MODAL ANALYSIS AND CUTOFF FREQUENCIES OF A DOUBLY CLAD PLASMA LOADED WAVEGUIDE HAVING A LEMNISCATES OF BERNOULLI TYPE CORE CROSS-SECTION

4.1 INTRODUCTION:

In the previous chapter (II and III), we have considered a simple and practical method based on scalar wave approximation to study the modal analysis and cutoff frequencies of a lemniscate singly and doubly clad optical waveguide. In this chapter, we have chosen a waveguide whose core cross section is of a lemniscate doubly clad shape and inner is bounded by plasma cladding.

It is well known that optical waveguide (fiber) is now a primary component of global and local communication systems. A few basic experiments in signal transmission in the 1960s have grown into a multibillion dollar industry; establish fiber-based communication networks around the world. Currently optical waveguides (fibers) play a central role in optical communication engineering and integral optical electronics. These waveguides utilize a wave phenomenon that traps the light locally and guides it in any direction, although their propagation lengths differ greatly. In order to develop new optical communication systems or optical devices many researchers and engineers continuously that forward new kinds of optical fibers with new technological routes. The study of unusual optical waveguide having unusual core cross section
has emerged as a new thrust area in light wave technology numerous research papers, review and technical articles [1-16] have appeared in this area. Some of the unusual geometries considered so far are triangular (including curvilinear triangular), elliptical, super elliptical, hypocycloidal, cardioidic cross sections etc [17-25].

Many other cross-sectional shapes can be studied, particularly by using powerful tools like the finite-element method [27]. However, apart from academic interest, some of these fibers of unusual cross-sections may occasionally find some practical relevance. In the present article, a very unusual cross-sectional shape is being considered, namely, the lemniscate of Bernoulli. This section resembles two non-circular loops joined together. This makes the shape relevant to a practical situation where more than one core is to be embedded in a common cladding. It is well known that if two circular cores of a fiber are embedded in a common cladding, the one influences the other resulting in a coupling between the two. When several cores are embedded in the same cladding the situation becomes more complex. If one attempts to make a theoretical analysis of the two cored waveguide, the mathematical difficulties become formidable. If, however one studies the modal properties of a waveguide with a core having a cross-section of the shape of a lemniscate of Bernoulli, one may attack the problem of two cores in a single cladding by using an integrated mathematical approach, although in this case the cores are no longer circular in shape. It is with this motivation that the present article reports an analytical study of a waveguide with a core cross-section of the shape of a lemniscate, attempting to correlate the results with a practical situation where a couple of cores are embedded in a common cladding.
So for the investigators has been concerned mainly with the geometrical aspect of the waveguide structure and limited the discussion to dielectric waveguides. The glass fiber, halide glass fibres, plastic clad glass fiber, and plastic fibres are common in use. Various anisotropic media have been introduced in fibres by several investigators and resulting modal behaviour have also been studied and lead to interesting consequences [28-33]. Some types of polymers, liquid crystal and plasma have also been tried [34-36, 37-38]. With the recent revival of interest in chiral media, several workers have studied the effect of chirality on the modal and the field characteristics of optical fiber waveguides and have published various papers [39-49].

In recent years the dielectric waveguide has attracted serious attentions because of its marvelous propagation properties for millimeter waves and sub millimeter waves, as well as for light wave and so on. As a new type of waveguide, plasma waveguide has also attracted a great deal of interest. Many scholars have been investigating it, but their studies are mainly connected with metal plasma waveguides. There are two causes for that. On one hand, the metal plasma waveguide possesses small attenuation properties. On the other hand, since binding conditions of a metal plasma waveguide are quite stronger, their eigen values of the characteristic equations are of a good separated spectrum. This results in analyzing them in quite a simple theory. Compared with metal plasma waveguide, plasma dielectric waveguide has weak biding conditions. So their theoretical analysis are quite complicated. However, since biding conditions in plasma waveguides are as a functions of many parameters, it makes them possess some unique properties. Shen has introduced a new concept of plasma dielectric waveguide [37-38], which can be used to
transfer electromagnetic energy in space. The waveguide he has studied is a cylindrical vacuum core surrounded by a plasma cladding. Such waveguide is no high frequency limitation for single-mode propagation. It can be used to transfer EM pulse at nearly to the speed of light and keep its profile and shape unchanged.

