CHAPTER II

FLOW BEHAVIOUR OF BLOOD TYPE SUSPENSIONS IN A PIPE

2.1. INTRODUCTION

The cardiovascular system provides the means for circulation of material throughout the body. Its function is to supply oxygen, metabolic fuels, hormones and vitamins to the organs and individual cells of the body, and also to remove metabolic products (such as carbon dioxide and water) from the cells. Blood is the primary circulating medium.

Blood is a suspension of cells in an aqueous solution, the 'plasma'. There are about \( 5 \times 10^9 \) cells in a millilitre of human blood [Ponder (1934)]. The cells are also called the corpuscles. The cells are of three types: red blood cells, the erythrocytes consisting about 95 percent (by number) of the total cells in the blood, white cells are present to the
extent of 0.13 percent (by number) and platelets, about 4.9 percent. The red cells transfer oxygen from the lungs to all the cells of the body and remove carbon dioxide formed by metabolic processes. White cells play a role in the resistance of the body from infection and platelets perform a function related to blood clotting. Most of cells in blood are red cells and its concentration in human blood called 'hematocrit value' is normally about 45 percent by volume. Therefore, from fluid mechanical point of view human blood can be considered as a suspension of red blood cells in plasma. Plasma is composed of 92 percent water, about 7 percent protein (mainly albumin, globulin and fibrinogen) and traces of inorganic and organic salts.

The human circulatory system consists of a complex network of blood vessels and performs important functions (respiratory, nutritive, excretory, protective and regulatory) in the human body. The blood vessels comprise of arteries, veins, capillaries and sinusoids. The internal diameter of these blood vessels lies in the range of 2.5 cm (aorta) to about 4 microns (capillary) [Keele and Neil (1968)]. From fluid mechanical point of view the circulatory system can be broadly classified into two parts viz., macrocirculation and microcirculation. The blood vessels with internal diameter more than 500 µm (1 µm = 10⁻⁶ m) such as aorta, arteries and veins are put under macrocirculation. The blood vessels with internal diameters less than 500 µm i.e. arterioles,
capillaries, venules are put under microcirculation. Merrill (1969), Cokelet (1972) have studied the rheology of human blood in microcirculation. Zweifach (1973) has also studied microcirculation in physiological conditions while Goldsmith and Skalak (1975) have studied hemodynamics of microcirculation.

In microvessels, the flow is characterized by very low Reynolds number (Re < 1), hence in governing equations inertial effects can be neglected.

From hydrodynamic point of view, viscosity of blood plays an important role in microcirculation. It is a well known fact that viscosity of blood varies with radial position. Due to velocity gradient across the vascular cross section, the blood cells tend to accumulate in the centre of the stream leaving a small plasma layer adjacent to the vessel wall. [Bugliarello and Sevilla (1970) Middleman (1972) and Charm and Kurland (1974)]. Gupta (1976) has also observed this fact by experiments. Hence, normal whole blood is a heterogeneous fluid, and its flow in tubes of the size (30 μm diameter < 300 μm) should be considered as two or three phase flow.

The viscosity of plasma is about 1.2 centipoise at 37°C and it does not vary with shear rate, i.e. plasma seems to behave in a Newtonian fashion. Such is not true for the whole blood. It is generally accepted that human blood is a
shear-dependent non-Newtonian fluid with finite but very small yield stress [Merrill (1969), Cokelet (1972)]. The plot of square root values of shear stress and shear rate is well known as Casson's plot. Casson's plot at low shear rates shows that blood has a yield stress [Cokelet (1963)] which is independent of temperature but varies with hematocrit. At high shear rates blood shows Newtonian behaviour, having a transition region in between. Merrill and Pelletier (1967) suggested a model for blood flow consisting of three regions: a Casson region for low shear rates, a Newtonian region for high shear rates and an intermediate transition region.

