CHAPTER-III

On Lorentzian Paracontact Structure Manifold
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ON LORENTZIAN PARACONTACT STRUCTURE
MANIFOLD

3.1 Preliminaries:

Matsumoto and Mihai [1] introduced the idea of Lorentzian paracontact structure and studied its several properties. The integrability conditions of an almost contact manifold was studied by Mishra [2]. The aim of the present chapter is to study certain properties and obtain the integrability conditions of a Lorentzian paracontact structure in a manifold.

Let $M^m$ be a $C^\infty$ differentiable manifold of dimension $m$. Let there exist on $M^m$ a $C^\infty$ tensor field $\phi$ of type $(1,1)$ and a Riemannian metric $G$ satisfying

\begin{align}
\phi^2 X &= X \tag{3.1.1} \\
\text{and} \\
G(\phi X, \phi Y) &= G(X, Y) \tag{3.1.2}
\end{align}

for arbitrary vector fields $X$ and $Y$. Then $M^m$ is said to be an almost product metric manifold and $\{\phi, G\}$ is called an almost product metric structure.
Let $M^n$ be a hyper surface of $M^m (n=m-1)$ and let $b : M^n \rightarrow M^m$ be the inclusion map such that $p \in M^n \Rightarrow bp \in M^m$. This mapping induces a linear transformation $B$ (called the jacobian map)

$$B : T_{(p)} \rightarrow T_{(bp)};$$

where $T_{(p)}$ is the tangent space to $M^n$ at $p$ and $T_{(bp)}$ is the tangent space to $M^m$ at $bp$.

Let $g$ be the metric tensor of $M^n$ induced by the metric tensor $G$ of $M^m$. Then

$$(3.1.3) \quad (a) \ G(BX, BY) \overset{def}{=} g(X, Y),$$

$$\quad (b) \ G(BX, N) = 0.$$  

Let us consider

$$(3.1.4) \quad \phi(BX) = BFX + u(X)N$$

and

$$(3.1.5) \quad \phi N = -BU.$$  

Where $N, u, U$ and $F$ are an unit normal, a 1-form a vector field and a tensor of the type $(1,1)$ respectively in $M^n$.

Operating $\phi$ on both the sides of (3.1.4), using (3.1.1) and separating the tangential and normal parts we obtain

$$(3.1.6) \quad \bar{X} = X + u(X)U,$$

$$(3.1.7) \quad u(\bar{X}) = 0,$$
where \( F(X) \equiv \overline{x} \). By virtue of (3.1.3) and (3.1.4) equation (3.1.2) yields

\[
\begin{align*}
(3.1.8) & \quad g(\overline{x}, y) \equiv g(x, y) + u(x)u(y), \\
(3.1.9) & \quad g(x, u) = u(x).
\end{align*}
\]

It is easy to show that

\[
\begin{align*}
(a) & \quad u(U) = -1, \\
(b) & \quad (\bar{U}) = 0 \quad \text{and} \\
(c) & \quad \text{rank}(F) = n - 1.
\end{align*}
\]

**Definition (3.1.1):** A \( C^\infty \) differentiable manifold \( M^n \) satisfying (3.1.6), (3.1.7), (3.1.8) and (3.1.10) is called Lorentzian paracontact metric manifold and the structure \{F, U, u, g\} is called Lorentzian paracontact metric structure [3].

**3.2. Some Results:**

**Definition (3.2.1):** A bilinear function \( A \) in \( M^n \) is said to be pure or hybrid according as [4]

\[
\begin{align*}
(3.2.1) & \quad A(x, y) + A(\overline{x}, \overline{y}) = 0 \\
(3.2.2) & \quad A(x, y) - A(\overline{x}, \overline{y}) = 0.
\end{align*}
\]

We define the fundamental 2-form in \( M^n \) as

\[
(3.2.3) \quad 'F(x, y) \equiv g(\overline{x}, y).
\]
Theorem (3.2.1): 'F (X,Y) is hybrid in both the slots in Lorentzian paracontact manifold.

Proof: Barring X in (3.1.8) and using (3.1.6), (3.1.7) we have

\[(3.2.4)\quad g(X, Y) = g(\bar{X}, \bar{Y}).\]

In view of (3.1.6), (3.1.7) (3.2.3) and (3.2.4) we obtain

\[(3.2.5)\quad 'F(\bar{X}, \bar{Y}) = g(\bar{X}, \bar{Y}) = g(X, Y) = g(X, Y) = 'F(X, Y).\]

Hence the result follows.