In this chapter we report an analytical investigation of the modal dispersion and cutoff properties for sustained modes in a plasma loaded doubly clad waveguide having a shape of the lemniscates of Bernoulli as its core cross section. This new unusual waveguide may be regarded as a collection of two cores embedded in a common cladding; the one influences the other, resulting in a coupling between the two. This is a practical situation whose one may expect a kind of cross talk due to coupling between the two cores. In the present chapter, we present a more general study of new unconventional plasma loaded doubly clad waveguide with double core cross section. It has been possible to find out in what way the modal dispersion and cutoff frequencies of guided modes as affected when the outer cladding is filled with plasma. It is seen that as the width of the plasma layer in inner cladding increases, the cutoff frequency also increases considerably in all considered cases. This shows that using plasma width as a new parameter we can control any particular mode on our choice. This complex problem needs a tedious numerical analysis for accurate results. Since our object is essentially to get insight into the modal dispersion properties, we adopt an analytical method using the scalar wave approximation. The present analysis is organized in following manner. Section 2 is devoted to the derivation of eigen value equation and cutoff equation having different parameters. Section 3 is devoted to the solution of eigen value equation taking some appropriate
parameter. In this section dispersion curves and cutoff frequency have been discussed in terms of the width of plasma in inner cladding and also in terms plasma frequency for fitted plasma width. Also variation of cutoff frequency against $\omega/\omega_p$ has been shown graphically. Finally, conclusions are given in section 4.

4.2 Theoretical Analysis and Characteristics Equation:

Fig 4.1(a) shows the transverse core cross section of a plasma loaded doubly clad waveguide having a lemniscates of Bernoulli type core cross-section with $n_1$ as refractive index of core, $n_2$ as refractive index of inner cladding of plasma material and $n_3$ as refractive index of outer cladding with air. The refractive index profile is given in Fig. 4.1(b). The shape of the cross section is represented by the equation as

![Diagram of proposed waveguide](image)

Fig 4.1(a) The cross sectional view of proposed waveguide.
Refractive index

Fig 4.1(b) The Refractive index with proposed waveguide.

\[ r^2 = a^2 \cos 2\theta \]  \hspace{1cm} (4.1)

where 'a' is the radius of the core cross section and 't' is the thickness of inner cladding. Since the curve is symmetrical both with respect to the x and the y-axis, it is sufficient to consider only one quadrant. We now want to choose an appropriate coordinate system \((\xi, \eta, z)\) suitable for the analysis of this cross section. The set of curves normal to above set may be written as

\[ r^2 = \eta^2 \sin 2\theta \]  \hspace{1cm} (4.2)

where \(\eta\) is another size parameter. We may now use the coordinates \((\xi, \eta, z)\) instead of \((x, y, z)\). The scale factors \(h_1\) and \(h_2\) corresponding to the new coordinates \(\xi\) and \(\eta\) are given by

\[ h_1 = \frac{\eta^3}{(\xi^4 + \eta^4)^{3/4}} \], \hspace{0.5cm} h_2 = \frac{\xi^3}{(\xi^4 + \eta^4)^{3/4}} \]

and the third coordinate 'z' remains unchanged.

The scalar wave equation in cartesian coordinate is written as

\[ \frac{d^2 E_z}{dx^2} + \frac{d^2 E_z}{dy^2} + \frac{d^2 E_z}{dz^2} + u^2 E_z = 0 \]  \hspace{1cm} (4.3)
where $E_z$ is $z$-component of the electric field. Assuming a harmonic variation of $E_z$ with respect to $z$ and $t$, we deduce

$$
\frac{\xi^3}{\eta^3} \left[ \frac{\partial}{\partial \xi} \left( \xi^3 \frac{\partial E_z}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \eta^3 \frac{\partial E_z}{\partial \eta} \right) + \frac{\partial}{\partial z} \left( \xi^3 \frac{\partial E_z}{\partial z} \right) \right] + \omega^2 \mu_0 \varepsilon E_z = 0 \quad (4.4)
$$

