CHAPTER - ONE

INTRODUCTION.

It is common knowledge learned from experience that cracks can be very detrimental to strength, even when small. We can chop wood into kindling with a small axe carefully aligned with the grain, we can split logs with wedges, tear two adhering sheets apart, cleave layers of mica easily, once a beginning is made, or even split an apple into halves with the two hands after cutting through the skin all round. Cracks running rapidly through hard structural materials (glass, metals, rock, and concrete) are also within common experience. But the slow extension of a crack in such materials is much more confined to the specialized experience of engineers. The rapid cracking or complete fracture is often so sudden that the unaided eye is unable to detect the process of rapid extension from some small initial crack, notch, hole or other irregularity.

Such irregularities, before cracking begins, are
important features, because they after the state of stress in their immediate neighbourhood, usually be introducing a local intensification. This can be very pronounced (as at the ends of the very slender elliptic hole), and, thus, a small volume of material in such a location can be more severely strained than any other.

So long as the material in question does not rupture or crack, the calculation of the fields of stress and strain is carried out by solving an ordinary value problem in some kind of idealized body, for instance, one obeying Hooke's law (the linearly elastic continuum), or one which becomes plastic at sufficiently large strain. The boundaries and the deformational behaviour (constitutive relations) of the material are regarded as given. The applied loads cause some deformation of the original boundary, as when a circular hole is pulled into a slightly oval shape. But particles of the material which are originally close neighbors remain close neighbors.

If on the other hand, rupture or cracking does
occur, this close neighbor relation no longer holds. The rupture or crack faces separate (even though, as in a small extension of a crack, the separation may be very small). There was a force distribution on them, but now there is none. In fact, the boundary form has changed. Then calculation of the stress and strain for a crack in process of extending now requires consideration of a sequence of ordinary boundary value problems. But it also requires some additional condition, a criterion to tell us when the boundary must undergo such a change, just as in the tensile fracture of an idealized brittle material we adopt the criterion that fracture must occur when the tensile stress reaches some value (determined by test) characteristic of the material.

Broadly speaking, fracture in solids is initiated by some flaw or imperfection, such as a microcrack, which, like a notch, causes the high elevation of stress in that region. For a sufficiently high value of the local stress, the atomic bonds at the crack edge may be broken. If this
should happen, the flaw may grow into a sizable fracture surface of causing complete failure of the solid. However, the analytical treatment of the strength of atomic bonds is not, in general, straightforward, and, hence, an alternative, continuum mechanics, approach to the problem of crack propagation will be adopted here.

The significance of intense and localized concentration of stress around sharp notches was first emphasized by Inglis [3]. He found that the stress near the tip of a flaw or notch can be many times greater than the applied stress at distances far away. Using the two-dimensional configuration of an elliptical hole in a plate under applied stress $P$ as the model in Fig. 1.1, Inglis obtained an expression for the maximum stress $\left(6_y\right)_{\text{max}}$ at the apex of the major axis of the ellipse where the radius of curvature $= b^2/a$ is a minimum. The equation is

$$\left(6_y\right)_{\text{max}} = P \left[ 1 + \left( \frac{2a}{b} \right) \right] \quad (1.1)$$

where $a$ and $b$ are the major and minor semiaxes of the ellipse. He further showed that if the flaw is in the
shape of a narrow ellipse or crack of length 2a having a notch radius, the stress concentration is approximately given by

$$\left(\sigma_y\right)_{\text{max}} = 2P \left(\frac{a}{\rho}\right)^{1/2}$$  \hspace{1cm} (1.2)

Since \(\rho\) is very small in comparison with \(a\), the actual stress \(\left(\sigma_y\right)_{\text{max}}\) at the root of the crack could be large enough to cause fracture.

