CHAPTER 5: Proposed Method

In this chapter a detailed insight into the research problem and how it is approach is provided, this chapter explains the problem formulation and its mathematical validation. This chapter provides a clear understanding about the proposed work and its significance which is demonstrated through the results explained in chapter 8.

5.1 Introduction:

Multiple-input multiple-output (MIMO) technology can be utilized to enhance the throughput and reliability of wireless communication links by introducing multiplexing and diversity gains to the overall system [123], [107]. Traditional single antenna transmission techniques pointing a maximum wireless system performance act at both the time domain and/or in the frequency domain. To overcome the adverse effects of multipath fading, typically channel coding is employed. Spatial domain can be exploited by using multiple antennas. For modern wireless communications multiple antenna techniques constitute a key technology. The advantages of multiple antennas are listed in [123]. As a result; MIMO systems are an effective means to meet the stringent requirements on today’s wireless communication systems that demand higher spectral efficiencies and throughputs [124]. On the other hand, phase noise severely deteriorates the performance of MIMO systems [125]. Accurate SNR estimate is required for measuring the channel quality for adaptive modulation schemes as well as for soft decoding procedures as shown in [126], [127] and [128]. In addition to low-complexity requirement, it is essential to assess the truthfulness of SNR estimators in term of their
statistical variances. For this purpose, the well-known CRLB is a prominent benchmark to evaluate the statistical variance performance of unbiased estimators.

Actually both data aided (DA) and non-data aided (NDA) trends are considered for either performance bounds derivation or estimation algorithms. Data aided approach, which relies on the transmission of known data streams such as training sequences and also pilot symbols, should expedite and ease the estimation process. Unfortunately, this approach limits the system throughput in the sense that adding known pilot symbols to the data stream should drop down the spectral efficiency of the communication system. Hence NDA SNR estimation approach receives substantial attention in recent literature. CRLB for NDA SNR estimation is derived in [12] from both BPSK and QPSK modulated signals with AWGN channel. Derived bounds are compared to those obtained for DA estimation. In [13], a straightforward approximation of the CRLB for NDA SNR estimation from BPSK modulated signals over AWGN channel is presented in efficient form that avoids tedious numerical integration. Authors, in [14], derive a lower bound for SNR estimation from general one/two dimensional modulation signals with axis/half plane symmetry over AWGN channel. Exact analytical CLRB of unbiased NDA SNR estimation from square QAM signals using I/Q received signal model is addressed in [15], where a generalization of the elegant CLRB expressions presented in [12] is also introduced.

In communication systems, analog components severely cause non linearity at high SNR’s. A huge level of phase noise is caused by cheap RF oscillators. Due to the maximum phase noise sensitivity, the communication system suffers severe performance degradation. The frequency synchronization is dominated by the rectification of the
oscillator’s phase noise. Phase noise is a time varying process that changes from symbol to symbol [129]–[131]. Moreover, the deteriorating effect of phase noise may be more severe in MIMO systems employing higher order modulations, given that in MIMO systems, independent oscillators may be used at each transmit and receive antenna resulting in multiple phase noise processes that need to be jointly estimated at the MIMO receiver [130]. In receiver, the phase noise correlation is exploited in time domain can be estimated and compensated. When dealing with phase noise in the MIMO system with each antenna having separate circuitry for carrier sequence the assignment is broadened to estimation and compensated [131].

The use of independent oscillators at each transmit and each receive antenna is well motivated in applications where antennas need to be placed far apart from one another, e.g., in the case of line-of-sight (LOS) MIMO systems. As a result, even though Cramér-Rao lower bounds (CRLBs) and algorithms for estimation of phase noise in single-input single-output (SISO) systems have been extensively and thoroughly studied in [129], these results cannot be applied to the case of MIMO systems. Similarly, phase locked loops that can be used in SISO systems for phase noise tracking, cannot be applied in LOS-MIMO systems where multiple phase noise parameters need to be tracked simultaneously at the receiver [14].

5.2 The Proposed NDA SNR Estimation:

Survey of literature points to the fact that although solutions based on Continuous-time or over sampled signals can be found in literature most SNR estimators in the literature assume the symbol timing to be established. The significance of the
envelope-based (EVB) estimators can be appreciated from the fact that they only make use of the received signal magnitude and thus can be applied even if the carrier phase has not been completely acquired. This is significant in applications like SNR estimation where it has to be estimated even when the value is low that it precludes accurate synchronization and decoding. The EVB maximum-likelihood (ML) SNR estimator requires the numerical solution of a set of nonlinear equations derived from the related likelihood function; the computational complexity motivates the development of the EVB expectation-maximization (EM) algorithm.