For the magnetic field $H_z$, we have a similar equation. In equation (4) we have $\mu^2 = \varepsilon_1 \mu_1 - \beta^2$, where $\omega$ is the angular frequency, $\varepsilon_1$ is the permittivity of the guiding region, $\mu_1 = \mu_0$ is the permeability and $\beta$ is the propagation constant. Now using the technique of separation of variables, we can obtain the solution of equation (4). We write

$$
E_z = E_1(\xi) E_2(\eta) \exp[j(\omega t - \beta z)]
$$

Equation (4) then yields the following two equations.

$$
\xi^6 \frac{\partial^2 E_z(\xi)}{\partial \xi^2} + 3 \xi^5 \frac{\partial E_z(\xi)}{\partial \xi} = \alpha \quad (4.5a)
$$

$$
\eta^6 \frac{\partial^2 E_z(\eta)}{\partial \eta^2} + 3 \eta^5 \frac{\partial E_z(\eta)}{\partial \eta} (\omega^2 \varepsilon \mu - \beta^2) \eta^6 E_z(\eta) = \alpha \quad (4.5b)
$$

where $k_0$ is free space wave number. Equation (4.5a) is the same for both regions and it does not contain any discriminating parameter such as 'u'. We concentrate on equation (4.5b) and choose $\alpha = 0$ for the low order modes. The fields must be finite at the center of the core.

We now want to choose an appropriate coordinate system ($\xi$, $\eta$, $z$) suitable for the analysis of this cross section. The direction of propagation is along $z$-axis, which is normal to the plane of paper in figure (1). We introduce new coordinate system by assuming that we have an infinite set of lemniscates of varying sizes $a$, now represented by the variable $\xi$. Next
we have another infinite set of orthogonal lemniscates of size $b$, now represented by variable $\eta$. The axial field components can be written as

$$E_{\text{Core}} = AJ_1(u\eta)\exp(j(\omega t - \beta z)) \text{ for } \eta < a$$

(4.6a)

$$E_{\text{Clad}} = BI_1(w_2\eta)\exp(j(\omega t - \beta z)) + CK_1(w_2\eta)\exp(j(\omega t - \beta z))$$

for $a < \eta < a_1$

(4.6b)

$$E_{\text{Clad}} = DK_1(w_3\eta)\exp(j(\omega t - \beta z)) \text{ for } \eta > a_1$$

(4.6c)

where $u^2 = \omega^2 n_1^2 \mu_1 - \beta^2$, $w_2^2 = \beta^2 - \omega^2 \mu_2 n_2^2$, $w_3^2 = \beta^2 - \omega^2 \mu_3 n_3^2$ and $n_2 = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$ with $\omega_p$ as plasma frequency. Also, A, B, C and D are unknown constants and $n_1, n_2, n_3$ is the refractive index of the core, inner cladding and outer claddings respectively and $\mu_2 = \mu_3 = \mu_0$ is the permeability of the claddings. $J_1$ is the Bessel function for the guiding region and $I_1, K_1$ are the modified Bessel functions for the inner cladding and outer cladding regions respectively.

Using the weak guidance approximation and remembering that tangential electric and magnetic field components and their derivatives are continuous across the boundaries at $\xi = a$ and $\xi = a_1$, therefore the boundary conditions are written as

$$E_{\text{Core}}|_{\eta = a} = E_{\text{Clad}}|_{\eta = a}$$

$$E_{\text{Clad}}|_{\eta = a} = E_{\text{Clad}}|_{\eta = a_1}$$

$$\frac{dE_{\text{Core}}}{d\xi}|_{\eta = a} = \frac{dE_{\text{Clad}}}{d\xi}|_{\eta = a}$$
We obtain the following characteristic equation for some low order modes guided by the waveguide under consideration:

$$\frac{dE_{\text{clad}1}}{d\xi} \Bigg|_{\eta=a_1} = \frac{dE_{\text{clad} II}}{d\xi} \Bigg|_{\eta=a_1}$$

$$\Delta_1 = \begin{bmatrix} J_1(ua) & -I_1(w_2a) & -K_1(w_2a) & 0 \\ wJ'_1(ua) & -w_2J'_1(w_2a) & -w_2K'_1(w_2a) & 0 \\ 0 & I_1(w_3a_1) & K_1(w_3a_1) & -K_1(w_3a_1) \\ 0 & w_3J'_1(w_3a_1) & w_3K'_1(w_3a_1) & -w_3K'_1(w_3a_1) \end{bmatrix} = 0 \quad (4.7)$$

The prime (') in equation (4.7) indicates differentiation with respect to its arguments.