2.2. FORMULATION OF THE PROBLEM

Following Merrill and Pelletier (1967), we have considered a three phase model for blood flow in the tubes of the size \(30 \leq \text{diameter} \leq 300 \text{ \mu m}\). In the present study Casson's model is used for blood cell regions where shear rate is low and blood has a yield stress. Next to the cell region, there is transition region, in which shear rate is higher and apparent viscosity is lower than in the cell region. Therefore, transition region is represented by power law model (with pseudoplastic or shear thinning behaviour). The layer of plasma near the tube wall has been taken as Newtonian fluid due to high shear rates in that layer.

One dimensional Casson's model may be described as
Figure 2.1 Geometric Representation of Microblood vessel.
\[ \tau^{1/2} = u_1^{1/2}(\dot{\varepsilon})^{1/2} + \tau_0^{1/2} \quad (2.1) \]

The power law model may be described as
\[ \tau = u_2(\dot{\varepsilon})^{1/2} \quad (2.2) \]

2.3. THE GOVERNING EQUATIONS

The geometric representation of micro-blood vessel in the form of a tube of the size \(30 \leq \text{diameter} \leq 300 \mu m\) is shown in the figure 2.1. \(u_1, u_2, u_3\) are the coefficients of viscosity and \(u_1, u_2, u_3\) are the velocities in Casson, transition and plasma regions respectively. Pressure gradient is supposed to be constant.

The equation of motion in Casson's region

\((0 \leq r \leq r_1)\) is

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( u_1^{1/2}(\frac{\partial u_1}{\partial r})^{1/2} + \tau_0^{1/2} \right)^2 \right] = \frac{dp}{dx} \quad (2.3) \]

The equation of motion for transition region \((r_1 \leq r \leq r_2)\) is

\[ \frac{1}{r} \frac{\partial}{\partial r} [r u_2(\frac{\partial u_2}{\partial r})^{1/2}] = \frac{dp}{dx} \quad (2.4) \]

The equation of motion for plasma region \((r_2 \leq r \leq R)\) is

\[ \frac{1}{r} \frac{\partial}{\partial r} [r u_3(\frac{\partial u_3}{\partial r})] = \frac{dp}{dx} \quad (2.5) \]

2.4. BOUNDARY CONDITIONS

Casson region:
\[ \frac{\partial u_1}{\partial r} = 0 \text{ at } r = 0 \quad (2.6) \]
Plasma region:
\[ u_3 = 0 \quad \text{at} \quad r = R \] (2.7)

Matching conditions:
\[
\left[ \left\{ \frac{1}{2} \left( \frac{\partial u_1}{\partial r} \right)^{1/2} + \tau_0^{1/2} \right\} \right]_{r=r_1} = \left[ \frac{\partial u_2}{\partial r} \right]_{r=r_1}^{1/2} \quad (2.8)
\]
\[
[u_1]_{r=r_1} = [u_2]_{r=r_1} \quad (2.9)
\]
\[
[u_2]_{r=r_2} = [u_3]_{r=r_2} \quad (2.10)
\]
\[
[u_2]_{r=r_2} = [u_3]_{r=r_2} \quad (2.11)
\]

2.5. SOLUTION OF THE PROBLEM

Solving the set of equations (2.3 to 2.5) satisfying the boundary conditions (2.6 to 2.11) and using the non-dimensional quantities.

\[
\bar{u}_1 = \frac{4 u_3 u_1}{(-dp/dx) R^2}, \quad \bar{u}_2 = \frac{4 u_3 u_2}{(-dp/dx) R^2},
\]
\[
\bar{u}_3 = \frac{4 u_3 u_3}{(-dp/dx) R^2}, \quad K_1 = \frac{(-dp/dx) u_3 R}{3 u_2^2},
\]
\[
K_2 = \frac{u_3}{u_1}, \quad K_3 = 4 \tau_0 / \frac{dp}{dx} R
\]
\[
\bar{r} = \frac{r}{R}, \quad \bar{r}_1 = \frac{r_1}{R}, \quad \bar{r}_2 = \frac{r_2}{R} \quad (2.12)
\]
we obtain
\[ \bar{u}_1 = [(1-\bar{r}_2^2) + K_1(\bar{r}_1^3 - \bar{r}_2^3) + K_2(\bar{r}_1^2 - \bar{r}_2^2) + K_2K_3(\bar{r}_1^3 - \bar{r}_2^3)] \]
\[ + \frac{4\sqrt{2}}{3} K_2K_3^{1/2} (\bar{r}_3^{3/2} - \bar{r}_2^{3/2}) ] \]  
(2.13)
\[ \bar{u}_2 = [(1 - \bar{r}_2^2) + K_1(\bar{r}_1^3 - \bar{r}_2^3)] \]  
(2.14)
\[ \bar{u}_3 = [(1 - \bar{r}_2^2)] \]  
(2.15)

