10.1 Composite Bearing:

The bearing made up of a combination of flat-land and tapered bearing is known as composite bearing (See Figure 10.1). Assume that the composite bearing is stationary at the lower surface transverse to the fluid film where pure sliding is absent and

\[ U = +U \text{ (constant)} \] (10.1.1)

Also assume that the pressure along the \( x \)-direction is very very small as compared to the pressure component along the \( y \)-direction and hence \( \frac{\partial P}{\partial x} \) becomes negligible in comparison with \( \frac{\partial P}{\partial y} \) and we may consider the pressure distribution as a function of \( y \) alone. The fluid film thickness is given by the equations as follows:

\[
\begin{align*}
\text{h} &= \alpha y, \\
\text{where} \\
\alpha &= \frac{h_1 - h_2}{L_1} \\
\text{in terms of taper for the region } L_1 \text{ } \{ \text{10.1.2} \}
\end{align*}
\]

and

\[ h = h_2 \text{ at } y = L_2 \]

for the region \( L_2 \).
Fig. 10.1 Composite bearing.
For the solution of such a composite bearing, all the results of Section (2.4) in one-dimensional form apply. Let the boundary conditions be considered as follows:

**Boundary conditions for the region L₁:**

(i) \[ P = 0 \text{ at } h = h₁ \]  \hspace{1cm} (10.1.3a)

(ii) \[ P = P_c \text{ at } h = h₂ \]

**Boundary conditions for the region L₂:**

Taking the exist edge as the origin, boundary conditions for this region are expressed as follows:

(i) \[ P = 0 \text{ at } y = 0 \]  \hspace{1cm} (10.1.3b)

(ii) \[ P = P_c \text{ at } y = L₂ \]

where \( P_c \) is the pressure common to the region \( L₁ \) and \( L₂ \).

10.2 **Determination of the pressure in the inertia-free rotatory frame of reference.**

With the help of the equations (10.1.1) and (10.1.2) the equation (2.4.3) is reduced to the following differentiation equation for determination of the pressure in the inertia-free rotatory frame of reference for both the regions \( L₁ \) and \( L₂ \):

\[
\frac{d}{dy} \left( h^3 \frac{dP}{dy} \right) = \frac{d}{dy} \left( - \frac{M \rho U}{2} h^3 \right) \]

(10.2.1)

Integrating the above equation with respect to \( y \) we get
\[
\frac{dP_0}{dy} = \frac{M \rho U}{2} \left( \frac{h^3}{h^3} - 1 \right) \tag{10.2.2}
\]

\(h_0\) being the film thickness where the pressure gradient vanishes.

\(P_0\) for the region \(L_1\):

Integrating the equation (10.2.2) with the help of the equation (10.1.2) for the region \(L_1\) we get

\[P_0(h) = -\frac{M \rho U}{2} \frac{L_1}{(h_1-h_2)} \left( \frac{h^3}{2h^3} + h \right) + C_1 \tag{10.2.3}\]

\(C_1\) being arbitrary constant of integration.

Applying the boundary conditions (10.1.3a) in the equation (10.2.3) we get

\[0 = -\frac{M \rho U}{2} \frac{L_1}{(h_1-h_2)} \left( \frac{h^3}{2h^3} + h_1 \right) + C_1 \tag{10.2.4}\]

and

\[P_c = -\frac{M \rho U}{2} \frac{L_1}{(h_1-h_2)} \left( \frac{h^3}{2h^2} + h_2 \right) + C_1 \tag{10.2.5}\]

The equations (10.2.3) and (10.2.5) yield

\[P_0 = P_c + \frac{M \rho U}{2} \frac{L_1}{(h_1-h_2)} \left\{ (h_2-h) + \frac{h^3}{2} \left( \frac{1}{h^2} - \frac{1}{h_2} \right) \right\} \tag{10.2.6}\]

where \(P_c\) is determined with the help of the equations (10.2.4) and (10.2.5) as follows:
\[ P_c = \frac{M \rho U}{2} L_1 \left[ 1 - \frac{h_0^3}{2h_1 h_2^2} (h_1 + h_2) \right] \] \hspace{1cm} (10.2.7)

