4.1. Basic Theory of Optical Thin-Film

Basic theory is necessary in order to make calculations of the properties of Multilayer thin-film coatings. A simple extension of the analysis occurs in the case of a thin, plane parallel film of material covering the surface of a substrate. The presence of two or more interfaces means that a number of beams will be produced by successive reflections and the properties of the film will be determined by the summation of these beams. It is said that the film is thin when interference effects can be detected in the reflected or transmitted light, that is, when the path difference between the beams is less than the coherence length of the light, and thick when the path difference is greater than the coherence length [1-3].

4.1.1 The Reflectance of a Thin-Film

In Figure 4.1, it is denoted waves in the direction of incidence by the symbol \( + \) (that is, positive-going) and waves in the opposite direction by \( - \) (that is, negative-going) [1].
The interface between the film and the substrate was denoted by the symbol \( b \). It is considered the tangential components of the fields. There is no negative-going wave in the substrate and the waves in the film can be added into one resultant positive-going wave and one resultant negative-going wave [1]. At this interface the tangential components of \( E \) and \( H \) are

\[
E_b = E_{1b}^+ + E_{1b}^-
\]

\[
H_b = \eta_1 E_{1b}^+ - \eta_1 E_{1b}^-
\]

(1-2)

Where it is being neglecting the common phase factors and where \( E_b \) and \( H_b \) represent the resultants. Hence
The fields at the other interface at the same instant and at a point with identical x and y coordinates can be determined by altering the phase factors of the waves to allow for a shift in the z coordinate from 0 to \(-d\). The phase factor of the positive going wave will be multiplied by \(\exp(i\delta)\) where

\[
\delta = 2\pi N_i d \cos\theta_1
\]  

and \(\theta_1\) may be complex, while negative-going phase factor will be multiplied by \(\exp(-i\delta)\) [1]. The values of E and H at the interface are using equations (3)-(6)
\[ E_{1a}^+ = E_{1b}^+ e^{i\delta} = \frac{1}{2} (H_b/\eta_l + E_b) e^{i\delta} \]
\[ E_{1a}^- = E_{1b}^- e^{-i\delta} = \frac{1}{2} (-H_b/\eta_l + E_b) e^{-i\delta} \]
\[ H_{1a}^+ = H_{1b}^+ e^{i\delta} = \frac{1}{2} (H_b + \eta_l E_b) e^{i\delta} \]
\[ H_{1a}^- = H_{1b}^- e^{-i\delta} = \frac{1}{2} (H_b - \eta_l E_b) e^{-i\delta} \]

\[ E_a = E_{1a}^+ + E_{1a}^- \]
\[ = E_b \left( \frac{e^{i\delta} + e^{-i\delta}}{2} \right) + H_b \left( \frac{e^{i\delta} - e^{-i\delta}}{2\eta_l} \right) \]
\[ = E_b \cos \delta + H_b \frac{i \sin \delta}{\eta_l} \]

\[ H_a = H_{1a}^+ + H_{1a}^- \]
\[ = E_b \eta_l \left( \frac{e^{i\delta} - e^{-i\delta}}{2} \right) + H_b \left( \frac{e^{i\delta} + e^{-i\delta}}{2} \right) \]
\[ = E_b i \eta_l \sin \delta + H_b \cos \delta \]

This can be written in matrix form

\[
\begin{bmatrix}
    E_a \\
    H_a
\end{bmatrix} =
\begin{bmatrix}
    \cos \delta & (i \sin \delta)/\eta_l \\
    i \eta_l \sin \delta & \cos \delta
\end{bmatrix}
\begin{bmatrix}
    E_b \\
    H_b
\end{bmatrix}.
\]

(8)
Since the tangential components of $E$ and $H$ are continuous across a boundary, and since there is only a positive-going wave in the substrate, this relationship connects the tangential components of $E$ and $H$ at the incident interface with the tangential components of $E$ and $H$ which are transmitted through the final interface. The $2 \times 2$ matrix on the right-hand side of equation (8) is known as the characteristic matrix of the thin film [4-6]. The input optical admittance of assembly is defined as

$$ Y = \frac{H_a}{E_a} \quad (9) $$

The reflectance of a simple interface between an incident medium of admittance $\eta_0$ and a medium of admittance $Y$ is i.e.

$$ \rho = \frac{\eta_0 - Y}{\eta_0 + Y} $$

$$ R = \left( \frac{\eta_0 - Y}{\eta_0 + Y} \right) \left( \frac{\eta_0 - Y}{\eta_0 + Y} \right)^* \quad (10) $$

Equation 8 can be written as

$$ E_a \begin{bmatrix} 1 \\ Y \end{bmatrix} = \begin{bmatrix} \cos \delta & \frac{\sin \delta}{\eta_1} \\ i \eta_1 \sin \delta & \cos \delta \end{bmatrix} \begin{bmatrix} 1 \\ \eta_2 \end{bmatrix} E_b \quad (11) $$

which gives

$$ Y = \frac{\eta_2 \cos \delta + i \eta_1 \sin \delta}{\cos \delta + i (\eta_2/\eta_1) \sin \delta} $$

Normally, $Y$ is the parameter which is of interest and the matrix product on the right hand side of equation (11) gives sufficient information for calculating it:
Where 

\[
\begin{bmatrix}
  B \\
  C
\end{bmatrix}
\]

is defined as the characteristic matrix of the assembly.