Total flux \( Q \) is given by
\[ \frac{Q}{2\pi} = \int_0^{\bar{r}_1} \bar{u}_1 \bar{r} d\bar{r} + \int_{\bar{r}_1}^{\bar{r}_2} \bar{u}_2 \bar{r} d\bar{r} + \int_{\bar{r}_2}^{\bar{r}_3} \bar{u}_3 \bar{r} d\bar{r} \]
\[ = \frac{1}{4}(1-\bar{r}_2^4) - \frac{3}{10} K_1(\bar{r}_2^5 - \bar{r}_1^5) + \frac{K_2}{4} \bar{r}_1^4 + \frac{2K_2K_3}{6} \bar{r}_1^3 \]
\[ - \frac{2\sqrt{2}}{7} K_3^{1/2} K_2 \bar{r}_1^{7/2} \]
(2.16)

2.6 RESULTS AND DISCUSSIONS

Bloch (1962) measured the thickness of the cell free layer in vessels of various sizes. In frog, where the mesenteric vessels were of the order of 100 \( \mu m \) or larger in diameter, the ratio of the plasma layer thickness to the vessel radius was found to be around 1:20. In the rat, in vessels of 10 to 50 \( \mu m \) in diameter, this ratio has been found to be about 1:5.

In a study of flow in the capillaries of the hamster, Copley and Staple observed ratios of the plasma layer : capillary radius of 1:4 to 1:10, consistent with Bloch's
Figure 2.2 Variation of Plasma layer thickness ($\delta/R$) with flux ($Q$) for different values of $\mu_3/\mu_1$ and $K_1 = 0.2$, $K_3 = 9.0$ and Transition region $(R_2 - R_3) = 0.2$.
FIGURE 2.3 Variation of flux (Q) with apparent viscosity
\( \left( \frac{\mu_1}{\mu_3} \right) \) for different plasma layer thickness \( \left( \delta/R \right) \)
and \( k_1 = 0.2, k_3 = 9.0 \); Transition region \( (\tilde{r}_2 - \tilde{r}_1) = 0.2 \)
observations for the rat. They further observed mechanically that the width of the plasma layer decreases as the flow decreases [Middleman (1972)].

Assuming the existence of a cell free plasma layer along the wall of a vessel as observed by Bloch (1962), Copley and Staple [Middleman (1972)] we have presented a mathematical model for micro-blood vessel of size $30 \leq$ diameter $\leq 300 \mu m$. In this model by varying ratio of plasma layer thickness to blood vessel radius as $3:10$, $1:5$, $1:10$ we have shown that the flux $Q$ increases with increasing width of plasma layer (Figure 2.2). In other words, width of plasma layer decreases as the flux decreases which is in accordance to the observations of Copley and Staple.

Haynes (1960), Thomas (1962) showed that thickness of the plasma layer decreases as the hematocrit value increases by fitting equation

$$
\eta = \eta_p[1 - (1 - \frac{\delta}{R})^4(1 - \frac{\eta_p}{\eta_c})]^{-1}
$$

(2.17)

In figure (2.3) we have shown that flux $Q$ decreases as apparent viscosity $\frac{\mu_1}{\mu_2}$ increases. But flux $Q$ decreases with decreasing plasma layer (as shown in figure 2.2) and a increase in apparent viscosity may be interpreted as increase in hematocrit. Hence we have concluded that thickness of plasma layer decreases as the hematocrit value increases in accordance to Haynes (1960) and Thomas (1962).