Theorem (3.2.2): 'F(X,Y) is symmetric in Lorentzian paracontact manifold.

Proof: By virtue of (3.2.3) and (3.2.4) we have the result.

Theorem: (3.2.3), On a Lorentzian paracontract manifold, we have

\[(3.2.6)\quad (D_x 'F)(\bar{Y}, Z)+(D_x 'F)(Y, \bar{Z}) = u(Y)(D_x u)(Z) + u(Z)(D_x u)(Y)\]

and

\[(3.2.7)\quad (D_x 'F)(\bar{Y}, \bar{Z})+(D_x 'F)(Y, Z) = u(Y)(D_x u)(\bar{Z}) + u(Z)(D_x u)(\bar{Y}).\]

Proof. In view of (3.1.9) and (3.2.3) we have

\[(3.2.8)\quad (D_x u)(Y) = g(D_x U,Y)\]

and

\[(3.2.9)\quad g((D_x F)(Y), Z) = (D_x 'F)(Y, Z),\]

where D is a Riemannian connection in M^n.

In consequence of (3.1.10) b and (3.2.3)

\[(3.2.10)\quad 'F(Y, U)=0.\]
Taking covariant derivative of equation (3.2.10) along the vector $X$ and using (3.1.10) b and (3.2.3) we obtain

$$\tag{3.2.11} (D'_{X}F)(Y,U) = -g(\bar{Y},D_{X}U).$$

Using (3.2.8) in (3.2.11), we have

$$\tag{3.2.12} (D'_{X}F)(Y,U) = -(D_{X}u)(\bar{Y}).$$

Again since $F(\bar{Y}) = F^{2}Y$, which gives

$$(D_{X}F)(\bar{Y}) = (D_{X}F^{2})(Y) - (D_{X}F)(Y).$$

Operating $g$ on both the sides and using (3.2.4), (3.2.9) we get

$$\tag{3.2.13} (D'_{X}F)(\bar{Y},Z) = g((D_{X}F^{2})(Y),Z) - (D'_{X}F)(Y,Z).$$

Equation (3.1.6) implies

$$F^{2}Y = Y + u(Y)U,$$

which by virtue of (3.1.6), (3.1.9) and (3.2.8) gives

$$\tag{3.2.14} g((D_{X}F^{2})(Y),Z) = u(Y)(D_{X}u)(Z) + (D_{X}u)(Y)u(Z).$$

In view of (3.2.13) and (3.2.14) we obtain (3.2.6). Again barring $Z$ in (3.2.6) and using (3.1.6), (3.1.7) and (3.2.12), we have (3.2.7).

**Theorem (3.2.4)** In order that an $n$-dimensional manifold $M^{n}$ admits a Lorentzian paracontact manifold, it is necessary and sufficient that $M^{n}$ contains distributions $D_{p}$, $D_{q}$ and $D_{1}$ of real dimensions $p, q$ and 1 respectively, $1 \leq p < n-1$, $p+q = n-1$ such that they have no common direction and span together the linear manifold of dimension $n$. 
Proof: Let $\lambda$ be the eigen value of $F$ and $P$ be the corresponding eigen vector. Then

\[ P = \lambda P. \]  

Barring (3.2.15) and using (3.1.6) and (3.2.15) we get

\[ P + u (P) U = \lambda^2 P. \]

Now two cases arise

Case-I Let $P = rU$ where $r$ is some scaler. Then by virtue of (3.1.10a) and (3.2.16) we have $\lambda = 0$. Hence the eigen value zero of $F$ is corresponding to the distribution $D_1$.

Case II Let $U$ and $P$ be linearly independent. Then in view of (3.2.16), $\lambda = \pm 1$. Let $p$ eigen values be $1$ and $(n-1-p)$ eigen values be $-1$. Also let $\eta_1, \eta_2, \ldots, \eta_p$ be the eigen vectors corresponding to the eigen value $1$, and $\xi_{p+1}, \xi_{p+2}, \ldots, \xi_{n-1}$ be those corresponding to the eigen value $-1$. Corresponding to eigen value $1$, the $p$ eigen vectors will lead to the distribution $D_p$ of real dimension $p$ and corresponding to the eigen value $-1$ the $(n-1-p)$ eigen vectors correspond to the distribution $D_q$ of real dimension $q$, such that $p + q = n - 1$. This completes the proof of the necessary part of the theorem.