Clearly, \( Y = C/ B \).

### 4.1.2 The Reflectance of an Assembly of Thin-Films

Let another film be added to the single film so that the final interface is denoted by \( c \), as shown in Figure 4.2. The characteristic matrix of the film nearest the substrate is

\[
\begin{bmatrix}
  \cos \delta & (i \sin \delta)/\eta_i \\
  i\eta_i \sin \delta & \cos \delta
\end{bmatrix}
\]

(13)

And from equation (4), equation (13) are given by

\[
\begin{bmatrix}
  E_b \\
  H_b
\end{bmatrix} = \begin{bmatrix}
  \cos \delta & (i \sin \delta)/\eta_2 \\
  i\eta_2 \sin \delta & \cos \delta
\end{bmatrix} \begin{bmatrix}
  E_c \\
  H_c
\end{bmatrix}.
\]

\[
\begin{array}{c}
\eta_0 \\
\eta_1 \\
\eta_2 \\
\eta_3
\end{array}
\]

\[
\begin{array}{c}
\text{incident medium} \\
a \\
b \\
c
\end{array}
\]

\[
\begin{array}{c}
\text{thin films} \\
d_1 \\
d_2
\end{array}
\]

\[
\text{substrate}
\]

**Figure 4.2** Notation for two films on a surface [1]

66
It can be applied equation (4) again to give the parameters at interface a, i.e.

\[
\begin{bmatrix}
E_a \\
H_a
\end{bmatrix} = \begin{bmatrix}
\cos\delta_1 & (i\sin\delta_1)/\eta_1 \\
\eta_1 \sin\delta_1 & \cos\delta_1
\end{bmatrix} \begin{bmatrix}
\cos\delta_2 & (i\sin\delta_2)/\eta_2 \\
\eta_2 \sin\delta_2 & \cos\delta_2
\end{bmatrix} \begin{bmatrix}
E_e \\
H_e
\end{bmatrix}
\]  

(14)

and the characteristic matrix of the assembly is

\[
\begin{bmatrix}
B \\
C
\end{bmatrix} = \begin{bmatrix}
\cos\delta_1 & (i\sin\delta_1)/\eta_1 \\
\eta_1 \sin\delta_1 & \cos\delta_1
\end{bmatrix} \begin{bmatrix}
\cos\delta_2 & (i\sin\delta_2)/\eta_2 \\
\eta_2 \sin\delta_2 & \cos\delta_2
\end{bmatrix} \begin{bmatrix}
1 \\
\eta_3
\end{bmatrix}
\]  

(15)

\(Y\) is \(C/B\) and the amplitude reflection coefficient and the reflectance are, from equation (10),

\[
\rho = \frac{\eta_0 - Y}{\eta_0 + Y}
\]

and

\[
R = \left(\frac{\eta_0 - Y}{\eta_0 + Y}\right) \left(\frac{\eta_0 - Y}{\eta_0 + Y}\right)^*.
\]

This result can be immediately extended to the general case of an assembly of \(q\) layers, when the characteristic matrix is simply the product of the individual matrices taken in the correct order, that is

\[
\begin{bmatrix}
B \\
C
\end{bmatrix} = \left(\prod_{r=1}^{q} \begin{bmatrix}
\cos\delta_r & (i\sin\delta_r)/\eta_r \\
\eta_r \sin\delta_r & \cos\delta_r
\end{bmatrix}\right) \begin{bmatrix}
1 \\
\eta_z
\end{bmatrix}
\]  

(16)
i.e.

\[
\begin{bmatrix}
B \\
C
\end{bmatrix} = M \begin{bmatrix}
1 \\
\eta_s
\end{bmatrix}
\]

Where M is product matrix given by

\[
M = [M_1][M_2]...[M_s][M_b][M_c]...[M_p][M_q]
\]

And where

\[
\delta_r = \frac{2\pi N_r d_r \cos \theta_r}{\lambda}
\]

\[
\eta_r = \frac{\omega}{N_r \cos \theta_r} \quad \text{for s-polarization (TE)}
\]

\[
\eta_r = \frac{\omega}{N_r / \cos \theta_r} \quad \text{for p-polarization (TM)}
\]

and where it has been used the suffix s to denote the substrate or exit medium [1].

\[
\eta_s = \frac{\omega}{N_s \cos \theta_s} \quad \text{for s-polarization (TE)}
\]

\[
\eta_s = \frac{\omega}{N_s / \cos \theta_s} \quad \text{for p-polarization (TM)}
\]

If \( \theta_0 \), the angle of incidence, is given, the values of \( \theta_r \) can be found from Snell’s law, i.e.

\[
N_0 \sin \theta_0 = N_r \sin \theta_r = N_s \sin \theta_s.
\]  \hspace{1cm} (17)

A useful property of the characteristic matrix of a thin film is that the determined is unity. This means that the determinant of the product of any number of these matrices is also unity [1].