**P \_c** for the region \( L_2 \):

Integrating the equation (10.2.2) with the help of the equation (10.1.2) for the region \( L_2 \) we get

\[ P_0(y) = \frac{M \rho U}{2} \left( \frac{h_0^3}{h_2^3} - 1 \right)y \] \hspace{1cm} (10.2.8)

On applying the boundary conditions (10.1.3b)

where, \[ P_c = \frac{M \rho U}{2} \left( \frac{h_0^3}{h_2^3} - 1 \right) L_2 \] \hspace{1cm} (10.2.9)

The equations (10.2.7) and (10.2.9) determine \( h_0 \) and \( P_c \) by the following equations:

\[ h_0^3 = \frac{2(L_1 + L_2) h_2^2 h_1^3}{2L_2 h_1^2 + L_2 h_2 (h_1 + h_2)} \] \hspace{1cm} (10.2.10)

and

\[ P_c = \frac{M \rho U L_1 L_2 (h_1 - h_2)(2h_1 + h_2)}{2[2L_2 h_1^2 + L_1 h_2 (h_1 + h_2)]} \] \hspace{1cm} (10.2.11)

On substituting the values of \( h_0^3 \) and \( P_c \) from the equations (10.2.10) and (10.2.11) in the equation (10.2.6) we determine \( P_0 \) for the region \( L_1 \) as follows:
From the equation (10.2.9), it is obvious that the expression for $P_c$ is a function of local $h$, i.e., $h_0$ given by the equation (10.2.10) above and thus $P_o$ will be continuous at the boundary.

### 10.3 DETERMINATION OF THE PRESSURE IN THE INERTIO-ROTATORY FRAME OF REFERENCE

Pressure for the region $L_2$.

With the help of the equations (10.1.1), (10.1.2) and (10.2.8) the equation (2.4.2) assumes the following form:

$$\frac{d}{dy} \left( h^3 \frac{dP}{dy} \right) + \frac{d}{dy} \left( \frac{M \rho U h^3}{2} \right) = 0$$  \hspace{1cm} (10.3.1)

because inertia-contributory terms vanish.

Integrating the equation (10.3.1) with the help of the equation (10.1.2) for the region $L_2$ and application of the boundary conditions (10.1.3b) we get

$$P(L_2) = \frac{M \rho U}{2} \frac{h^3}{h_o^3} \left( \frac{y}{h_2^2} - 1 \right)$$  \hspace{1cm} (10.3.2)

where

$$P_c = \frac{M \rho U}{2} \frac{h^3}{h_o^3} \left( \frac{y}{h_2^2} - 1 \right)L_2$$  \hspace{1cm} (10.3.3)
and \( h_0 \) is determined by the equation (10.2.10).

Thus the equations (10.3.2) and (10.3.3) coincide respectively with the equations (10.2.8) and (10.2.9) of the inertia-free rotatory frame of reference. Hence the pressure distribution in the inertia-free rotatory frame of reference remains unaffected in the inertia-rotatory frame of reference for region \( L_2 \).

Pressure for the region \( L_1 \):

With the help of the equations (10.1.1), (10.1.2) and (10.2.6) the equation (2.4.2) assumes the following form:

\[
\frac{d}{dh} \left( \frac{\alpha \rho h^3}{\mu} \frac{dp}{dh} \right) + \frac{d}{dh} \left( \frac{M \rho^2 U h^3}{2\mu} \right) + \frac{d}{dh} \left[ \frac{M^2 \rho^4 U^2 \alpha}{6720 \mu} \right] \left( -61h^6 + 19h_0^3 h^3 - 12h_0^6 \right) = 0 \quad (10.3.4)
\]

where \( \alpha \) and \( h_0 \) are respectively given by the equation (10.1.2) and (10.2.10).

Integrating the equation with the help of the boundary conditions (10.1.3a) and (10.3.3) we get

\[
P(L_1) = \left[ P_c \left\{ 1 - \frac{\frac{1}{2} - \frac{1}{h_2^2}}{\frac{1}{h_2^2} - \frac{1}{h_1^2}} \right\} + \frac{M \rho U}{2\alpha} \times \right. \\
\left. \frac{(h_2 - h_1)(\frac{1}{h_2^2} - \frac{1}{h_1^2})}{((h_2 - h) - \frac{(\frac{1}{h_2^2} - \frac{1}{h_1^2})}{\frac{1}{h_2^2} - \frac{1}{h_1^2}}} \right].
\]
where \( h_0^3 \) and \( P_c \) are given by equations (10.2.10) and (10.2.11) respectively.