Similarly, it can also be sproved that the condition is sufficient.
Now we put

\begin{align*}
(a) \quad & 2L(X) = \overline{X} + \overline{X}, \\[3.2.17] (b) \quad & 2M(X) = \overline{X} - \overline{X}, \\[3.2.17] (c) \quad & G(X) = \overline{X} - X.
\end{align*}

Then we have

\begin{align*}
(a) \quad & L(X) + M(X) = \overline{X}, \\[3.2.18] (b) \quad & L(X) - M(X) = \overline{X}, \\[3.2.18] (c) \quad & L(X) + M(X) - G(X) = X.
\end{align*}

Let \( \{A_x, B_y, u\} \) be the set inverse to \( \{\eta^x, \xi^y, U\} \);

where \( x = 1,2,\ldots,p \), \( y = p+1, p+2,\ldots,n-1 \). Then

\begin{align*}
(a) \quad & A_x(\eta^x) = \delta^x_{x'}, 1 \leq x, x' \leq p, \\[3.2.19] (b) \quad & B_y(\xi^y) = \delta^y_{y'}, p + 1 \leq y, y' \leq n - 1, \\[3.2.19] (c) \quad & u(U) = -1, \\[3.2.19] (d) \quad & A_x(\xi^y) = B_y(\eta^x) = A_x(U) = B_y(U) = u(\eta^x) = u(\xi^y) = 0, \\[3.2.19] (e) \quad & A_x(X)\eta^x + B_y(X)\xi^y - u(X)U = X.
\end{align*}

Where \( \delta^x_{x'} \) denotes the kronecker delta and \( x, x' \) being integers.

Barring (3.2.19)e and making use of the fact that \( \eta^x \) and \( \xi^y \) are the eigen vectors corresponding to the eigen values 1 and -1 respectively, we get

\begin{align*}
(3.2.20) \quad & A_x(X)\eta^x - B_y(X)\xi^y = \overline{X}.
\end{align*}

Again barring both sides of (3.2.20) we have

\begin{align*}
(3.2.21) \quad & A_x(X)\eta^x + B_y(X)\xi^y = \overline{X}.
\end{align*}
In view of equations (3.2.19)e, (3.2.20) and (3.2.21) we have

\[ 2 A_x(X) \eta^x = \overline{X} + \overline{X}, \]
\[ 2 B_y(X) \xi^y = \overline{X} - \overline{X}, \]

and \( u(X)U = \overline{X} - \overline{X}. \)

Comparing these equations with (3.2.17) we get

\[(a) \quad L(X) = A_x(X) \eta^x, \]
\[(b) \quad M(X) = B_y(X) \xi^y, \]
\[(c) \quad G(X) = u(X)U. \]

Now in consequence of (3.2.19) and (3.2.22) we also have

\[(3.2.23) \quad L(\overline{X}) = \eta^x, M(\overline{X}) = \xi^y, G(U) = U, \]
\[(3.2.24) \quad L(\overline{X}) = M(\overline{X}) = L(U) = M(U) = G(\overline{U}) = G(\overline{X}) = 0. \]

Also

\[(3.2.25) \quad L^2(X) = L(X), \quad M^2(X) = M(X), \quad G^2(X) = G(X) \text{ and} \]
\[(3.2.26) \quad L(M(X)) = M(L(X)) = L(G(X)) = M(G(X)) = G(L(X)) = G(M(X)) = 0. \]

The Nijenhuis tensor \( N(X,Y) \) of \( F \) is given by [2]

\[(3.2.27) \quad N(X,Y) = \overline{X,Y} - \overline{X,Y} - \overline{X,Y} + \overline{X,Y}. \]

In view of (3.1.7) and (3.2.27) we have

\[(a) \quad u(N(X,Y)) = u([X,Y]), \]
\[(b) \quad u(N(\overline{X},\overline{Y})) = u([\overline{X},\overline{Y}]), \]
\[(c) \quad u(N(\overline{X},\overline{Y})) = u([\overline{X},\overline{Y}]), \]
\[(d) \quad u(N(\overline{X},\overline{Y})) = u([\overline{X},\overline{Y}]). \]
3.3 **Integrability conditions:**

In this section we shall obtain the integrability conditions of the Lorentzian paracontact manifold \( M^n \)