It avoids difficulties over signs and quadrants if, in the case of absorbing media, the scheme used for computing phase thicknesses and admittances is:
\[
\delta_r = (2\pi/\lambda) n r (n_r^2 - k_r^2 - n_0^2 \sin^2 \theta_0 - 2im\kappa, r)^{1/2}
\]

the correct solution being in the fourth quadrant. Then

\[
\eta_{rz} = \mathcal{Y} \left( n_r^2 - k_r^2 - n_0^2 \sin^2 \theta_0 - 2im\kappa, r \right)^{1/2}
\]

(18)

and

\[
\eta_{rp} = \frac{y_r^2}{\eta_{rz}} = \frac{\mathcal{Y}^2 (n_r - ik_r)^2}{\eta_{rz}}
\]

(19)

(20)

It is useful to examine the phase shift associated with the reflected beam. Let \( Y = a + ib \).

Then with \( \eta_0 \) real

\[
\rho = \frac{\eta_0 - a - ib}{\eta_0 + a + ib} = \frac{(\eta_0^2 - a^2 - b^2) - i(2b\eta_0)}{(\eta_0 + a)^2 + b^2}
\]

i.e

\[
\tan \phi = \frac{-2b\eta_0}{(\eta_0^2 - a^2 - b^2)}
\]

(21)

where \( \phi \) is the phase shift [1,5].

4.1.3 Reflectance, Transmittance and Absorptance

Sufficient information is included in equation (16) to allow the transmittance and absorptance of a thin film assembly to be calculated. To have a physical meaning, the incident medium should be transparent, that is, \( \eta_0 \) must be real. First of all, it is calculated
the net intensity at the exit side of the assembly, which it is taken as the $k^{th}$ interface. This is given by

$$I_k = \frac{1}{2} \text{Re}(E_k H_k^*)$$  \hspace{1cm} (22)$$

where it is being dealt with the component of intensity normal to the interfaces

$$I_k = \frac{1}{2} \text{Re}(E_k \eta_k^* E_k^*)$$

$$= \frac{1}{2} \text{Re}(\eta_k^*) E_k E_k^*$$  \hspace{1cm} (23)$$

If the characteristic matrix of the assembly is

$$[B]$$

$$[C]$$

then the net intensity at the entrance to the assembly is

$$I_s = \frac{1}{2} \text{Re}(BC^*) E_k E_k^*.$$  \hspace{1cm} (24)$$

Let the incident intensity be denoted by $I_i$; then equation (24) represents the intensity actually entering the assembly, which is (1-R) $I_i$

$$(1-R)I_i = \frac{1}{2} \text{Re}(BC^*) E_k E_k^*$$

i.e.

$$I_i = \frac{\text{Re}(BC^*) E_k E_k^*}{2(1-R)}$$  \hspace{1cm} (25)$$

Equation (23) represents the intensity leaving the assembly and entering the substrate and so the transmittance $T$ in equation (26) is
The absorptance $A$ in the multilayer is connected with $R$ and $T$ by the relationship

$$ R + T + A = 1 $$

So that

$$ A = 1 - R - T = (1 - R) \left(1 - \frac{\text{Re}(\eta_z)}{\text{Re}(BC^*)}\right). $$

(27)

In the absence of absorption in any of the layers it can readily be shown that the above expressions are consistent with $A = 0$ and $T + R = 1$, for the individual film matrices will have determinants of unity and the product of any number of these matrices will also have a determinant of unity. The product of the matrices can be expressed as

$$ \begin{bmatrix} a & i\beta \\ i\gamma & \delta \end{bmatrix} $$

where $a\delta + i\beta = 1$ and, because there is no absorption, $a$, $\beta$, $\gamma$ and $\delta$ are all real.

$$ \begin{bmatrix} B \\ C \end{bmatrix} \begin{bmatrix} \alpha & i\beta \\ i\gamma & \delta \end{bmatrix} \begin{bmatrix} 1 \\ \eta_z \end{bmatrix} = \begin{bmatrix} \alpha + i\beta \eta_z \\ \delta \eta_z + i\gamma \end{bmatrix} $$

$$ \text{Re}(BC^*) = \text{Re}\left[(\alpha + i\beta \eta_z)(\delta \eta_z^* - i\gamma)\right] = (\alpha\delta + \gamma\beta)\text{Re}(\eta_z) $$

$$ = \text{Re}(\eta_z) $$

(28)

and the result follows. It can be manipulated equations (26) and (27) into slightly better forms.

From equation (10)
\[ R = \left( \frac{\eta_0 B - C}{\eta_0 B + C} \right) \left( \frac{\eta_0 B - C}{\eta_0 B + C} \right) \]  

So that

\[ (1 - R) = \frac{2\eta_0 \left( BC^* + B^* C \right)}{(\eta_0 B + C)(\eta_0 B + C)^*}. \]

Inserting this result in equation (26) it is obtained

\[ T = \frac{4\eta_0 \text{Re} \left( \eta_z \right)}{(\eta_0 B + C)(\eta_0 B + C)^*} \]  

(30)

and in equation (27)

\[ T = \frac{4\eta_0 \text{Re} \left( \eta_z \right)}{(\eta_0 B + C)(\eta_0 B + C)^*} \]  

(31)

Equations (29), (30) and (31) are the most useful forms of the expressions for \( R \), \( T \) and \( A \).