The equation (10.3.5) clearly reproduces the pressure distribution by the quantity within the first bracket \([\ldots]\) in the inertia-free rotatory frame of reference for the region \( L_1 \) determined by the equation (10.2.12) and yield the pressure distribution by the quantity within the second bracket \(<\ldots\>) in the inertia-rotatory frame of reference for the region \( L_1 \) which is directly proportional to \( M^2 \rho^3 U^2 \) and inversely proportional to \( \mu^2 \).

### 10.4 LOAD CAPACITY

The normal load capacity \( W \) of the composite bearing per unit length is given by

\[
W = W(L_1) + W(L_2)
\]  

(10.4.1)

where \( W(L_1) \) and \( W(L_2) \) are loads on the regions \( L_1 \) and \( L_2 \) respectively given by the following:
\[ W(L_1) = \int \frac{h_2}{h_1} BP(L_1) \frac{dy}{dh}dh \tag{10.4.2} \]

and

\[ W(L_2) = \int_{0}^{L_2} BP(L_2) dy \tag{10.4.3} \]

\(B\) being the dimensionless breadth of the bearing.

On substituting the value of \(P(L_1)\) from the equation (10.3.5) in the equation (10.4.2) and using the equations (10.2.10) and (10.2.11) we get

\[
W(L_1) = \left[ \frac{-M \rho U B L_1^2 (h_1 - h_2) [2L_2 (h_1 + h_2) + L_1 h_2]}{4[2L_2 h_1^2 + L_1 h_2 (h_1 + h_2)]} \right] \\
+ \left[ \frac{M^2 \rho^3 U^2 BL_1}{67200 \mu^2 (h_1 + h_2) [2L_2 h_1^2 + L_1 h_2 (h_1 + h_2)]} \right] x
\]

\[
[L_1 h_2 (122 (h_1^6 + h_2^6) + h_1 h_2 x) \\
(61h_1^4 - 312h_1^3 h_2 + 258h_1^2 h_2^2 - 312h_1 h_2^3 + 61h_2^4)] \\
+ 2L_2 h_1^2 (122h_1^5 + 27h_2) \\
- h_1 h_2 (61h_1^3 + 61h_1^2 h_2 + 156h_1 h_2^2 - 129h_2^3)] > \tag{10.4.4}
\]

On substituting the values of \(P(L_2)\) from the equation (10.3.2) in the equation (10.4.3) and using the equation (10.2.10) we get
On substituting the values of \( W(L_1) \) and \( W(L_2) \) from the equations (10.4.4) and (10.4.5) in the equation (10.4.1) we obtain

\[
W(L_2) = \frac{M \rho U L_1^2 L_2^2 B(h_1-h_2)(2h_1+h_2)}{4[2L_2 h_1^2 + L_1 h_2 (h_1+h_2)]} \quad (10.4.5)
\]

which reproduces the load capacity by the quantity within the first bracket \([\ldots]\) in the inertia-free rotatory frame of reference and yields the load capacity by the quantity within the second bracket \(<\ldots>\) in the inertio-rotatory frame of reference.

Inertio-rotatory correction in load capacity defined by the equation (6.2.3) is given by
\[ W_{\text{correc.}} = \frac{M \rho^2 U}{16300 \mu^2 (h_1 + h_2)} x \]

\[ [L_1 h_2 \{122(h_1^6 + h_2^6) + h_1 h_2 \times \]

\[ (61h_1^4 - 312h_1^3 h_2 + 259h_1^2 h_2^2 \]

\[ - 312h_1^2 h_2^3 + 61h_2^4) \} \]

\[ + 2L_2 h_1^2 \{122h_1^5 + 27h_2^5 - h_1 h_2 \times \]

\[ (61h_1^3 + 61h_1^2 h_2 + 156h_1 h_2^2 \]

\[ - 129h_2^3) \}] x \]

\[ l(h_1 - h_2)\{L_2^2(2h_1 + h_2) - 2L_1 L_2 (h_1 + h_2) - L_1^2 h_2^2 \} \}

10.5 SHEAR STRESS, FRICTIONAL FORCE AND FRICTIONAL COEFFICIENT

SHEAR STRESS FOR L_2:

The aggregate of the equations (2.4.4), (2.4.5), (10.1.1), (10.2.8), (10.2.10) and (10.3.2) yields the components of the shear stress for the region L_2 as follows:

\[ (\tau_x)_{L_2} = -\frac{\mu U}{h_2} \frac{M^2 \rho^2 U L_1 h_2^3 (h_1 - h_2)(2h_1 + h_2)}{48\mu [2L_2 h_1^2 + L_1 h_2(h_1 + h_2)]} \quad (10.5.1) \]

and

\[ (\tau_y)_{L_2} = \frac{(-M \rho U h_2)}{12[2L_2 h_1^2 + L_1 h_2(h_1 + h_2)]} \{8L_2 h_1^2 + L_1 (6h_1^2 + h_2) + h_2^2)\} \quad (10.5.2) \]
On substituting the values of $\tau_x$ and $\tau_y$ from the equations (10.5.1) and (10.5.2) in (2.3.3), the shear stress is completely determined for the region $L_2$.

**SHEAR STRESS For $L_1$**:

The aggregate of the equations (2.4.4), (2.4.5), (10.1.1), (10.2.6), (10.2.10), (10.2.11) and (10.3.5) yields the components of the shear stress for the region $L_1$ as follows:

$$(\tau_x)_{L_1} = -\frac{\mu U}{h} + \frac{M \rho^2 U^2 \alpha h^3}{120 \mu h} - \frac{M \rho}{24\mu} x$$

$$< \frac{M \rho U}{2} (h^3 - h^3) + \frac{M^2 \rho^3 U^2 \alpha}{13440 \mu^2} x$$

$$[2(169h^6 - 130h^3 + 3h^6) + h_1^2h_2^2]$$

$$\frac{76h^3}{(h_1+h_2)} - 61(h_1^2 + h_2^2) >$$

and

$$(\tau_y)_{L_1} = -\frac{M \rho U}{12h^2} (3h^3 + h^3) + \frac{M^2 \rho^3 U^2 \alpha}{80640 \mu^2(h_1+h_2)^2h^2} x$$

$$[(h_1+h_2)\{183h^2_1h^2_2(h_1^2+h_2^2) + 94h^6_0$$

$$+ 22h^3_0h^3_1 - 94h^6_0$$

$$- 22h^2_1h^2_2h^3_0 \}$$

On substituting the values of $\tau_x$ and $\tau_y$ from the equations (10.5.3) and (10.5.4) in (2.3.3), the shear stress is completely determined for the region $L_1$. 
The frictional force on the composite bearing is given by

\[ F_j = (F_j)(L_1) + (F_j)(L_2) \]  \hfill (10.5.5)

where \((F_j)(L_1)\) and \((F_j)(L_2)\) are the frictional forces on the regions \(L_1\) and \(L_2\) respectively given by the following:

\[
(F_j)(L_1) = \int_{h_1}^{h_2} (\tau_y)_{L_1} \frac{dy}{dh} \]  \hfill (10.5.6)

and

\[
(F_j)(L_2) = \int_{h_0}^{h_2} (\tau_y)_{L_2} dy \]  \hfill (10.5.7)

On substituting the value of \((\tau_y)_{L_1}\) from the equation (10.5.4) in the equation (10.5.6) we get

\[
(F_j)(L_1) = \frac{M \rho U L_1}{12} \left\{ \frac{3h_0^3}{h_1 h_2} + \frac{(h_1 + h_2)}{2} \right\} + \frac{M^2 \rho^3 u^2 (h_2 - h_1)}{80640 \mu^2} \times \]

\[
[183h_1 h_2 (h_1^2 + h_2^2) + h_0^3 \{ 11(h_1 + h_2) - \frac{228 h_1 h_2}{(h_1 + h_2)} \} \]

\[- \frac{94}{5} \left\{ h_1^4 h_2^3 + h_1^2 h_2^4 + h_1^3 h_2^2 + h_1 h_2^4 \right\} + \frac{84 h_0^6}{h_1 h_2} \} \]  \hfill (10.5.8)