**Theorem (3.3.1)** In a Lorentzian paracontact manifold \( M^n \), we have

\[
\begin{align*}
(3.3.1) \quad & (a) \quad (dL)(G(X), G(Y)) = 0, \\
& (b) \quad (dM)(G(X), G(Y)) = 0
\end{align*}
\]

or equivalently

\[
\begin{align*}
(c) \quad & (dA_x)(G(X), G(Y)) = 0, \\
(d) \quad & (dB_y)(G(X), G(Y)) = 0.
\end{align*}
\]

**Proof.** In consequence of (3.2.22)c and (3.3.1)a we have

\[
(3.3.2) \quad (dL)(G(X), G(Y)) = (dL)(u(X)U, u(Y)U),
\]

\[
= (u(X), u(Y))(dL)(U, U),
\]

\[
= (dL)(G(Y), G(X)).
\]

Also, we have [2]

\[
(dL)(G(X), G(Y)) = G(X)\text{L}(G(Y)) - G(Y)\text{L}(G(X)) - \text{L}([G(X), G(Y)]),
\]

which in view of (3.2.26) gives

\[
(3.3.3) \quad (dL)(G(X), G(Y)) = -\text{L}([G(X), G(Y)])
\]

\[
= L([G(Y), G(X)])
\]

\[
= -(dL)(G(Y), G(X)).
\]

Thus from equations (3.3.2) and (3.3.3) we have (3.3.1)a.

Similarly equation (3.3.1)b follows. Also from equations (3.3.1)a,b and making use of (3.2.22) we obtain equations (3.3.1)c,d respectively.
Theorem (3.3.2) In $M^n$ we have

(3.3.4)  
(a) $2 (dM)(L(X),L(Y)) = \overline{[L(X),L(Y)] - [L(X),L(Y)]}$,  
(b) $2 (dL)(M(X),M(Y)) = -\overline{[M(X),M(Y)] - [M(X),M(Y)]}$;

(3.3.5)  
(a) $2 (dM)(L(X),L(Y)) = A_x(X)A_y(Y) \left( \overline{[\eta^*,\eta^*]} - \overline{[\eta^*,\eta^*]} \right)$,  
(b) $2 (dL)(M(X),M(Y)) = B_x(X)B_y(Y) \left( \overline{[\xi^*,\xi^*]} - \overline{[\xi^*,\xi^*]} \right)$

and  

(3.3.6)  
(a) $8(dM)(L(X),L(Y)) = \overline{N(X,Y) - N(X,Y)}$,  
(b) $8(dL)(M(X),M(Y)) = -\overline{N(X,N) - N(X,Y)}$.

Proof: we have [2]

$$2 (dM)(L(X),L(Y)) = 2 \{L(X)M(L(Y)) - L(Y)M(L(X))\} - 2 M([L(X),L(Y)]),$$

which in view of (3.2.26) becomes

$$2(dM)(L(X),L(Y)) = -2M([L(X),L(Y)]).$$

Making use of (3.2.17)b we get

(3.3.7)  
$$2(dM)(L(X),L(Y)) = -\overline{[L(X),L(Y)] - [L(X),L(Y)]}.$$  

From this the equation (3.3.4)a follows. The proof of (3.3.4) b is similar to that of the proof of (3.3.4)a. In consequence of equations (3.2.22) and (3.3.4) we have (3.3.5).

Now in view of (3.2.17)a and (3.3.4)a we have
\[8(dM)(L(X), L(Y)) = [X+X, Y+Y] - [X+X, Y+Y] \]
\[= [X, Y] + [X, Y] + [X, Y] + [X, Y] \]
\[= ([X, Y]) + [X, Y] + [X, Y] + [X, Y] \]
\[= [X, Y] - [X, Y] - [X, Y] + [X, Y] \]
\[= ([X, Y] - [X, Y] - [X, Y] + [X, Y]) \]

By virtue of (3.2.27) the above equation yields (3.3.6)a

Similarly we can prove (3.3.6)b. Hence the theorem.

