:

\[
[H(n)]_{k,k} = \frac{1}{n} \sum_{l=1}^{L} [a_l^{(n)}] \times e^{-j2\pi \left( \frac{k-1}{N} - \frac{1}{2} \right) n l}
\]

(7)

\( L \) is the total number of propagation paths; \( a_l \) is the \( l \)th complex gain of variance with \( \sum_{l=1}^{L} a_l^2 = 1 \).

The \( L \) individual elements are uncorrelated with respect to each other. They are wide sense stationary narrowband complex Gaussian processes, with the so-called Jakes’ power spectrum with Doppler frequency \( f_d \).

It means that \( a_l \) are correlated complex Gaussian variables with zero means and correlation coefficients given by:

\[
R_{kl} = E \left[ a_l^{(n)} a_{(n-p)}^{(n)} \right] = \sigma^2 a_l f_0 \left( 2\pi f_d TK \right)
\]

(8)

Then, the observation model for the \( n \)th OFDM symbol can be re-written as:

\[
y_{(n)} = diag(x_{(n)}) F a_{(n)} + \omega_{(n)}
\]

(9)

where \( a_{(n)} = [a_1^{(n)}, \ldots, a_L^{(n)}] \) is \( L \times 1 \) vector and \( F \) is the \( N \times L \) Fourier matrix.

IV. Proposed NDA SNR Estimation and Phase Noise Estimation

The high SNR approximation results in a significant bias in the estimates of EM estimator in the low SNR region. The moments based estimators perform better at this region. Hence, estimators are proposed to overcome the limitations. The SNR is estimated by the moments based estimators up to a certain threshold point, \( \rho_0 \). EM estimator is used to estimate SNR greater than \( \rho_0 \).

Similarly a modification method is proposed to reduce the bias of conventional first and second order moments based SNR estimator. Although the moment orders can be any two different values, only the second and fourth order moments based estimator (M2M4) has a close form solution as we know.

The M2M4 algorithm can be expressed as:

\[
\rho_{2,4} = \frac{-2\lambda_{2,4} + 1 - \sqrt{2\lambda_{2,4}^2 - \lambda_{2,4}}}{\lambda_{2,4} - 1}
\]

(10)

The expression of the first and second moments based estimator is given by:

\[
\rho_{1,2} = f_{1,2}^{-1}(\lambda_{1,2})
\]

(11)

where \( f_{1,2}^{-1} = M / M \), and \( I_m(\cdot) \) is Bessel function of the first kind with the order \( m \):

\[
\lambda_{1,2} = f_{1,2}(\rho) = \frac{\pi e^{-\rho}}{4(1 + \rho)} \left[ (1 + \rho) I_{0} \left( \frac{\rho}{2} \right) + \rho I_{1} \left( \frac{\rho}{2} \right) \right]^2
\]

(12)

In practice, the first, second and fourth order moments are estimated by their respective time averages as:

\[
\hat{\lambda}_{2,4} = \frac{M_2}{M_4} = \frac{\frac{1}{N} \sum_{n=0}^{N-1} u_n^2}{\frac{1}{N} \sum_{n=0}^{N-1} u_n^4}
\]

\[
\hat{\lambda}_{1,2} = \frac{M_1}{M_2} = \frac{\frac{1}{N} \sum_{n=0}^{N-1} u_n}{\frac{1}{N} \sum_{n=0}^{N-1} u_n^2}
\]

(13)

(14)

where \( N \) is the observation length. As \( u_n \) is known in the receiver, the SNR of the received samples can be estimated utilizing above two statistics with equations (10) and (11). All of above mentioned algorithms use these statistics to replace corresponding parameters directly. However, we find that there is bias for the \( M_1 M_2 \) estimator especially when the observation length is short.