On substituting the value of \((\tau_y)_{L_2}\) from the equation (10.5.2) in the equation (10.5.7), we get

\[
(F_j)(L_2) = \frac{-M \rho U h_2 L_2 \left[ 8L_2 h_2^2 + L_1 (6h_1^2 + h_1 + h_2) \right]}{12 \{ 2L_2 h_1^2 + L_1 h_2 (h_1 + h_2) \}} \]  \hfill (10.5.9)
The equation (10.5.5), (10.5.8) and (10.5.9) yield the frictional force on the composite bearing as follows:

\[
P_j = < \frac{M \rho U}{12} \left[ L_1 \left\{ \frac{3h_1^3}{h_1 h_2} + \frac{(h_1 + h_2)}{2} \right\} \right.
\]
\[
- \frac{h_2 L_2 \left[ 6L_2 h_1^2 + L_1 (6h_1^2 + h_1 h_2 + h_2^2) \right]}{\left\{ 2L_2 h_1^2 + L_1 h_2 (h_1 + h_2) \right\}} \left] > \right.
\]
\[
+ \frac{M^2 \rho^3 u^2 (h_2 - h_1)}{30640 \mu^2} \times
\]
\[
\left[ 183h_1 h_2 (h_1^2 + h_2^2) + h_0 \left\{ 11(h_1 + h_2) - \frac{228}{(h_1 + h_2)} \right\} \right.
\]
\[
- \frac{94}{5} \left\{ h_1^4 + h_1^2 h_2 + h_2^4 + h_1 h_2^3 + h_1^3 h_2 \right\},
\]
\[
+ \frac{84h_6}{h_1 h_2} \right] > \) \tag{10.5.10}
\]

which reproduces the frictional force by the quantity within the first bracket \(<\cdot>\) in the inertia-free rotatory frame of reference and yields the frictional force by the quantity within second bracket \(<\cdot>\) in the inertia-rotatory frame of reference.

The frictional coefficient for the composite bearing is determined on substituting the values of \(W\) and \(P_j\) respectively from the equations (10.4.6) and (10.5.10) in the equation (3.3.5) as follows:
\[ f = \frac{\{2L_2h_1^2 + L_1h_2(h_1 + h_2)\}}{3BL_1} \times \]

\[ < \left[ L_1 \left\{ \frac{3h_0^3}{h_1h_2} + \frac{(h_1 + h_2)}{2} \right\} \right. \]

\[ - \frac{h_2L_2}{2} \frac{8L_2h_1^2 + L_1(6h_1^2 + h_1h_2 + h_2^2)}{\{2L_2h_1^2 + L_1h_2(h_1 + h_2)\}} \]

\[ M \rho^2 \left( \frac{U(h_2 - h_1)}{6720 \mu^2} \times \right. \]

\[ \left[ 183h_1h_2(h_1^2 + h_2^2) + h_0^3 \{11(h_1 + h_2) - \frac{228h_1h_2}{(h_1 + h_2)^3}\} \right. \]

\[ - \frac{94}{5} \{h_1^4 + h_1^3h_2 + h_1^2h_2^2 + h_1h_2^3 + h_2^4\} \]

\[ + \frac{84h_0^6}{h_1h_2} \left] > x \right. \]

\[ < \{L_2^2(2h_1 + h_2) - 2L_1L_2(h_1 + h_2) - L_1^2h_2(1 + h_2)\} \]

\[ M \rho^2 \left( \frac{U}{16800 \mu^2(h_1 + h_2)} \times \right. \]

\[ \left[ L_1h_2 \{122(h_1^6 + h_2^6) + h_1h_2 \times \right. \]

\[ (61h_1^4 - 312h_1^3h_2 + 259h_1^2h_2^2 - 312h_1h_2^3 + 61h_2^4)\} \]

\[ + 2L_2h_1^2 \{122h_1^5 + 27h_2^5 \]

\[ - h_1h_2(61h_1^3 + 61h_1^2h_2 + 156h_1h_2^2 - 129h_2^3)\} \left] > -1 \right. \]

\[ (10.5.11) \]
The conclusions similar to those of the Section (3.4) are easily derived for the composite bearings also. The important exceptional conclusion for this bearing is that though the flat-land portion of the bearing is independent of the effect of the pressure in the inertio-rotatory frame of reference yet other ingredients like load capacity, frictional force and frictional coefficient do affect this portion in the inertio-rotatory frame of reference.