4.2 Antireflection Coatings

4.2.1 Antireflection Coatings on High-Index Substrates

The term high-index cannot be defined precisely in the sense of a range with a definite lower bound. It simply means that the substrate has an index sufficiently higher than the available thin-film materials to enable the design of high performance antireflection coatings consisting entirely, or almost entirely, of layers with indices lower than that of the substrate \([7-10]\). These high-index substrates are principally of use in the infrared. Semiconductors are common, and it would be completely impossible to use them in the vast majority of applications without some form of antireflection coating. For many purposes, the reduction of a 30% reflection loss to one of a few percent would be
considered adequate. It is only in a limited number of applications where the reflection loss must be reduced to less than one percent.

4.2.1.1 The Single Layer Antireflection Coatings

The simplest form of antireflection coating is a single layer [11-15]. Consider figure 4.3, if the incident medium is air, provided the index of the film is lower than the index of substrate, the reflection coefficient at each interface will be negative, denoting a phase change of $180^\circ$. The resultant locus is a circle with a minimum at the wavelength for which the phase thickness of the layer is $90^\circ$, that is, a quarter-wave optical thickness, when the two vectors are completely opposed. Complete cancellation at this wavelength, that is, zero reflectance, will occur if the vectors are of equal length.

![Vector diagram of a single-layer antireflection coating](image)

**Figure 4.3.** Vector diagram of a single-layer antireflection coating [1].
This condition in the notation of Figure 4.3 is,

\[ \rho_a = \rho_b \]

\[ \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} = \frac{\eta_1 - \eta_s}{\eta_1 + \eta_s} \]

which requires

\[ \frac{\eta_1}{\eta_0} = \frac{\eta_s}{\eta_1} \]

or

\[ \eta_1 = \left( \eta_0 \eta_s \right)^{1/2} \]

which can also be written for normal incidence.

The condition for a perfect single layer antireflection coating is, therefore, a quarter-wave optical thickness of material with optical admittance equal to the square root of the product of the admittances of substrate and medium.

The optical admittance of a substrate coated with a quarter-wave optical thickness of material is,

\[ Y = \eta_f^2 / \eta_s \]

where \( \eta_f \) is the admittance of the film material and \( \eta_s \) that of the substrate. Therefore the reflectance is given by equation (36)
\[ R = \left( \frac{\eta_0 - Y}{\eta_0 + Y} \right)^2 = \left( \frac{\eta_0 - \eta_s^2/\eta_s}{\eta_0 + \eta_s^2/\eta_s} \right)^2 \]  

(36)

It is used the matrix method [5]. The characteristic matrix of a single film on a substrate is given by

\[
\begin{bmatrix}
B \\
C
\end{bmatrix} = \begin{bmatrix}
\cos \delta_1 & (i \sin \delta_1)/\eta_1 \\
\eta_1 \sin \delta_1 & \cos \delta_1
\end{bmatrix} \begin{bmatrix}
1 \\
\eta_s
\end{bmatrix}
\]

(37)

i.e

\[
\begin{bmatrix}
B \\
C
\end{bmatrix} = \begin{bmatrix}
\cos \delta_1 + i(\eta_s/\eta_1) \sin \delta_1 \\
\eta_s \cos \delta_1 + i\eta_1 \sin \delta_1
\end{bmatrix}
\]

(38)

Where

\[
\begin{cases}
\eta_p = \eta/\cos \theta \\
\eta_s = \eta \cos \theta
\end{cases}
\]  

for each material

\[
\delta_1 = 2\pi \eta_1 d_1 \cos \theta_1 / \lambda
\]

where \( \theta \) is the angle of incidence, \( \eta_s \) is optical admittance for s-polarization (TE), \( \eta_p \) is optical admittance for p-polarization (TM) and

\[ \eta_0 \sin \theta_0 = \eta_1 \sin \theta_1 = \eta_s \sin \theta_s \]  

(39)

If \( \lambda_0 \) is the wavelength for which the layer is a quarter-wave optical thickness at normal incidence, then

\[ n_1 d_1 = \lambda/4 \]

and
\[ \delta_1 = \frac{\pi}{2} \left( \frac{\lambda_0}{\lambda} \right) \cos \theta_1 \] (40)

So that the new optimum wavelength is \( \lambda_0 \cos \theta_1 \) [16-20]. The amplitude reflection coefficient in equation (41) is

\[
\rho = \frac{\eta_0 - Y}{\eta_0 + Y} = \frac{\eta_0 - C/B}{\eta_0 + C/B} = \frac{(\eta_0 - \eta_s) \cos \delta_1 + i \left[ \left( \eta_0 \eta_s / \eta_1 \right) - \eta_1 \right] \sin \delta_1}{(\eta_0 + \eta_s) \cos \delta_1 + i \left[ \left( \eta_0 \eta_s / \eta_1 \right) + \eta_1 \right] \sin \delta_1}
\] (41)