From a different viewpoint, Griffith [1,2] approached the problem of fracture by appealing to the first law of thermodynamics. He postulated that a necessary condition for a crack to spread under the action of external loads is that the energy \(T\) used in creating new fracture surface is supplied from the released strain energy \(W_1\) in the elastic solid. Both \(T\) and \(W_1\) depend on the size of the crack. The stationary value of the free energy \(F = T - W_1\) corresponds to certain critical crack length \(a_{cr}\). Referring to Fig. 1.2 Griffith's energy criterion assumes that crack extension takes place when \(a\), the half crack length, exceeds the critical value \(a_{cr}\).
Further, if the surface energy of the material and crack size are known, the criterion of failure can yield, in principle, an inequality defining the minimum load for fracture. In the energetic approach, however, analytical difficulties arise when the direction of crack growth is not a priori evident. Because of this, critical stress calculations have been limited to a few crack problems processing a high degree of symmetry. This restriction can be somewhat relaxed if attention is directed to a particular region in the system where the failure criterion is based on some limiting value of the local stress field.

Instead of considering the energy of the entire crack system, Irwin ([4], [5]) proposed to examine the stress field in the immediate vicinity of the crack tip. With the knowledge of Sneddon's result [7] for the stress distribution around a penny-shaped crack, he pointed out that the crack tip stresses owing to the conditions of generalized plane stress or plane strain can be expressed
by a two-parameter set of equations. These parameters, called the stress-intensity factors, are functions of the crack dimensions and applied loads. The critical values of the stress-intensity factors, which can be determined experimentally for different materials, govern the condition of unstable crack propagation. A typical curve representing those combinations of applied stress and crack length at the onset of rapid crack extension is shown in Fig. 1.3.

The concept of stress-intensity factor can also be associated with the idea of strain energy release rate of crack extension force, which represents the loss of energy from the strain energy of the crack system as the crack advances by a short segment. While the determination of this energy release rate requires only the stress and displacement fields in a small zone around the crack tip, it can be related to the strain energy derivative $\delta W_1/\delta a$ of the Griffith theory which is established by considering the system as a whole.
In an effort to establish the equivalence of the energy and stress criteria for fracture, Sanders [6] reformulated the two-dimensional theory of Griffith and obtained an equivalent criterion involving a certain integral around any contour enclosing the crack tip. His result suggests that the energy criterion for fracture is potentially equivalent to postulating the existence of a critical strength of inverse square root stress singularity for crack growth to occur.

It should be mentioned that the linear elasticity solution for a shape crack gives rise to infinite stresses at the singular crack point where the radius of curvature is zero. In reality, of course, the deformed shape of the crack adopts a finite curvature at the tip, and the stress levels always remain smaller in magnitude than some ultimate stress. Hence, it is likely that any large deformation theory would predict finite stresses at the crack tip. In addition, the occurrence of local plastic deformation also tends to reduce the stress concentrating
effect of the crack. If the zone of plastic flow is small, in comparison with the crack length then the stress distribution, in the large, will not be seriously disturbed. In other words, the sum of the energy dissipated in the crack tip region during loading and the elastic strain energy in the same region after loading will not differ appreciably from predicted by the linear elasticity solution. The singular solution should provide some measure with fracture.

The quantitative development of the continuum theory of fracture is primarily concerned with the size and shape of the crack and of its orientation relative to the applied load. Such a theory assumes the existence of cracks either on the surface or in the interior of the solid and does not account for the formation or initiation of new cracks. The assessment of the theoretical strength of cracked bodies is basically a problem of the mathematical theory of fracture. Recent advancements in the application of special mathematical techniques and continuum mechanics theories have cleared the way for more general application of the current fracture...
concepts. Effective solutions to three-dimensional crack
in a general state of stress are available. Moreover,
it is now possible to include such effects as anisotropy,
nonhomogeneity, couple stresses, etc., into the theory of
crack propagation.

Over the last decade, a considerable number of
papers have been published on the statics and dynamics
of elastic bodies with cracks. An account of some of
these papers shall be given in the next chapter.

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