4.3. Numerical Results and Discussion:

We now consider the implication of the eigen value equation and the cutoff equation derived above. For this, we do some illustrative numerical calculations. We take $n_1 = 1.5$, $n_3 = 1.3$, $\lambda = 1.55 \mu m$. The eigen value equation [4.7] can now be considered as showing the functional dependence of $\beta$ on "a" in an implicit form. In view of the fact that $n_1k_0 \geq \beta \geq n_2k_0$ a graphical solution of equation [4.7] is obtained. Instead of showing the $\beta$ verses "a" graphs, we show in Fig. 2 to Fig.7 the $b'$ verses $V$ graphs, where $b' = \frac{\beta^2}{k_0^2 - n_2^2}$ and $V = 2\pi a \lambda_0^{-1} (n_1^2 - n_2^3)^{\frac{1}{2}}$. These are the dispersion curves for some low order modes for different values of plasma width $t$. The variation of cutoff frequencies (cutoff $V$-values) along with $\omega/\omega_p$ is also given in Fig. 8. Further, the cutoff frequencies obtained from equation (4.7) for different value of plasma thickness "t" of the inner cladding region are given in table 1. Now we are in a position to discuss some interesting features of our illustrative plots from Fig. 2 to
Fig. 7. We observe that all dispersion curves are of the expected standard shape. Thus we infer that plasma width does not affect the nature of the dispersion curves. By examining all dispersion curves we notice that the first curve (on the left) has a slightly steep rise but then $b'$ reaches the saturation values slowly. The average slope decreases as we move from curve to curve from low to higher order modes. We also find that the curves are more or less regularly spaced.

Now we want to observe the effect of plasma width $t$ of inner cladding on the number of sustained guided modes. Considering Fig. 2 to Fig. 6, it is observed that as $t$ is increased from $t = 0.01 \mu m$ to $t = 1.0 \mu m$, the cutoff frequencies (V-value) also increases considerably and after that it remain constant when $t$ increases from $1.0 \mu m$ to $2.0 \mu m$. This is very important characteristic of the proposed waveguide because the greater the cutoff frequencies, the fewer will be the number of sustained guided modes. This is an important factor for clear picture or clear sound. Further, this behavior of the proposed waveguide is very important to control the cutoff frequency on our wish. If we observed the Fig. 6 and Fig. 7, we notice that there is no effect of plasma width on dispersion curves. From this we infer that cutoff frequency remains unchanged as higher plasma width.

Fig 8 shows the plot of cutoff V-values against $\omega/\omega_p$ at $t=0.01 \mu m$. We observe that $V_c$ increases very slowly from lower to higher values of $\omega/\omega_p$. We may assume that variation of $V_c$ against $\omega/\omega_p$ remains constant. This is another important feature of the proposed waveguide that it can be used for single mode operation below $V \approx 2.0$. 
Next, we come to table 1 and table 2 which show the dependence of cutoff V-values on plasma width $t$ of inner cladding at fixed plasma frequency $0.4\omega$ and $0.8\omega$ respectively. Observing both tables we notice the similar behavior as discussed above for dispersion curves. Further, we infer particularly from this tables that as plasma frequency increases from $0.4\omega$ to $0.8\omega$ keeping $t$ as fixed, the cutoff frequencies (V-value) also increases considerably. This is another interesting features point to be noted.