**Theorem (3.3.3)** we have

\[(3.3.8) \ (a) \ (dG)(L(X), L(Y)) = -u([L(X), L(Y)])U \]
\[= -A_x(X) A_x(Y) u(\eta^x, \eta^{x'}) U \]
\[= [L(X), L(Y)] - [L(X), L(Y)], \]
\[(b) \ (dG)(M(X), M(Y)) = -u([M(X), M(Y)])U \]
\[= -B_y(X) B_y(Y) u(\xi^{y'}, \xi^{y}) U \]
\[= [M(X), M(Y)] - [M(X), M(Y)] \]

and

\[(3.3.9) \ (a) \ 4(dG)(L(X), L(Y)) = \]
\[= -\{u(N(X, Y)) + u(N(X, Y)) + u(N(X, Y)) + u(N(X, Y))\}U, \]
\[(b) \ 4(dG)(M(X), M(Y)) = -\{u(N(X, Y)) - u(N(X, Y)) - u(N(X, Y)) + u(N(X, Y))\}U. \]

**Proof:** we have [2]

\[(dG)(L(X), L(Y)) = L(X)G(L(Y)) - L(Y)G(L(X)) - G([L(X), L(Y)]). \]

From (3.2.26) we get

\[(3.3.10) \ (dG)(L(X), L(Y)) = -G([L(X), L(Y)]). \]

In view of (3.2.22) c we have
(3.3.11) \((dG) (L(X), L(Y)) = -u([L(X), L(Y)]) U.\)

Now making use of (3.2.22)a we obtain
\[
(dG) (L(X), L(Y)) = -A_x(X) A_y(Y) u([\eta^x, \eta^y]) U.
\]

Also from equations (3.2.17)c and (3.3.10) we obtain
\[
(dG) (L(X), L(Y)) = [L(X), L(Y)] - [L(X), L(Y)].
\]

This proves (3.3.8)a. Similarly (3.3.8)b can be proved.

Again from equation (3.3.11) we have
\[
4(dG)(L(X), L(Y)) = -u([2L(X), 2L(Y)]) U.
\]

Which in view of (3.2.17) a becomes
\[
4(dG)(L(X), L(Y)) = -u([X + X, Y + Y]) U
\]
\[
= -u([X, Y] + [X, Y] + [X, Y] + [X, Y]) U.
\]

From equation (3.2.28) we have (3.3.9)a. The proof of (3.3.9)b is similar to the proof of equation (3.3.9)a. Thus we have the theorem.

**Theorem (3.3.4)** The distribution \(D_1\) is completely integrable.

**Proof:** In view of (3.2.18)c the distribution \(D_1\) is given by [2]

(3.3.12) \(L(X) = 0\) and \(M(X) = 0.\)

A necessary and sufficient condition for the distribution \(D_1\) to be completely integrable is that \(L(X) = 0\) and \(M(X) = 0\) be completely integrated i.e.

(3.3.13) \((a)\) \((dL)(X,Y) = 0\) and \((b)\) \((dM)(X,Y) = 0\)
be satisfied for any vector field $X$ satisfying

$$G(X) = -X. \tag{3.3.14}$$

From (3.3.13) and (3.3.14) we have

$$\tag{3.3.15} \text{(a) } (dL) (G(X), G(Y)) = 0 \quad \text{and} \quad \text{(b) } (dM) (G(X), G(Y)) = 0.$$  

But in view of theorem (3.3.1), equation (3.3.15) is satisfied. Which completes the proof of the theorem.

**Theorem (3.3.5).** A necessary and sufficient condition for the distributions $D_p$ and $D_q$ to be completely integrable is that

$$\tag{3.3.16} \begin{align*}  
(a) & \quad [L(X), L(Y)] = [L(X), L(Y)], \\
(b) & \quad [M(X), M(Y)] = -[M(X), M(Y)], \\
(c) & \quad u([L(X), L(Y)]) = 0, \\
(d) & \quad [L(X), L(Y)] = [L(X), L(Y)], \\
(e) & \quad u([M(X), M(Y)]) = 0, \\
(f) & \quad [M(X), M(Y)] = [M(X), M(Y)].
\end{align*}$$

or equivalently

$$\tag{3.3.17} \begin{align*}  
(a) & \quad A_x (X) A_x (Y) ([\eta^x, \eta^x]) U = 0, \\
(b) & \quad B_y (X) B_y (Y) ([\xi^y, \xi^y]) U = 0, \\
(c) & \quad A_x (X) A_x (Y) u ([\eta^x, \eta^x]) U = 0, \\
(d) & \quad B_y (X) B_y (Y) u ([\xi^y, \xi^y]) U = 0.
\end{align*}$$

These equations can also be expressed in the forms

$$\tag{3.3.18} \begin{align*}  
(a) & \quad N(X, Y) = 0 \quad \text{and} \quad (b) \quad u(N(X, Y)) + u(N(X, Y)) = 0.
\end{align*}$$

**Proof:** In view of (3.2.18)c the distribution $D_p$ is given by
(3.3.19) \[ M(X) = 0 \text{ and } G(X) = 0. \]

In order that the distribution \( D_p \) be completely integrable it is necessary and sufficient that \( M(x) = 0 \) and \( G(x) = 0 \) be completely integrable i.e.

