CHAPTER IV

EFFECTS OF VISCOSITY VARIATION IN BLOOD
IN PULMONARY SHEET FLOWS

4.1 INTRODUCTION

The main function of the lung is to oxygenate the blood and remove carbon-dioxide gas from it. This part of blood circulation in lungs is known as the pulmonary circulation [Fung (1984)] and the process is known as diffusion. In this process, the blood is spread into very thin layers, so that blood-gas interfacial area becomes very large. In mammals [Maloney and Castle (1960); Zhung and Fung (1983a)], blood flows from right ventricle to the pulmonary artery, then to the capillary blood vessels, then to veins and finally to the left atrium. In the lung, the smallest unit is alveolus. The alveolus is bounded by network of capillary blood vessels. The walls of each alveolus are shared by neighbouring alveoli, and are called interalveolar septa. Each sheet of blood is bounded by thin membrane which is double layered [Fung et. al. (1962), (1972), (1974)]. In lung, since the complexities of the flow are the result of the path, which the air must take as it moves in and out of the lung. It is the neglect of this influence that led to naive description of bronchial air flow. The portion of bronchial tree which extends from trachea to terminal
bronchioles-air ways approximate 0.07 cm in diameter [Horsfield and Cumming (1968)] . The trachea conducts all the inspired air from larynx to the carina the main bifurcation formed by the junction of right and left main branches. These divisions go on increasing and like this, there are 20 divisions or bifurcations in between the trachea and gas-exchanging region of lung. The earlier attempts to measure the lengths and diameters of various airways were made by Rohrer (1915) and Findensen (1935). They used their results for simple calculations of fluid flow and its influence on pressure drop and particle deposition. These results are not very useful because they only provide direct probing of airways or bronchial casts.

The pressure-diameter relation for the pulmonary arteries is linear because the pulmonary arteries are embedded in lung parenchyma consisting of alveolar walls attached to the outside of blood vessels as its integral part [Fung (1981)] . When pulmonary pressure changes the degree of lung inflation also for pulmonary alveoli [Fung and Sobin (1972a)], the pressure of blood and the change in alveolar air volume have been related to the changes in area and the thickness of sheet such that the blood volume in an alveolar sheet is equal to the product of thickness x area x VSTR (i.e. vascular space tissue ratio). Thus, the elasticity problems are related to thickness and area [Fung and Sobin (1972b)] .
To understand the flow of blood in pulmonary capillary sheet, an attempt should be made to introduce the interaction of red blood cells with capillary sheet. Figure 1 describes a pulmonary sheet through which many red cells flow in the space. For the problem, the variables of interest are the pressure $p$, the pressure gradient $\frac{\partial p}{\partial x}$ and coefficient of viscosity $\mu_0$, mean velocity $U_0$, angular frequency of oscillation $\omega$, the sheet thickness $h$ and width of sheet $W$, the diameter of post $\varepsilon$ and the distance between the post $a$, and the angle the mean flow and a reference line defining the postal pattern as shown in figure is $\theta$. For such flows, it is assumed that the Reynold's number is much smaller than 1 as the motion is very slow.

4.2 FORMULATION OF THE PROBLEM

We consider blood as the suspension of red blood cells and through a one-dimensional diffusion equation, the viscosity is determined using Einstein's formula

$$\mu = \mu_0 (1 + \alpha C) \quad (4.1)$$

The velocity in $x$ and $y$ direction are obtained through the momentum equation and the continuity equation and finally differential equation for the pressure distribution is obtained.

The one dimension diffusion equation for the distribution of cells, dimensional equation of motion and continuity is:

$$D \frac{d^2C}{dy^2} = m \quad (4.2)$$
\[
\frac{\partial u}{\partial x} = 0 \quad (4.3)
\]

\[
\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} \right) = 0 \quad (4.4a)
\]

\[0 = \frac{\partial u}{\partial y} \text{ i.e. } v = 0 \quad (4.4b)\]

where, \( m \) = Rate of production of cells

\( C \) = Concentration of red cells in blood,

\( D \) = Diffusion coefficient,

\( \bar{a} \) = Compliance constant of the capillary sheet,

\( \rho \) = Density of the fluid,

\( U_0 \) = Velocity in pulmonary sheet,

\( \alpha \) = Characteristic constant representing the viscosity-variations,

\( \bar{h} \) = Sheet thickness.