In this work we have proposed a new a robust NDA SNR estimation method, which is developed on the basis of the Envelope Based Estimator. We have adopted a hybrid approach based on EM and simpler moments-based techniques capable of performing CRLB (Cramer-Rao lower bound) over a wide SNR range. The high SNR approximation results in a significant bias in the estimates of EM estimator in the low SNR region. The moments based estimators perform better at this region. Hence, estimators are proposed to overcome the limitations. The SNR is estimated by using moments based estimators up to a certain threshold point, then after EM estimator is used to estimate the SNR. Similarly a modification method is proposed to reduce the bias of conventional first and second order moments based SNR estimator. Although the moment orders can be any two different values, only the second and fourth order moments based estimator (M2M4) has a close form solution as we know.

The received symbol-spaced samples at the matched filter output can be represented by

\[ r_n = \mathcal{A} e^{j(2\pi f_n T + \theta + 2\pi c_n/M)} + \omega_n \]  \hspace{1cm} (5.1)
\( n = 0, 1, 2 \ldots N-1 \) time index in the observation interval,

\( \mathcal{A} \) = amplitude of the transmitted signal, \( f \) = frequency of carrier, \( \theta \) = initial phase,

\( T \) = symbol space, \( M \) = modulation order, \( C_n \) = Modulating data, \( \omega_n \) = Complex white Gaussian random variable.

SNR of the received samples

\[
\rho = \frac{S}{N_0} = \frac{\mathcal{A}}{2\sigma^2}
\]  

5.2

Signal envelope \( u_n = |r_n| \)

5.3

\( u_n \) has no relationship with the phase of received samples

\[
M_k = E [u_n^k]
\]  

5.4

Estimator equation \( \lambda_{k,l} = \frac{M_k}{M_l} = f_{k,l}(\rho) \)

5.5

Which depends only on \( \rho \) but not on \( \sigma^2 \).

The envelope based estimator that depends on the \( k \)th & \( l \)th order moment.

\[
\rho = f_{k,l}^{-1}\left(\frac{M_k}{M_l}\right)
\]  

5.6

The M2M4 algorithm can be expressed as

\[
\rho_{2,4} = \frac{-2\lambda_{2,4} + 1 - \sqrt{2\lambda_{2,4}^2 - \lambda_{2,4}}}{\lambda_{2,4} - 1}
\]  

5.7
The expression of the first and second moments based estimator is given by

$$\rho_{1,2} = f_{1,2}^{-1}(\lambda_{1,2})$$ \hspace{1cm} 5.8

Where $f_{1,2}^{-1} = \frac{M_2^2}{M_4^2}$, and $Im(\cdot)$ is Bessel function of the first kind with the order $m$.

$$\lambda_{1,2} = f_{1,2}(\rho) = \frac{\pi e^{-\rho}}{4(1+\rho)} \left((1 + \rho)I_0 \left(\frac{\rho}{2}\right) + \rho I_1 \left(\frac{\rho}{2}\right)\right)^2$$ \hspace{1cm} 5.9

In practice, the first, second and fourth order moments are estimated by their respective time averages as

$$\hat{\lambda}_{2,4} = \frac{\hat{M}_2^2}{\hat{M}_4} = \frac{\frac{1}{N} \sum_{n=0}^{N-1} u_n^2}{\frac{1}{N} \sum_{n=0}^{N-1} u_n^4}$$ \hspace{1cm} 5.10

$$\hat{\lambda}_{1,2} = \frac{\hat{M}_1^2}{\hat{M}_2} = \frac{\frac{1}{N} \sum_{n=0}^{N-1} u_n}{\frac{1}{N} \sum_{n=0}^{N-1} u_n^2}$$ \hspace{1cm} 5.11

Where $N$ is the observation length. As $u_n$ is known in the receiver, the SNR of the received samples can be estimated utilizing above two statistics.