(3.3.20) \( (a) \ (dM)(X,Y) = 0 \quad \text{and} \quad (b) \ (dG)(X,Y) = 0 \)

be satisfied for arbitrary vector fields \( X, Y \) satisfying

(3.3.21) \[ L(X) = X. \]

From equations (3.3.20) and (3.3.21) we have

(3.3.22) \( (a) \ (dM)(L(X), L(Y)) = 0 \quad \text{and} \quad (b) \ (dG)(L(X), L(Y)) = 0. \)

Thus in consequence of (3.3.4)a, (3.3.5)a, (3.3.8)a and (3.3.22) we have (3.3.16) a,c,d. and (3.3.17) a,c.

Similarly for the integrability of the distribution \( D_q \), from equation (3.2.18)c, the distribution \( D_q \) is given by

(3.3.23) \[ L(X) = 0 \quad \text{and} \quad G(X) = 0. \]

In order that \( D_q \) be completely integrable it is necessary and sufficient that \( L(X) = 0 \) and \( G(X) = 0 \) be completely integrable i.e.

(3.3.24) \( (a) \ (dL)(X,Y) = 0 \quad \text{and} \quad (b) \ (dG)(X,Y) = 0 \)

be satisfied for arbitrary vector fields \( X, Y \) satisfying

(3.3.25) \[ M(X) = X. \]

In consequence of (3.3.24) and (3.3.25) we have

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(3.3.26) (a) \((dL)(M(X), M(Y)) = 0\) and 
(b) \((dG)(M(X), M(Y)) = 0\).

In view of (3.3.4)b, (3.3.5)b, (3.3.8)b and (3.3.26) we have (3.3.16) b,e,f and (3.3.17) b,d.

Again for the integrability of \(D_p\), from (3.3.6)a, (3.3.9)a and (3.3.22) we have

(3.3.27) (a) \(N(X, Y) - N(\bar{X}, \bar{Y}) = 0\) and 
(b) \(\{u(N(\bar{X}, \bar{Y}) + u(N(X, Y)) + u(N(X, \bar{Y})) + u(N(X, Y))\} U = 0\).

And for the integrability of \(D_q\), from equations (3.3.6)b, (3.3.9)b and (3.3.26) we get

(3.3.28) (a) \(N(\bar{X}, \bar{Y}) + N(X, Y) = 0\) and 
(b) \(\{u(N(\bar{X}, \bar{Y}) - u(N(X, Y)) - u(N(X, \bar{Y})) + u(N(X, Y))\} U = 0\).

In view of (3.3.27) and (3.3.28) equation (3.3.18) follows.

3.4. Pseudo D- Conformal Transformation

Definition (3.4.1): Let us consider a transformation \(b\) and the corresponding Jacobian map \(B\) which transforms the structure \(\{F, U, u, g\}\) to the structure \(\{F, T, A, h\}\) such that

(3.4.1) \(B \bar{X} = B \bar{X}\).

(a) \(h(BX, BY) \circ b = e^g (\bar{X}, \bar{Y}) - e^{2u} u(X) u(Y)\),

(b) \(T = e^{-\sigma} BU\),

(c) \(T(BX) \circ b = e^u(X)\),
where $\sigma$ is a differential function, $T$ is vector field and $A$ is a 1-form on $M^3$. Then the transformation is called Pseudo D-conformal transformation [4]

**Theorem (3.4.1).** The structure $\{F,T,A,h\}$ is a Lorentzian paracontact structure.

**Proof:** Barring (3.4.1) and using (3.1.6), (3.4.1) and (3.4.2)c we have

\[ (3.4.3) \quad BX = BX + A(BX) \circ oT. \]

Also in view of (3.1.10)b

\[ (3.4.4) \quad \overline{T} = \overline{\sigma} B \overline{U} = 0. \]

Again barring $X$ and $Y$ in (3.4.2)a and using (3.1.6), (3.1.7), (3.4.1) and (3.4.2)a we get

\[ (3.4.5) \quad h(BX, BY) = h(BX, BY) + A(BX)A(BY). \]

Hence the theorem follows.
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