Boundary and Matching Conditions are

\[\frac{\partial C}{\partial y} = 0 \text{ at } y = 0\]

\[C = 0 \text{ at } y = h_a\]

\[u = 0 \text{ at } y = h_a\]

\[x = 0 \text{ at } h = h_a\]

\[x = L \text{ at } h = h_v\]

Introducing non-dimensional quantities are:

\[y = \frac{\bar{y} h_a}{\bar{h} h_a}, \quad C = \overline{C}, \quad x = \bar{x} h_a, \quad u = U_0 \bar{u}, \quad \mu = \mu_0 \mu_r\]

\[p = U_o \bar{p}, \quad h = \bar{h} h_a\]

\( h_a \) = thickness of sheet at the alveolar-end,

\( h_v \) = thickness of sheet at the venule-end.

\[\beta = h_v/h_a\]
and $\bar{h} = 1 + \bar{a} \bar{p}$ \hfill (4.5)

where,

$$\bar{a} = \frac{\alpha \rho \bar{u}_0^2}{h_a}$$

The governing equation of motion, equation of continuity and equation for rate of flow is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$ \hfill (4.6)

and

$$\frac{\partial \bar{p}}{\partial x} = \frac{\partial}{\partial y} \left( \mu \frac{\partial \bar{V}}{\partial y} \right)$$ \hfill (4.7)

The expression for the flux is obtained by the formula:

$$Q = \int_0^h \bar{u} \partial \bar{y}$$ \hfill (4.8)

Boundary and matching conditions are:

(1) $\frac{\partial \bar{C}}{\partial \bar{y}} = 0$ at $\bar{y} = 0$

(2) $\bar{C} = 0$ at $\bar{y} = 1$

(3) $\bar{u} = 0$ at $\bar{y} = 1$ \hfill (4.9)

(4) $\bar{x} = 0$ at $\bar{h} = 1$

(5) $\bar{x} = 1$ at $\bar{h} = \beta$

4.3 SOLUTION OF THE PROBLEM

The expression for the concentration of red cells subject to the boundary conditions (1) and (2) is given below:

$$\bar{C} = (1 - \bar{y}^2)$$
Introducing the value of \( c \) in (1) the local variation of the viscosity is given by

\[
\mu_r = 1 + \frac{\alpha}{c_0} (1 - y^2) \tag{4.10}
\]

where

\[
c_0 = \frac{m h_a^2}{2D}
\]

Introducing the expression of \( \mu_r \) in the equation of motion (4.11) and finally solving it for the velocity distribution, one may obtain the expression for the velocity is

\[
\tilde{u} = -\frac{\partial P}{\partial x} E \left[ \log \left( \frac{K - \tilde{h}}{K - y} \right) - \log \left( \frac{K + \tilde{y}}{K + h} \right) \right] \tag{4.11}
\]

**Flow Rate:** The expression for the flow rate is given by

\[
Q = E \frac{\partial P}{\partial x} \left[ \left( \tilde{h} - K \right) \log \left( K - \tilde{h} \right) + \left( \tilde{h} + K \right) \log \left( K + \tilde{h} \right) - \tilde{h} \log \left( K - \tilde{h} \right) - \log \left( K + \tilde{h} \right) \right] \tag{4.12}
\]

where

\[
E = \frac{u_0^2 h_a c_0}{\frac{\mu o}{\alpha} + 1}, \quad K = \sqrt{\frac{c_0}{\alpha} + 1}
\]

Introducing the expression of velocity in equation (4.5) the expression for the normalised flow rate is given below:

\[
\bar{Q} = \frac{\rho u_0^2 c_0}{h_a} \left[ \left( h - K \right) \log \left( K - \tilde{h} \right) - \left( \tilde{h} + K \right) \log \left( K + \tilde{h} \right) - \tilde{h} \log \left( K - \tilde{h} \right) - \log \left( K + \tilde{h} \right) \right] \tag{4.13}
\]

where,
\[ \bar{Q} = \frac{Q}{\left( \frac{\partial p}{\partial x} \frac{h^2}{\mu_0 u_0} \right)} \]

For one-dimensional flow for incompressible fluid, the conservation of mass equation \( \bar{Q} \) to be constant, i.e.