And the reflectance

\[
R = \frac{(\eta_0 - \eta_s)^2 \cos^2 \delta_1 + \left[ (\eta_0 \eta_s / \eta_1) - \eta_1 \right]^2 \sin^2 \delta_1}{(\eta_0 + \eta_s)^2 \cos^2 \delta_1 + \left[ (\eta_0 \eta_s / \eta_1) + \eta_1 \right]^2 \sin^2 \delta_1}
\] (42)

4.2.1.2 Double Layer Antireflection Coatings

A vector diagram of one possibility is shown in Figure 4.4. It is used the matrix method.

The characteristic matrix of the assembly is

\[
B = \begin{bmatrix}
\cos \delta_1 & (i \sin \delta_1) / \eta_1 \\
\eta_1 \sin \delta_1 & \cos \delta_1
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
\cos \delta_2 & (i \sin \delta_2) / \eta_2 \\
\eta_2 \sin \delta_2 & \cos \delta_2
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \delta_1 \left[ \cos \delta_2 + i \left( \eta_5 / \eta_2 \right) \sin \delta_2 \right] + i \sin \delta_1 \left( \eta_5 \cos \delta_2 + i \eta_2 \sin \delta_2 \right) / \eta_1 \\
\eta_1 \sin \delta_1 \left[ \cos \delta_2 + i \left( \eta_5 / \eta_2 \right) \sin \delta_2 \right] + \cos \delta_1 \left( \eta_5 \cos \delta_2 + i \eta_2 \sin \delta_2 \right)
\end{bmatrix}
\] (43)
Figure 4.4 Vector diagram for double-layer antireflection coating [1].

The reflectance will be zero if the optical admittance \( Y \) is equal to \( \eta_0 \), i.e.

\[
\begin{align*}
\eta_1 \sin \delta_1 \left[ \cos \delta_2 + i \left( \eta_s / \eta_2 \right) \sin \delta_2 \right] + \cos \delta_1 \left( \eta_s \cos \delta_2 + i \eta_2 \sin \delta_2 \right) \\
= \eta_0 \left\{ \cos \delta_1 \left[ \cos \delta_2 + i \left( \eta_s / \eta_2 \right) \sin \delta_2 \right] + i \sin \delta_1 \left( \eta_s \cos \delta_2 + i \eta_2 \sin \delta_2 \right) / \eta_1 \right\}
\end{align*}
\]

(44)

The real and imaginary parts of these expressions must be equated separately giving

\[
- \left( \eta_s / \eta_2 \right) \sin \delta_1 \sin \delta_2 + \eta_s \cos \delta_1 \cos \delta_2 = \eta_0 \cos \delta_1 \cos \delta_2 - \left( \eta_s / \eta_2 \right) \sin \delta_1 \sin \delta_2
\]

\[
\eta_s \sin \delta_1 \cos \delta_2 + \eta_s \cos \delta_1 \sin \delta_2 = \left( \eta_s / \eta_2 \right) \cos \delta_1 \sin \delta_2 + \left( \eta_0 \eta_s / \eta_1 \right) \sin \delta_1 \cos \delta_2
\]

i.e.

\[
\tan \delta_1 \tan \delta_2 = \left( \eta_s - \eta_0 \right) / \left[ \left( \eta_s / \eta_2 \right) - \left( \eta_0 \eta_s / \eta_1 \right) \right]
\]

(45)

\[
= \eta_1 \eta_2 \left( \eta_s - \eta_0 \right) / \left( \eta_1^2 \eta_s - \eta_0 \eta_2^2 \right)
\]

and
\[
\tan \delta_2 / \tan \delta_1 = n_2 \left( \frac{n_0 n_s}{n_1^2} \right) / n_1 \left( n_2^2 - n_0 n_s \right)
\]

(46)

Giving

\[
\tan^2 \delta_1 = \frac{(n_s - n_0)(n_1^2 - n_0 n_s) n_1^2}{(n_1^2 n_s - n_0 n_s^2)(n_0 n_s - n_1^2)}
\]

\[
\tan^2 \delta_2 = \frac{(n_s - n_0)(n_0 n_s - n_1^2) n_2^2}{(n_1^2 n_s - n_0 n_s^2)(n_2^2 - n_0 n_s)}
\]

(47-48)

The values of \(\delta_1\) and \(\delta_2\) found from these equations must be paired. The right hand sides of equations (47) and (48) must be positive. \(\delta_1\) and \(\delta_2\) are then real. This requires that, of the expressions either all

\[
(n_2^2 - n_0 n_s)
\]

(49)

\[
(n_1^2 n_s - n_0 n_s^2)
\]

(50-51)

three must be positive, or, any two are negative and the third positive.