Finally, it is appropriate that we compare the above discussed features of the proposed waveguide with those for a double clad optical waveguide having no plasma in inner cladding [26]. It is seen that in all considered cases the cutoff of frequency in present case in slightly greater than those in paper [26]. Also the proposed waveguide in the present paper can be used for single mode operation also. Thus the present study shows that considered waveguide in superior to the waveguide taken in paper [26].
Table-1: The cutoff frequencies (cutoff V-value) of the proposed waveguide under consideration for different values of plasma thickness $t$ with plasma frequency $\omega_p=0.4\omega$

<table>
<thead>
<tr>
<th>Cutoff V value</th>
<th>$t=0.01\ \mu m$</th>
<th>$t=0.1\ \mu m$</th>
<th>$t=0.2\ \mu m$</th>
<th>$t=0.5\ \mu m$</th>
<th>$t=1.0\ \mu m$</th>
<th>$t=2.0\ \mu m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>2.5026</td>
<td>2.8181</td>
<td>3.0062</td>
<td>3.1548</td>
<td>3.1730</td>
<td>3.1730</td>
</tr>
<tr>
<td>$V_2$</td>
<td>5.6484</td>
<td>5.9820</td>
<td>6.1883</td>
<td>6.3521</td>
<td>6.3703</td>
<td>6.3703</td>
</tr>
<tr>
<td>$V_5$</td>
<td>15.122</td>
<td>15.465</td>
<td>15.677</td>
<td>15.847</td>
<td>15.868</td>
<td>15.868</td>
</tr>
</tbody>
</table>

Table-2: The cutoff frequencies (cutoff V-value) of the proposed waveguide under consideration for different values of plasma thickness $t$ with plasma frequency $\omega_p=0.8\omega$

<table>
<thead>
<tr>
<th>Cutoff V value</th>
<th>$t=0.01\ \mu m$</th>
<th>$t=0.1\ \mu m$</th>
<th>$t=0.2\ \mu m$</th>
<th>$t=0.5\ \mu m$</th>
<th>$t=1.0\ \mu m$</th>
<th>$t=2.0\ \mu m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>2.5269</td>
<td>2.9668</td>
<td>3.1609</td>
<td>3.2701</td>
<td>3.2762</td>
<td>3.2762</td>
</tr>
<tr>
<td>$V_2$</td>
<td>5.6726</td>
<td>6.1398</td>
<td>6.3521</td>
<td>6.4674</td>
<td>6.4735</td>
<td>6.4765</td>
</tr>
<tr>
<td>$V_5$</td>
<td>15.146</td>
<td>15.625</td>
<td>15.844</td>
<td>15.965</td>
<td>15.974</td>
<td>15.974</td>
</tr>
</tbody>
</table>
Fig. 2. Dispersion curves ($b'$ versus $V$) of a few lowest modes for the proposed waveguide at $t=0.01 \mu m$ with $\frac{\omega_p}{\omega} = 0.4$ fixed.

Fig. 3. Dispersion curves ($b'$ versus $V$) of a few lowest modes for the proposed waveguide at $t=0.1 \mu m$ with $\frac{\omega_p}{\omega} = 0.4$ fixed.
Fig. 4. Dispersion curves ($b'$ versus $V$) of a few lowest modes for the proposed waveguide at $t=1\mu m$ with $\frac{\omega_p}{\omega} = 0.4$ fixed.

Fig. 5. Dispersion curves ($b'$ versus $V$) of a few lowest modes for the proposed waveguide at $t=0.01\mu m$ with $\frac{\omega_p}{\omega} = 0.8$ fixed.
Fig. 6. Dispersion curves ($b'$ versus $V$) of a few lowest modes for the proposed waveguide at $t=0.1\mu m$ with $\frac{\omega_p}{\omega} = 0.8$ fixed.

Fig. 7. Dispersion curves ($b'$ versus $V$) of a few lowest modes for the proposed waveguide at $t=1\mu m$ with $\frac{\omega_p}{\omega} = 0.8$ fixed.
Fig. 8: The variation of cutoff frequency $V_c$ for few lowest guided modes as a function of $\frac{\omega_p}{\omega}$ at $t=0.01 \mu m$. 
4.4. Conclusion:

In this paper an analytical study of the eigen modes a new unconventional optical waveguide filled with plasma in the inner cladding region is presented for the first time in our knowledge. This is new idea for a new waveguide and the paper has some new results to be noted. Our analysis shows that the introduction of thin plasma layer in the inner cladding of the proposed waveguide gives the following advantages:

1. We can have single mode operation below V=2.0 which is less than the standard weakly guiding step index fiber. It is to be noted that in the case of standard fiber V=2.40 for $LP_{11}$ mode.
2. We can get any particular mode of interest by adjusting the width of the plasma layer. These are the special features of the proposed waveguide.
REFERENCES