\[ \frac{d\bar{Q}}{dx} = 0 \]  

(4.14)

Introducing \( \bar{u} \) in equation (11) and on integration one finally obtains a relation in \( \bar{x} \) and \( \bar{h} \)

\[ \bar{x} = \frac{A - B - \bar{h}(C)}{G - h_a - (q)} - \frac{D - E - \bar{h}(C)}{D - E - \bar{h}(C)} \]  

(4.15)

where

\begin{align*}
A & = (\bar{h} - K) \log (K - \bar{h}) \\
B & = (\bar{h} + K) \log (K + \bar{h}) \\
C & = \log (K - \bar{h}) - \log (K + \bar{h}) \\
D & = (1 - K) \log (K - \bar{h}) \\
E & = (1 + K) \log (K + 1) \\
F & = \log (K - 1) - \log (K + 1) \\
G & = (\bar{h} - K) \log (K - \bar{h})
\end{align*}

**APPARENT VISCOSITY:**

In order to find the expression for apparent viscosity, the expression for the flow rate is rewritten in the following form:

\[ \bar{Q} = \frac{\rho \ u_0^2 \ c_0}{h_a} \left[ A - B - \bar{h}(C) \right] \]  

(4.16)
Comparing with the corresponding expression of flow rate in purely viscous fluid of constant viscosity \( \mu_0 \), we get the expression for the apparent viscosity as given below:

\[
\mu_r = \frac{\rho U_0^2 C_0}{h_a \gamma} [A - B - h(C)]
\]

(4.17)

4.4 RESULTS AND DISCUSSION

The problem of sheet flow in pulmonary capillary has been analysed by introducing the anomalous behaviour of blood. Introducing the plasma viscosity \( \mu_0 \) variation of apparent viscosity with sheet thickness \( \bar{h} \) and axial position \( \bar{x} \) have been described in Figures 4.2 and 4.3 for various values of parameters involved in the analysis. Figures 4.4 and 4.5 depict the variation of flow rate and sheet thickness \( \bar{h} \) with axial position \( \bar{x} \) and other parameter involved in the analysis of the physiological system under study. The Reynolds number being very low, the concentration of cells affects the local variation of the viscosity and other physiological events of the system. From the figures 4.3 and 4.4, one may conclude that the viscosity decreases as the sheet thickness increases from point .2 to 1. The viscosity also decreases with axial distance from the entry. This is due to the fact that the sheet thickness \( \bar{h} \) increases with \( \bar{x} \) and naturally the viscosity should decrease because of the dilation of the walls of the sheet. Figure 4.3 describes the variation of sheet thickness with axial position. The sheet thickness increases with axial
distance from the entry. The effect of diffusivity is to further increase the sheet thickness and the effect of \( \alpha \) is also to increase the sheet thickness. This may be due to the fact that as \( \alpha \) increases, the viscosity also increases and this, in turn, increases the pressure within the blood. As the sheet thickness increases flow rate increases and since the sheet thickness increases with axial portion flow rate also increases with axial portion.

One may observe that \( \bar{h} \) increases with \( x \) and \( \bar{Q} \) increases with \( \bar{h} \) i.e. \( \bar{Q} \) increases with axial distance. The diffusion coefficient increases the flow rate as well as sheet thickness.

**CONCLUDING REMARKS:**

Viscosity variations in pulmonary sheet flows may be associated with various diseases. The results of the above analysis predict diseases like anemia depending on viscosity of pulmonary blood.
Fig. 4.1 Flow-sheet model, Plan view cross section through X-X of (a)
$M = 1$
$\alpha = 0.5$
$\rho = 1$
$\beta = 0.1$
$\mu_0 = 1$
$\gamma_0 = 1$
$\alpha = 2.5$
$\alpha = 4.5$

Results for viscosity

Fig. 4.2: Variation of viscosity with $\bar{h}$ for various values of $D$ and $\alpha$. 

Apparent Viscosity

$\bar{h}$
Fig. 4.3 Variation of viscosity with axial position for different values, D and α.
Fig. 4.4 Variation of flow rate with $\bar{h}$ for different values of $D$ and $\alpha$.
Fig 45 Variation sheet thickness with \( \bar{h} \) for different values of \( D \) and \( \alpha \)