All of above mentioned algorithms use these statistics to replace corresponding parameters directly. However, we find that there is bias for the M1M2 estimator especially when the observation length is short. Let’s evaluate the expectation of $\hat{M}_1^2$, which can be expressed as

$$E[\hat{M}_1^2] = E \left[ \left(\frac{1}{N} \sum_{n=0}^{N-1} u_n\right)^2 \right]$$

$$= E \left[ \frac{1}{N^2} \sum_{n=0}^{N-1} u_n^2 \right] + E \left[ \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0, m \neq n}^{N-1} u_n u_m \right]$$ \hspace{1cm} 5.12
If \( n \) is not equal to \( m \), \( u_n \) is independent of \( u_m \) so we can further express 5.12 as

\[
E[\tilde{M}_1^2] = \frac{1}{N^2} \sum_{n=0}^{N-1} E[u_n^2] + \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0; m \neq n}^{N-1} E[u_n]E[u_m]
\]

\[
= \frac{1}{N} M_2 + \frac{N-1}{N} \frac{M_1^2}{M_2}
\]

5.13

Ignoring the effect of the divider in 5.10, the expectation of the statistic 1.2 can be expressed using 5.11 and 5.12

\[
E[\hat{\lambda}_{1,2}] = \frac{1}{N} + \frac{N-1}{N} \frac{M_1^2}{M_2}
\]

\[
= \frac{1}{N} + \frac{N-1}{N} \hat{\lambda}_{1,2}
\]

\( \hat{\lambda}_{1,2} \) is a biased estimation of \( \lambda_{1,2} \)

With a reduced bias as

\[
\tilde{\lambda}_{1,2} = \frac{N}{N-1} \hat{\lambda}_{1,2} - \frac{1}{N-1}
\]

5.14

EM estimation is an iterative procedure, the convergence tolerance shall be set to a constant value, \( \tau \ll 1 \) and let \( m \) be the maximum number of iterations.

\( C_N \) = Unknown data sequence / transmitted binary sequence.

\( R_N \) = Elements \([r_0r_1 ... r_{N-1}]^T\) represent the received baseband signal sequence.

The EM algorithm involves four steps:

Step 1: calculation of expectation
Find the expectation of $\Lambda_{EM}(R_N ; \mathcal{A}; \sigma^2)$ w.r.t $C_k$ as

$$\psi(R_N ; \mathcal{A}; \sigma^2) = E_{C_k}[\Lambda_{EM}(R_N ; \mathcal{A}; \sigma^2)]$$

$$= \ln(2\sigma^2) - \frac{1}{2\sigma^2} \left[ \sum_{k=0}^{N-1} r_k + \mathcal{A}^2 \sum_{k=0}^{N-1} E_{C_k}[C_k^2] \right] + \sum_{k=0}^{N-1} E_{C_k} \left[ \ln \left( I_0 \left( \frac{\sqrt{r_k} \cdot A_{C_k}}{\sigma^2} \right) \right) \right]$$

5.15

At the start of $n^{th}$ iteration

$$E_{C_k}[C_k^2] = \sum_{i=0}^{l} P_{i,k,n-1} i^2$$

$$= P_{l,k,n-1}$$

5.16

$$E_{C_k} \left[ \ln \left( I_0 \left( \frac{\sqrt{r_k} \cdot A_{C_k}}{\sigma^2} \right) \right) \right] = \sum_{i=0}^{l} P_{i,k,n-1} \ln \left( I_0 \left( \frac{\sqrt{r_k} \cdot A_{i}}{\sigma^2} \right) \right)$$

$$= P_{l,k,n-1} \ln \left( I_0 \left( \frac{\sqrt{r_k} \cdot \mathcal{A}}{\sigma^2} \right) \right)$$

5.17

Where $P_{i,k,n-1}$ is the probability of ‘$C_k = i$’ at the $(n-1)^{th}$ iteration is given by

$$P_{i,k,n-1} = P(C_k = i|r_k,A_{n-1},\sigma_{n-1})$$

$$= \frac{p(r_k|C_k=i,A_{n-1},\sigma_{n-1}) \cdot p(c_k=i)}{p(r_k|A_{n-1},\sigma_{n-1})}$$

Where $P(C_k = i) = \left\{ \begin{array}{ll} \alpha & \text{if } i = 1 \\ 1 - \alpha & \text{if } i = 0 \end{array} \right.$
Combining 5.15, 5.16 & 5.17

\[ P(r_k; A_{n-1}, \sigma_{n-1}) = \sum_{i=0}^{l} P(r_k | c_k = i, A_{n-1}, \sigma_{n-1}) P_i \]

Step 2: Maximization of expectation

Maximize \( \psi(R_N; \mathcal{A}; \sigma^2) \) w.r.t arguments \( \mathcal{A} \) & \( \sigma^2 \).