An effective coating is one consisting of two quarter-wave layers (Figure 4.5). The appearances of the vector diagram at three different wavelengths are shown (a), (b) and (c). At \(\lambda = 3/4 \lambda_0\) and \(\lambda = 3/2 \lambda_0\) the three vectors in the triangle are inclined at 60° to each other. Provided the vectors are all of equal length, the triangles will be closed and the reflectance will be zero at these wavelengths [21-23]. This condition can be written

\[
\frac{n_1}{n_0} = \frac{n_2}{n_1} = \frac{n_1}{n_2}
\]

(52)
and solved for $\eta_1$ and $\eta_2$:

$$n_1^3 = n_0^2 n_i$$

$$n_2^3 = n_0 n_i^2$$

The reflectance at the reference wavelength $\lambda_0$ where the layers are quarter-waves is given by

$$R = \frac{\eta_0 - (n_i^2 / n_2) \eta_s^2}{\eta_0 + (n_i^2 / n_2) \eta_s}$$

$$= \left( \frac{1 - (n_s / \eta_0)^2}{1 + (n_s / \eta_0)^2} \right)$$

which is a considerable improvement over the bare substrate [1].

**Figure 4.5** Vector diagrams for quarter-quarter antireflection coatings on a high index substrate [1].
The coating described is a special case of a general coating where the layers are of equal thickness. To compute the general conditions it is easiest to return to the analysis leading up to equations (47) and (48).

Let $\delta_1$ be set equal to $\delta_2$ and denoted by $\delta$, where it is recalled that if $\lambda_0$ is the wavelength for which the layers are quarter-waves then

$$\delta = \frac{\pi}{2} \left( \frac{\lambda_0}{\lambda} \right).$$

(56)

From equation (46)

$$\eta_2 \left( \eta_0 \eta_s - \eta_1^2 \right) = \eta_1 \left( \eta_2^2 - \eta_0 \eta_s \right).$$

(57)

$$\eta_0 \eta_s = \eta_1 \eta_2$$

(58)

which is a necessary condition for zero reflectance.

From equation (45) it is found the wavelengths $\lambda$ corresponding to zero reflectance:

$$\tan^2 \delta = \frac{\eta_1 \eta_2 \left( \eta_s - \eta_0 \right)}{\eta_1^2 \eta_s - \eta_0 \eta_2^2} = \frac{\eta_0 \eta_s \left( \eta_s - \eta_0 \right)}{\eta_1^2 \eta_s - \eta_0 \eta_2^2}$$

(59)

If $\delta'$ is the solution in the first quadrant, then there are two solutions and the two values of $\lambda$ are:

$$\lambda = \left( \frac{\pi/2}{\delta} \right) \lambda_0$$

(60)

In all practical cases, $\eta_s$ will be greater than $\eta_0$ and the above equation for $\tan^2 \delta$ will have a real solution provided $\delta'$ is positive or zero.
\[ \eta_1^2 \eta_5 - \eta_0 \eta_2^2 \]  

(61)

This expression is identical to expression (50).

At the reference wavelength \( \lambda_0 \), \( \delta = \pi/2 \) and the layers are quarter-waves. The optical admittance is given by

\[ \frac{\eta_1^2}{\eta_2^2} \eta_5 \]  

(62)

and the reflectance by

\[ R = \left( \frac{\eta_0 - \left( \frac{\eta_1^2}{\eta_2^2} \right) \eta_5}{\eta_0 + \left( \frac{\eta_1^2}{\eta_2^2} \right) \eta_5} \right)^2 \]  

(63)

4.2.1.3 Multilayer Antireflection Coatings

Figure 4.6 shows a vector diagram for a three layer coating. Each layer is a quarter-wave thick at \( \lambda_0 \). If \( \eta_3 > \eta_2 > \eta_1 > \eta_0 \), then the vectors will oppose each other at \( 2/3 \lambda_0 \), \( \lambda_0 \), and \( 2 \lambda_0 \) and provided the vectors are all of equal length, will completely cancel at these wavelengths, giving zero reflectance [1].

This coating is similar to the quarter-quarter coating, but where the two zeros of the two-layer coating are situated \( 3/4 \lambda_0 \) and \( 3/2 \lambda_0 \), those of this three layer coating stretch from \( 2/3 \lambda_0 \) to \( 2 \lambda_0 \), a much broader region [24-27].

The condition for the vectors to be of equal length is
\[
\frac{n_1}{n_0} = \frac{n_2}{n_1} = \frac{n_3}{n_2} = \frac{n_4}{n_3} = \frac{n_5}{n_4}
\]

which becomes with some manipulation

\[
\begin{align*}
\eta_1^4 &= \eta_0^3 \eta_3 \\
\eta_2^4 &= \eta_0^2 \eta_3^2 \\
\eta_3^4 &= \eta_0 \eta_3^3
\end{align*}
\]

Figure 4.6. Vector diagram for a quarter-quarter-quarter coating on a high-index substrate [1, 3].
4.3. Method

There are many different methods for the design of multilayer antireflection coatings. Here some of the commonly used methods in thin film design will be mentioned briefly.\[28]\n
4.3.1. The Vector Method

The vector method is usually associated with the design of antireflection coatings. Two assumptions are involved: first, that the layers are all non-absorbing, and second, that the behavior of a multilayer can be understood by considering one reflection of the incident wave at each interface.