Let’s evaluate the expectation of \( 2 M \), which can be expressed as:

\[
E[M_2] = E \left[ \frac{1}{N} \sum_{n=0}^{N-1} u_n^2 \right]
\]

\[
= E \left[ \frac{1}{N^2} \sum_{n=0}^{N-1} u_n^2 \right] + E \left[ \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0; m \neq n}^{N-1} u_n u_m \right]
\]

(15)

If \( n \) is not equal to \( m \), \( u_n \) is independent of \( u_m \). So we can further express (14) as:

\[
E[M_2] = \frac{1}{N^2} \sum_{n=0}^{N-1} E[u_n^2] + \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0; m \neq n}^{N-1} E[u_n u_m] = \frac{1}{N} M_2 + \frac{N-1}{N} M_1^2
\]

(16)
Ignoring the effect of the divider in (13), the expectation of the statistic 1, 2 can be expressed using (15) and (16) with a reduced bias as:

\[
\tilde{\lambda}_{1,2} = \frac{N}{N-1} \lambda_{1,2} - \frac{1}{N-1}
\]  

(17)

EM estimation is an iterative procedure, the convergence tolerance shall be set to a constant value, \( \tau \ll 1 \) and let \( m \) be the maximum number of iterations.

The joint pdf of the elements of \( RN \) conditioned on the elements of \( CN \) is given as:

\[
\rho(R_{N} | C_{N}) = \prod_{k=0}^{N-1} \frac{1}{2 \sigma^2} e^{-\frac{r_k^2}{2 \sigma^2}} I_0 \left( \frac{\sqrt{r_k A C_k}}{\sigma} \right)
\]  

(18)

The log likelihood of \( p(R_{N} | C_{N}) \) is given as:

\[
\Lambda_{EM} (R_{N}; A; \sigma^2) = -N \ln(\sigma^2) + \left( -1 \frac{1}{2 \sigma^2} \sum_{k=0}^{N-1} r_k^2 + A^2 \sum_{k=0}^{N-1} c_k^2 \right) + \sum_{k=0}^{N-1} \ln \left( I_0 \left( \frac{\sqrt{r_k A C_k}}{\sigma} \right) \right)
\]  

(19)

The BCRLB assumes no knowledge of the transmitted symbols, and can be derived by taking into account the a priori distribution of the transmitted symbols.

Thus, to evaluate the BCRLB, the likelihood function \( p(y|\emptyset) \) is achieved by averaging out \( p(y|\emptyset, S) \) over a priori distribution of \( S \). Accordingly, assuming all elements of \( A \) are equiprobable, the log likelihood function is given by:

\[
\log p(y(k)|\emptyset(k)) = \log \sum_{i=0}^{S-1} \frac{1}{5(\pi \sigma_c^2)^5} e^{-\frac{||y(k) - \hat{y}^{[i]}(k)H\hat{\theta}^{[i]}(k)c_i||^2}{\sigma_c^2 \omega}}
\]  

(20)

The estimates are calculated using both analytical and numerical methods. A 2 × 2 MIMO system is used for estimating the phase noise variation.

V. Results and Discussion

Bayesian Cramér-Rao Lower Bounds (BCRLB) is derived for analysing and estimating phase noise. The offline and online BCRLBs are evaluated analytically using the closed-form expression.

The online (filtering) BCRLB is plotted for frame length \( K=20 \) and offline (smoothening) BCRLB is plotted for frame lengths \( K=[3; 5; 10; 15; 20] \). Fig. 1 and Fig. 2 show the plot of the offline and online BCRLBs for frame lengths of \( K=[3; 5; 10; 15; 20] \) and a 2 × 2 MIMO system with an SNR of 1=2×2 w = 5 dB.

The online BCRLB is plotted for frame length \( K=20 \) and offline BCRLB is plotted for frame lengths \( K=[3; 5; 10; 15; 20] \). Fig. 1 and Fig. 2 show the plot of the offline and online BCRLBs for frame lengths of \( K=[3; 5; 10; 15; 20] \) and a 2 × 2 MIMO system with an SNR of 1=2×2 w = 5 dB.

The results in Fig. 1 and Fig. 2 reveal that the minimum and maximum values of the offline BCRLB are achieved at the middle and end-points of a frame, respectively, for any frame length, \( K \). This implies that the best phase noise estimate can be achieved for the middle symbol within the frame, whereas the estimates get poorer as one move to the boundary points.

This behaviour is expected since the phase noise for the symbol in the middle of the frame is followed by the largest number of past and future symbols with highly correlated phase noise values.

Thus, by exploiting the observed symbols and correlations, the phase noise values corresponding to the middle symbol can be estimated with the highest accuracy. This can be observed from the time dependency of the BCRLB.
Moreover Fig. 1 and Fig. 2 show that the online BCRLB decreases with increasing observation length, K, since the longer the length of the observation sequence the more information is available for estimation of the kth symbol’s phase noise. The Validity of the proposed, modified moments based estimator is tested against a theoretical SNR value. The proposed method is validated using mean square error, standard deviation of the snr values and change in the bias value in comparison to a theoretical value. The results are depicted in the Figs. 3-5. Such figures clearly indicate that the proposed method closely follows the values produced by theoretical calculations and there by validate the suitability of the proposed method in analysis.

The results of the proposed method for SNR estimation using a hybrid method is discussed in the following section. SNR estimation method is developed on the basis of the Envelope Based Estimator. We have adopted a hybrid approach based on EM and the modified simpler moments-based techniques. The results are compared with that of CRLB (Cramér-Rao lower bound) based estimator, the bench mark any lower bounds based estimation.

Maximum Likelihood estimation (MLE) is an important tool in determining the actual probabilities of the assumed model of communication. Maximum likelihood estimation is a method to determine these unknown parameters associated with the corresponding chosen models of the communication channel. The Cramér-Rao Lower Bound is widely used in statistical signal processing as a benchmark to evaluate unbiased estimators. It can be observed from Fig. 6 that the performance of both the estimators almost converge at high SNR but at low SNR the mse performance of CRLB is much better than ML estimator.

This makes it possible the deployment of CRLB estimators for highly sensitive MIMO OFDM systems. Fig. 7 gives the MSE performance of SNR estimation by subspace based channel estimation and the CRLB based estimator. From Fig. 6 and Fig. 7 it can be observed that the proposed method performs better than the ML Data Aided Estimator and Subspace based Blind Channel Estimator. Such figures clearly show that the performance of the proposed hybrid estimator is better compared to these techniques at both high and low SNR values. From Fig. 8 it can be observed that the BER performance based on the proposed estimation algorithm outperforms the subspace based channel estimation over all the SNR range considered and provides BER performance close to that of CRLB. The reason is that the more accurate channel estimate obtained by the proposed algorithm can provide more accurate phase information about the channel fading.

More accurate phase information can provide better coherent detection performance and increased robustness.

At high SNR, the BER characteristics of both the estimation algorithms are close to each other because of the fact at high SNR, both estimation algorithms can achieve nearly the same channel estimation accuracy.

The results validate the fact that the proposed estimator closely follows and converges with estimate as given by the CRLB based estimator. It can be concluded

Fig. 3. Mean Square Error plot of theoretical SNR and SNR calculated using the Modified Method

Fig. 4. Standard Deviation plot of theoretical SNR and SNR calculated using the Modified Method

Fig. 5. Plot of Normalised bias for theoretical SNR and SNR calculated using the Modified Method
that the proposed estimator is capable of giving a performance as close to that of CRLB.

![Graph showing MSE Performance versus SNR for ML, CRLB Estimator and the Proposed Estimator](image)

**Fig. 6.** MSE Performance versus SNR for ML, CRLB Estimator and the Proposed Estimator

![Graph showing MSE Performance versus SNR for Subspace, CRLB Estimator and the Proposed Estimator](image)

**Fig. 7.** MSE Performance versus SNR for Subspace, CRLB Estimator and the Proposed Estimator

![Graph showing BER Performance for Subspace, CRLB Estimator and the Proposed Estimator](image)

**Fig. 8.** BER Performance for Subspace, CRLB Estimator and the Proposed Estimator

### VI. Conclusion

In this paper, a NDA SNR estimation method is developed on the basis of the Envelope Based Estimator. We have adopted a hybrid approach based on EM and simpler moments-based techniques capable of performing CRLB (Cramér-Rao lower bound) over a wide SNR range. Similarly we have derived a Bayesian Cramér-Rao Lower Bound Estimator for estimating phase noise. It can be concluded from the results that the proposed estimator performs much better when compared to both ML and Subspace based channel estimation techniques. The performance of the proposed estimator is uniform across the entire spectrum of SNR and closely follows the performance of CRLB estimator.

Also in this work Bayesian Cramér-Rao Lower Bound estimator for estimating phase noise which is very crucial in avoiding signal deterioration and improving signal quality is derived.

The performance of BCRLB for both online and offline for 2×2 MIMO System using both analytical and numerical approaches is evaluated. It can be concluded that a high degree of estimate can be achieved for phase noise of middle symbol within a frame and the estimates get poorer as moved towards the boundary.

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### References


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