Let \( \theta_n = [A_n \sigma_n^2] \) be the value of the arguments at the end of \( n^{th} \) iteration are obtained by partially differentiating 5.18 w.r.t \( \mathcal{A} \) & \( \sigma^2 \) & equating to 0 as given below.

\[ \frac{\partial \psi(R_N; \mathcal{A}; \sigma^2)}{\partial \mathcal{A}} = \frac{-A \sum_{k=0}^{N-1} P_{l, k, n-1}}{\sigma^2} + \sum_{k=0}^{N-1} P_{l, k, n-1} \frac{l_1(\sqrt{r_k} \mathcal{A})}{l_0(\sqrt{r_k} \mathcal{A})} \frac{\sqrt{r_k}}{\sigma^2} \]

5.19

\[ \frac{\partial \psi(R_N; \mathcal{A}; \sigma^2)}{\partial \sigma^2} = \]

\[ \frac{-N}{\sigma^2} + \frac{1}{2(\sigma^2)^2} \left[ \sum_{k=0}^{N-1} r_k + \mathcal{A}^2 \sum_{K=0}^{N-1} P_{l, k, n-1} \right] - \frac{\mathcal{A}}{(\sigma^2)^2} \sum_{K=0}^{N-1} P_{l, k, n-1} \frac{l_1(\sqrt{r_k} \mathcal{A})}{l_0(\sqrt{r_k} \mathcal{A})} \sqrt{r_k} \]

5.20
After high SNR approximation, equating 5.19 & 5.20 to 0 gives.

\[
A_n = \frac{\sum_{k=0}^{N-1} r_{l,k,n-1} \sqrt{r_k}}{\sum_{k=0}^{N-1} p_{l,k,n-1}} \tag{5.21}
\]

\[
2\sigma^2 = \frac{1}{N} \left[ \sum_{k=0}^{N-1} r_k - A_n^2 \sum_{k=0}^{N-1} p_{l,k,n-1} \right] \tag{5.22}
\]

Step3:

Compute the SNR estimate \( \rho_n = \frac{A_n^2}{2\sigma_n^2} \) at the end of \( n^{th} \) iteration. If \( \tau \) or \( m \) is reached, recalibrate the noise estimation. Otherwise, continue with the iteration procedure.

Step4:

After convergence, noise is re estimated from the knowledge of the vector

\[
\mathcal{P}_{0,n} = \begin{bmatrix} p_{00,n} & p_{02,n} & \ldots & p_{0,N-1,n} \end{bmatrix}^T \quad \text{The structure of the elements of } \mathcal{P}_{0,n} \text{ is given below.}
\]

\[
\mathcal{P}_{0k,n} \approx \begin{cases} 1 & \text{if } c_k = 0 \\ 0 & \text{if } c_k = 1 \end{cases}
\]

Due to the above structure, the noise power \( 2\sigma_n^2 \) shall be recalibrated as shown below

\[
2\sigma_n^2 = \frac{\sum_{k=0}^{N-1} \mathcal{P}_{0k,n} r_k}{\sum_{k=0}^{N-1} \mathcal{P}_{0k,n}} \tag{5.23}
\]

Using 5.21 & 5.23 the SNR estimate can be evaluated

\[
\hat{\rho} = \frac{\hat{A}^2}{2\hat{\sigma}_n^2}.
\]
The joint probability density function of the elements of $RN$ conditioned on the elements of $CN$ is given as,

$$\rho(R_N|C_N) = \prod_{k=0}^{N-1} \frac{1}{2\sigma^2} e^{-\frac{r_k^2 + c_k^2}{2\sigma^2}} I_0 \left(\frac{\sqrt{r_k^2 c_k}}{\sigma^2}\right)$$  \hspace{1cm} (5.24)

The log likelihood of $p(RN|CN)$ is given as,

$$\Lambda_{EM}(R_N; A; \sigma^2) = -N \ln(\sigma^2) - \frac{1}{2\sigma^2} \left[\sum_{k=0}^{N-1} r_k + A^2 \sum_{k=0}^{N-1} c_k^2\right]$$

$$+ \sum_{k=0}^{N-1} \ln \left(I_0 \left(\frac{\sqrt{r_k c_k}}{\sigma^2}\right)\right)$$  \hspace{1cm} (5.25)