The reflection coefficient at each interface is represented by

\[ R_r = \frac{n_{r-1} - n_r}{n_{r-1} + n_r} \]  

(66)

and the phase thicknesses of the layers are given by

\[ \delta_r = 2\pi n_r \cos \theta_r d_r / \lambda \]  

(67)

where \( n_r \) is the refractive index and \( d_r \) is the thickness of the \( r^{th} \) layer. The resultant amplitude coefficient of the stack is given by the vector sum of coefficients for each interface [1].

4.3.2. Alternative Method of Calculation

The success of the vector method prompts one to ask whether it can be made more accurate by considering second and subsequent reflections at the various boundaries instead of just one. In fact, an alternative solution of the thin film problem can be obtained in this way. It is simpler to consider normal incidence only.
4.3.3. Smith’s Method

In 1958, Smith formulated a useful design method which is known as the method of effective interfaces. It consists of choosing any layer in the multilayer and then considering multiple reflections within it. The reflection and transmission coefficients at the boundaries of the layer are considered to be the resultant coefficients of the complete structures on either side. This technique can be extended to deal with absorbing layers also. This method provides an insight into the properties of a particular type of filter, but is highly complicated [5].

4.3.4. The Smith Chart

The Smith chart is a device which is intended to simplify calculations involved in the design of thin film filters. This chart represents circles of constant amplitude coefficient and circles of constant real part and constant imaginary part of the reduced optical admittance. A scale, calibrated in terms of optical thickness is provided around the outside of the chart. This enables the calculation to be very simply carried out by rotating the point corresponding to the amplitude reflection coefficient of the particular layer, around the center of the chart through the appropriate angle [28].

4.3.5. Circle Diagrams

In this method, the multilayer is considered to be gradually built up layer by layer, immersed all the time in the final incident medium. As each layer increases from zero to its final value, some parameter of the multilayer at that stage, like reflectance or admittance, is calculated and the locus is plotted. The loci for these dielectric layers take the form of a series of circular arcs or even complete circles, each corresponding to a
single layer, which are connected at points corresponding to interfaces between different layers [28].

4.3.6 Solar spectrum modeling with SMARTS2

The accuracy of solar cell measurement under natural sunlight is determined mostly by the spectrum of the sunlight. In this work, the spectrum of natural sunlight is modeled using the model SMARTS2. This model accurately predicts the solar spectrum at the surface of the earth, under clear skies, and runs on a desktop computer.

A reasonable amount of detail is given in this section, as few solar cell researchers (the target audience for this work) are likely to know much about atmospheric modeling [5].

4.3.7 Cell modeling with PC1D

The spectral mismatch when testing a cell is dependent on both the source spectrum and the characteristics of the cell. To investigate the effect of cell characteristics, a group of cells was simulated with the one-dimensional semiconductor-modeling package PC1D.

PC1D was used to calculate cell currents, rather than spectral responses. The spectral response formulation of spectral mismatch was not used because the spectral response of a solar cell is not a well defined quantity. It is difficult to measure and in fact varies depending on the spectrum and intensity of the light illuminating the cell. Consequently, in this work $I_{sc}$ was directly simulated with PC1D [5].

4.4. FORMULATION

4.4.1. Four-Layer Antireflection Coating
The arrangement is illustrated in Figure 4.7. The characteristic matrix of the assembly is

\[
\begin{pmatrix}
B \\
C
\end{pmatrix} = \left\{ \prod_{r=1}^{4} \begin{bmatrix}
\cos \delta_r & \sin \delta_r / \eta_r \\
in_r \sin \delta_r & \cos \delta_r
\end{bmatrix} \right\} \begin{bmatrix}
1 \\
\eta_s
\end{bmatrix}
\]

(68)

where \( \eta_s \) is substrate admittance, \( \delta_r \) is the phase thickness of the \( r \)th layer, i.e.

\[
\delta_r = \frac{2 \pi N_r d_r \cos \theta_r}{\lambda}
\]

(69)

where \( d_r \) is the physical thickness of the \( r \)th layer, and \( \eta_r \) is the optical admittance of the \( r \)th layer, i.e.

\[
\eta_r = \frac{\eta Y N_r \cos \theta_r}{\cos \theta_r} \quad \text{for s-polarization (TE)}
\]

\[
\eta_r = \frac{\eta Y N_r}{\cos \theta_r} \quad \text{for p-polarization (TM)}
\]

(70)

Where \( Y \) is the admittance of free space, \( N_r \) is the refractive index of the \( r \)th layer, \( \theta_0 \) is the angle of incidence and \( \theta_r \) which is incidence angle of the \( r \)th layer can be found from Snell's Law, i.e.
\[ N_0 \sin \theta_0 = N_r \sin \theta_r = N_s \sin \theta_s \quad (71) \]

We can calculate equation (68) and found the matrix elements B and C from the result of this calculation in equation (72)

\[
B = \left\{ \begin{array}{l}
\cos \delta_1 \cos \delta_2 \cos \delta_3 \cos \delta_4 - \frac{n_1}{n_3} \cos \delta_1 \cos \delta_2 \sin \delta_4 - \frac{n_2}{n_2} \cos \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 \\
- \frac{n_2}{n_2} \cos \delta_2 \sin \delta_3 \cos \delta_4 - \frac{n_1}{n_1} \sin \delta_2 \sin \delta_3 \cos \delta_4 + \frac{n_2}{n_2} \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 \\
- \frac{n_1}{n_1} \sin \delta_1 \cos \delta_2 \sin \delta_3 \cos \delta_4 - \frac{n_1}{n_1} \sin \delta_1 \cos \delta_2 \sin \delta_3 \cos \delta_4 \\
+ \left[ \frac{n_2}{n_2} \cos \delta_1 \cos \delta_2 \cos \delta_3 \sin \delta_4 + \frac{n_1}{n_1} \cos \delta_1 \cos \delta_2 \sin \delta_3 \cos \delta_4 + \frac{n_1}{n_1} \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 \\
- \frac{n_1}{n_1} \sin \delta_4 \sin \delta_2 \sin \delta_3 \cos \delta_4 - \frac{n_2}{n_2} \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 - \frac{n_1}{n_1} \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 \\
+ \frac{n_1}{n_1} \sin \delta_1 \cos \delta_2 \cos \delta_3 \cos \delta_4 - \frac{n_2}{n_2} \sin \delta_1 \cos \delta_2 \cos \delta_3 \cos \delta_4 \\
\end{array} \right\} \quad (72)
\]

And

\[
C = \left\{ \begin{array}{l}
- \frac{n_1 n_4}{n_3} \sin \delta_1 \cos \delta_2 \cos \delta_3 \cos \delta_4 - \frac{n_1 n_4}{n_3} \sin \delta_1 \cos \delta_2 \cos \delta_3 \cos \delta_4 - \frac{n_1 n_4}{n_3} \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 \\
+ \frac{n_1 n_4}{n_4} \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 - \frac{n_1 n_4}{n_4} \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 - \frac{n_1 n_4}{n_4} \cos \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 \\
+ \frac{n_1 n_4}{n_4} \cos \delta_1 \cos \delta_2 \cos \delta_3 \cos \delta_4 - \frac{n_1 n_4}{n_4} \cos \delta_1 \cos \delta_2 \cos \delta_3 \cos \delta_4 \\
+ \frac{n_1 n_4}{n_4} \sin \delta_2 \cos \delta_3 \sin \delta_4 - \frac{n_2}{n_2} \sin \delta_2 \cos \delta_3 \sin \delta_4 - \frac{n_2}{n_2} \sin \delta_2 \sin \delta_3 \cos \delta_4 + \frac{n_1}{n_1} \cos \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 \\
- \frac{n_1 n_4}{n_3} \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 + n_1 \cos \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 + n_1 \cos \delta_1 \cos \delta_2 \sin \delta_3 \cos \delta_4 \\
- \frac{n_1 n_4}{n_3} \cos \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 + n_1 \cos \delta_1 \cos \delta_2 \sin \delta_3 \cos \delta_4 \end{array} \right\} \quad (73)\]
The amplitude reflection coefficient and the reflectance are, respectively

$$\rho = \left( \frac{n_0 - Y}{n_0 + Y} \right) = \left( \frac{n_0B - C}{n_0B + C} \right)$$

(74)

where the input optical admittance is \( Y = C / B \).

The effective coating is one consisting of four quarter-wave layers [35-39]. Let \( \delta_1 \) be set equal to \( \delta_2 \) and denoted by \( \delta \), and if \( \lambda_0 \) is the wavelength for which the layers are quarter-waves then

$$\delta = \frac{\pi}{2} \left( \frac{\lambda_0}{\lambda} \right)$$

(75)

Now \( \delta = \delta_1 = \delta_2 = \delta_3 = \delta_4 = 90^\circ \) at \( \lambda = \lambda_0 \). By substituting \( \sin 90^\circ = 1 \), \( \cos 90^\circ = 0 \) into the equations (72) and (73) to obtain the expression of the optical admittance \( Y \).

Using \( Y \), the reflectance of our assembly is written as

$$R_5 = \left( \frac{n_0 - \frac{n_1^2 n_3^2}{n_2^2 n_4^2} n_5}{n_0 + \frac{n_1^2 n_3^2}{n_2^2 n_4^2} n_5} \right)^2$$

(76)

Inserting the optical admittance into equation (30) the transmittance of the assembly is

$$T_5 = \frac{4n_0 n_1^2 n_3^2 \Re(n_5)}{n_2^2 n_4^2 \left( n_0 + \frac{n_1^2 n_3^2}{n_2^2 n_4^2} n_5 \right)}$$

(77)

and into equation (31) the absorption of the assembly is
The purpose of this chapter is to achieve a broader band visible antireflection coating design with multilayer structure which is consisted of insulator thin films. In order to design the normal incidence wideband visible multilayer AR coatings we used different types of layers which are more than two-different materials for even folded and three materials for odd folded.

References:


