CHAPTER V

SCALE PARAMETER ESTIMATION IN EXPOENTIAL AND NORMAL DISTRIBUTIONS

The problem of statistical inference in life time data has attracted several researchers and in the present investigation so far we considered the problem of estimation and testing of parameters for exponential distributions. However, many instances have been cited when normal distribution provides quite a good fit to the life data. Therefore this Chapter is divided into two parts; the first one deals with estimation of common scale parameter in $k$ exponential distributions and the other investigates the problem of scale parameter estimation in normal distribution case when coefficient of variation is assumed to be known.
5A. ESTIMATION OF COMMON SCALE PARAMETER OF TWO-PARAMETER EXPONENTIAL DISTRIBUTIONS

5A.1 INTRODUCTION

In the field of life testing experiments when a producer of a costly electronic item wants to find out the average excess life of his products on the basis of \( k \) sets of items, the estimation of common scale parameter (common average excess life) requires due attention. It has been found in many problems that there are occasions when a two-parameter exponential distribution is most appropriate for fitting life test data.

Consider that \( x_{(11)} \leq x_{(12)} \leq \ldots \leq x_{(1n_1)} \); \( x_{(21)} \leq x_{(22)} \leq \ldots \leq x_{(2n_2)} \); \( \ldots; x_{(k1)} \leq x_{(k2)} \leq \ldots \leq x_{(kn_k)} \) be \( k \) samples \( (S_1, S_2, \ldots, S_k) \) from distributions having pdf’s \( f(x, A_1, \theta) \), \( f(x, A_2, \theta) \), \ldots, \( f(x, A_k, \theta) \), respectively, where

\[
f(x, A_j, \theta) = \frac{1}{\theta} \exp \left[ -\frac{(x - A_j)}{\theta} \right], \quad x > A_j, \quad \theta > 0
\]

Here \( A_j \)'s are the location parameters and \( \theta \) is the common scale parameter. Let \( S_{r_1}, S_{r_2}, \ldots, S_{r_k} \) be the sets of the first \( r_1, r_2, \ldots, r_k \) smallest observations of \( S_{n_1}, S_{n_2}, \ldots, S_{n_k} \).
respectively and the constants $r_j$'s and $n_j$'s are fixed and preassigned i.e. the samples are type II censored from exponential distributions with common scale parameter $\theta$.

Three different situations are considered:

Situation I: When location parameters are known,

Situation II: When location parameters are equal but common value is unknown,

Situation III: When location parameters are unknown.

Epstein and Sobel (1954) proposed a minimum variance unbiased estimator for common scale parameter $\theta$, using $k$ sets of observations under each situation as follows:

$$\hat{\theta}_1 = \frac{\sum_{j=1}^{k} V_j}{R}, \text{ for situation I}$$

$$\hat{\theta}_2 = \frac{\sum_{j=1}^{k} V_j^*}{(R-1)}, \text{ for situation II}$$

and

$$\hat{\theta}_3 = \frac{\sum_{j=1}^{k} V_j}{(R-k)}, \text{ for situation III}$$

where

$$V_j = \sum_{i=1}^{r_j} (x_{ji} - A_j) + (n_j - r_j)(x_{jr_j} - A_j), \text{ for } j=1,2,\ldots,k$$
\[
V_j^* = \sum_{i=1}^{r_j} (x_{ji} - \hat{\Lambda}) + (n_j - r_j)(x_{j\cdot} - \hat{\Lambda}) \quad , j=1,2,\ldots,k
\]

\[
V_j = \sum_{i=1}^{r_j} (x_{ji} - \hat{\Lambda}_j) + (n_j - r_j)(x_{j\cdot} - \hat{\Lambda}_j) \quad , j=1,2,\ldots,k
\]

\[
\hat{\Lambda} = \min(x_{11}, x_{21}, \ldots, x_{k1}),
\]

\[
\hat{\Lambda}_j = x_{j1},
\]

and

\[
R = \sum_{j=1}^{k} r_j
\]

They have shown that the distribution of \( \hat{\Theta} \) (i.e. \( \hat{\Theta}_1, \hat{\Theta}_2 \) and \( \hat{\Theta}_3 \)) depend only on \( R, \Theta \) (and in case 3 also on \( k \)). They have also shown that

\[
\sum_{j=1}^{k} 2V_j / \Theta, \quad \sum_{j=1}^{k} 2V_j^* / \Theta \quad \text{and} \quad \sum_{j=1}^{k} 2V_j / \Theta
\]

are distributed as chi-square with \( 2R, 2(R-1) \) and \( 2(R-k) \) degrees of freedom in situation I, situation II and situation III respectively.
For improving an existing estimator $T$, Goodman (1953) considered a class of estimators $dT$, $d$ being a scalar and determined $d$ such that MSE of $dT$ is minimum. Many authors including Searls (1964) and Singh et al. (1973) also suggested simple methods for improving some estimators. The minimum mean squared error (MMSF) estimators of common scale parameter $\theta$ using two sets of observations have been given by Pandey and Singh (1979).

In this part of the Chapter, we have considered the problem of estimation of common scale parameter $\theta$ of $k$ two-parameter exponential distributions. In section 5A.2 improved estimator of $\theta$ under each of the three situations has been proposed and the expression for MSE has been derived. Consideration of efficiency of proposed estimators relative to $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$ under each situation, respectively and conclusions have also been discussed.
5A.2 THE PROPOSED ESTIMATORS AND THEIR PROPERTIES

5A.2.1 Estimator for Situation I:

Let us consider a general class of linear estimators

\[ P_1 = \sum_{j=1}^{k} l_j v_j, \]  

(5A.2.1.1)

where \( l_j \)'s are unknown scalars to be determined such that the MSE of \( P_1 \) is minimum.

Noting that \( \sum_{j=1}^{k} 2v_j/\theta \) is chi-square variate with \( 2R \) degrees of freedom whose distribution depends on \( \theta \) and \( R \) only. Hence

\[ \text{MSE}(P_1) = \left[ \sum_{j=1}^{k} l_j^2 r_j (1+r_j) + 2 \sum_{j<j'}^{k} l_j l_{j'} r_j r_{j'} - 2 \sum_{j=1}^{k} l_j r_j + 1 \right] \theta^2 \]  

(5A.2.1.2)

Differentiating \( \text{MSE}(P_1) \) with respect to \( l_j \)'s and equating them to zero, we find the values of \( l_j \)'s that minimize the MSE\((P_1)\). These are

\[ l_1 = l_2 = \ldots = l_k = 1/(R+1) \]  

(5A.2.1.3)

Substituting these values of \( l_j \)'s in (5A.2.1.1), we get the proposed estimator for this situation as
and the corresponding MSE is given by

\[ \text{MSE}(P^*_1) = \frac{\sigma^2}{(R+1)} \]  

(5A.2.1.5)

The relative efficiency of \( P^*_1 \) with respect to \( \hat{\theta}_1 \) (minimum variance unbiased estimator) is

\[ \text{RE}(P^*_1, \hat{\theta}_1) = \frac{\text{MSE}(\hat{\theta}_1)}{\text{MSE}(P^*_1)} = 1 + \frac{1}{R} \]  

(5A.2.1.6)

From equation (5A.2.1.6), we conclude that the proposed estimator \( P^*_1 \) is better than \( \hat{\theta}_1 \) in the sense of minimum mean square. The gain is more for smaller values of \( R \).

5A.2.2 Estimator for Situation II:

Let us consider a general class of linear estimators

\[ P^*_2 = \sum_{j=1}^{k} m_j \sqrt{v_j} \]  

(5A.2.2.1)

where \( m_j \)'s are \( k \) unknown scalars. To obtain an improved estimator we determine \( m_j \)'s such that \( \text{MSE}(P^*_2) \) is minimum.

Since \( \sum_{j=1}^{k} \frac{2v_j^*}{\theta} \) is a chi-square variate with \( 2(R-1) \)
degrees of freedom and its distribution is depends on only $R$ & $\theta$. Hence

$$
\text{MSE}(P_2) = \left[ \frac{2}{m^2_i} r_i (r_i - 1) + \sum_{j \neq 1}^{k} \frac{2}{m^2_j} r_j (r_j + 1) + 2m_i (r_i - 1) \sum_{j \neq 1}^{k} m_j r_j 
+ 2 \sum_{(j \neq j')}^{k} \sum_{i=1}^{k} m_j m_j' r_j r_j' - 2 \left\{ m_1 (r_1 - 1) + \sum_{j \neq 1}^{k} m_j r_j \right\} + 1 \right] \theta^2
$$

(5A.2.2.2)

Differentiating MSE($P_2$) with respect to $m_j$'s and equating it to zero, we find the values of $m_j$'s ($j=1,2,\ldots,k$) which minimize the MSE($P_2$). The $m_j$'s so obtained are

$$
m_1 = m_2 = \ldots = m_k = 1/R
$$

(5A.2.2.3)

Substituting these optimum values of $m_j$'s in (5A.2.2.1) we get the proposed estimator for $\theta$ is as

$$
P_2^* = \sum_{j=1}^{k} v_j^*/R
$$

(5A.2.2.4)

with

$$
\text{MSE}(P_2^*) = \frac{\theta^2}{R}
$$

(5A.2.2.5)

The relative efficiency $P_2^*$ with respect to minimum variance unbiased estimator $\hat{\theta}_2$ is

$$
\text{RE}(P_2^*, \hat{\theta}_2) = \frac{\text{MSE}(\hat{\theta}_2)}{\text{MSE}(P_2^*)} = 1 + 1/(R-1)
$$

(5A.2.2.6)
Since $R > 1$, the relative efficiency is always greater than 1 and the larger gains in relative efficiency are achieved for smaller values of $R$. Thus the proposed estimator $\hat{P}_2^*$ is better than the minimum variance unbiased estimator $\hat{\theta}_2$ in the sense of minimum mean square.

5A.2.3 Estimator for Situation III:

Let us consider a general class of linear estimators

$$P_3 = \sum_{j=1}^{k} b_j V_j,$$  \hspace{1cm} (5A.2.3.1)

which involves $k$ unknown scalars $b_j$'s and is a class of estimators of type $d\hat{\theta}_3$. We determine the scalars $b_j$'s so as to minimize the MSE of $P_3$.

Since $\sum_{j=1}^{k} 2V_j/\theta$ is a chi-square variate with $2(R-k)$ degrees of freedom and its distribution is depends only on $\theta$, $k$ and $R$.

Hence

$$\text{MSE}(P_3) = \sum_{j=1}^{k} b_j r_j (r_j - 1) + 2 \sum_{j<k}^{k} b_j b_j (r_j - 1)(r_j - 1)$$

$$-2 \sum_{j=1}^{k} b_j (r_j - 1) + 1 \theta^2$$ \hspace{1cm} (5A.2.3.2)
Differentiating $\text{MSE}(P_3^*)$ with respect to $b_j$'s and equating them to zero, we find the values of $b_j$'s which minimize the $\text{MSE}(P_3^*)$. Thus we get

$$b_1 = b_2 = \ldots = b_k = 1/(R-k+1)$$

(5A.2.3.3)

Substituting these values of $b_j$'s in (5A.2.3.1), we get

$$P_3^* = \sum_{j=1}^{k} \frac{V_j}{(R-k+1)}$$

(5A.2.3.4)

and the corresponding MSE is

$$\text{MSE}(P_3^*) = \sigma^2/(R-k+1)$$

(5A.2.3.5)

The relative efficiency of $P_3^*$ with respect to minimum variance unbiased estimator $\hat{\theta}_3$ is

$$\text{RE}(P_3^*, \hat{\theta}_3) = \frac{\text{MSE}(\hat{\theta}_3)}{\text{MSE}(P_3^*)} = 1 + 1/(R-k)$$

(5A.2.3.6)

If $R > k$, $\text{RE}(P_3^*, \hat{\theta}_3) > 1$ and the larger gains in relative efficiency are obtained for smaller values of $R$. Thus the proposed estimator $P_3^*$ is better than $\hat{\theta}_3$ in the sense of minimum mean square.

Remarks: 1. For $k=2$ we get the same results as that of Pandey and Singh (1979).

2. In case if we consider complete samples instead of censored ones, the results obtained above are still true with the change that $r_j$'s are replaced by $n_j$'s.
5B. EFFICIENT ESTIMATION OF VARIANCE OF A NORMAL DISTRIBUTION USING ITS KNOWN COEFFICIENT OF VARIATION

5B.1 INTRODUCTION

In previous Chapter we considered the exponential distribution as a model for life time distributions whose parameters estimation and testing problems have been widely studied. On the other hand, suitability of normal distribution as a life time model has also been highlighted by many authors including Davis (1952) and Bazovsky (1961), among others. It gives quite a good fit for failure time data relating to, for example, failure times for a large number of incandescent lamps. Therefore, the present part of the Chapter deals with the problem of estimation of scale parameter \( \sigma^2 \) in \( N(\theta, \sigma^2) \) when CV is known.

Let us consider a random sample of \( n \) observations \( x_i, i=1,2,\ldots,n \) from a life test experiment assuming that the failure time distribution is \( N(\theta, \sigma^2) \). The CV, say, \( C=\sigma/\theta \) is assumed to be known.

Let

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i,
\]

and

\[
s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2,
\]
be the sample mean and the sample variance respectively. The maximum likelihood estimators of $\theta$ and $\sigma^2$ respectively are $\bar{x}$ and $(n-1)s^2/n$. It is well known that $\bar{x}$ and $s^2$ are UMVU estimators of $\theta$ and $\sigma^2$ respectively.

The present investigation is concerned with the situation where the CV is known. Snedecor (1946, pp 47-49) gave several examples when the CV's are known. For instance, the status of girls or boys in a certain age group has a constant CV of 3.75%, or the body weight of male albino rats between 90 and 243 days of age has an average CV of about 14%. Davis and Goldsmith (1976, pp 40-41) find that the weight variation along the length of a synthetic fibre has a constant CV. Further example are also available in Snedecor (1946) and Hald (1952).

The knowledge of CV is of great practical interest and should be used in statistical analyses, wherever feasible. In fact, it is often used in planning future experiments. Searls (1964) introduced the idea of utilizing known CV as an apriori information in the estimation of mean, and several other authors discussed the problem of estimating the parameters $\theta$ and $\theta^2$ as well in $N(\theta,C\theta^2)$. (cf. Khan (1968), Govindarajulu & Sahai (1972), Singh et al. (1973), Das (1975), Gleser and Healy (1976), Sen (1978,1979), Prasad & Sahai(1983), Singh and Upadhyaya(1985), Singh and Katyar (1988), etc.).
Singh and Pandey (1975) provided an estimate of the variance in a general population. Considering, in particular, a normal population, they suggested the following four estimators of $\sigma^2$.

\[
\hat{\sigma}^2 = \frac{nC^2}{(n+C^2)} x^2 ,
\]

\[
\tilde{\sigma}^2 = C^2 \left( x^2 - \frac{s^2}{n} \right) ,
\]

\[
\hat{\sigma}^*^2 = k^* C^2 \left( x^2 - \frac{s^2}{n} \right) ,
\]

and:

\[
\hat{\sigma}^**^2 = \tilde{\alpha} \hat{\sigma}^2 + (1 - \tilde{\alpha}) s^2 ;
\]

where

\[
k^* = \left[ \frac{2C^4}{n(n-1)} + \frac{4C^2}{n} + 1 \right]^{-1} ,
\]

and $\tilde{\alpha}$ lies between 0 and 1 and the value of $\tilde{\alpha}$ is given by

\[
\tilde{\alpha} = \lambda_2 / (\lambda_1 + \lambda_2) ,
\]

where

\[
\lambda_1 = \left[ \left( \frac{n^2 (3b^2 - 2)}{(n+C^2)^2} \right) - 1 \right] ; b = 1 + C^2/n
\]
and

\[ \lambda_2 = \frac{2}{(n-1)}. \]

With such an optimum choice of \( \tilde{\alpha} \), it has been shown by Singh and Pandey (1975) that for small \( n \) and \( C \), the estimator \( \sigma^{**2} \) will be more efficient than all others as well as the usual estimator \( s^2 \).

We may write a family of estimators of \( \sigma^2 \) as

\[ T = a_1 \tilde{\alpha}^2 + a_2 s^2 \]  \hspace{1cm} (58.1.1)

where \( a_1 \) and \( a_2 \) are scalars.

It is easily noted that all estimators proposed by Singh and Pandey (1975) are members of (58.1.1).

In the present part of this Chapter, we have consider the problem of estimation of variance of a normal distribution exploiting the apriori information in terms of CV 'C'. In section (58.2), another family of estimators for \( \sigma^2 \) has been proposed and the MMSE in it has been obtained. The expressions for Bias and MSE have also been developed. The evaluation of efficiency of this estimator vis-a-vis \( \sigma^{**2} \) of Singh and Pandey (1975) constitutes section (58.3).
5B.2. THE PROPOSED ESTIMATOR

Let us consider a class of estimators, linear in \( x^2 \), \( s^2 \) and \( x^3/s \), as follows

\[ Y = W_1 x^2 + W_2 s^2 + W_3 x^3/s , \]  

(5B.2.1)

where \( W_1 \), \( W_2 \) and \( W_3 \) are suitably chosen constants to be determined such that MSE of \( Y \) is minimum. This has been achieved by introducing one more linearly additive component in (5B.1.1).

To evaluate the Bias and MSE of \( Y \), we use the knowledge that \( \bar{x} \) and \( s^2 \) are independently distributed and \( (n-1)s^2/\theta^2 \) is a chi-square variate with \( (n-1) \) degrees of freedom while \( \bar{x} \) is a normal variate with mean \( \theta \) and variance \( C^2\theta^2/n \). Therefore, we have

\[ E(s^m) = \left( \frac{2}{n-1} \right)^{m/2} \Gamma((n+m-1)/2) \left[ \left( \frac{C^2\theta^2}{\theta^2} \right) \right]^{m/2} / \Gamma((n-1)/2) ; m=\pm1,\pm2,4. \]  

(5B.2.2)

Writing

\[ P(m) = \left( \frac{2}{n-1} \right)^{m/2} \Gamma\left((n+m-1)/2\right) / \Gamma((n-1)/2) , \]

\[ \sigma^m = \left[ \left( \frac{C^2\theta^2}{\theta^2} \right) \right]^{m/2} \]
We easily note that

\[
\begin{align*}
E(x^2) &= \beta_1 \sigma^2, \quad E(x^3) = \beta_2 \sigma^3, \quad E(x^4) = \beta_3 \sigma^4 \\
E(x^5) &= \beta_4 \sigma^5, \quad E(x^6) = \beta_5 \sigma^6
\end{align*}
\]

(5B.2.3)

where

\[
\beta_1 = \frac{b}{C^2}, \quad \beta_2 = \frac{(3b - 2)}{C^3}, \quad \beta_3 = \frac{(3b^2 - 2)}{C^4},
\]

\[
\beta_4 = \frac{(15b^2 - 20b + 6)}{C^5},
\]

and

\[
\beta_5 = \frac{(15b^3 - 30b + 16)}{C^6}.
\]

Applying (5B.2.2) and (5B.2.3) for expectation of \( Y \), we get

\[
E(Y) = \left[ W_1 \beta_1 + W_2 P(2) + W_3 \beta_2 P(-2) \right] \sigma^2
\]

(5B.2.4)

The Bias(\( Y \)), expressed as a fraction of \( \sigma^2 \) is given by

\[
\text{Bias}(Y)/\sigma^2 = \left[ W_1 \beta_1 + W_2 P(2) + W_3 \beta_2 P(-1) \right] - 1
\]

(5B.2.5)

Similarly, the MSE(\( Y \)) is given by

\[
\text{MSE}(Y) = \left[ 1 + W_1^2 \beta_3 + W_2^2 P(4) + W_3^2 \beta_5 P(-2) + 2W_1 W_2 \beta_1 P(2) + 2W_1 W_3 \beta_4 P(-1) \right. \\
+ 2W_2 W_3 \beta_2 P(1) - 2W_1 \beta_1 - 2W_2 P(2) - 2W_3 \beta_2 P(-1) \] \sigma^4
\]

(5B.2.6)
Differentiating (5B.2.6) with respect to $W_1$, $W_2$, and $W_3$ partially and equating them to zero, we get the following system of normal equations,

\[
\begin{align*}
W_1 &= \frac{\partial}{\partial W_1} \beta_1 + \frac{\partial}{\partial W_1} \beta_1^p(2) + \frac{\partial}{\partial W_1} \beta_4^p(-1) = \beta_1 \\
W_2 &= \frac{\partial}{\partial W_2} \beta_1^p(2) + \frac{\partial}{\partial W_2} \beta_2^p(4) + \frac{\partial}{\partial W_2} \beta_2^p(1) = \beta_2^p(4) \\
W_3 &= \frac{\partial}{\partial W_3} \beta_1^p(4) + \frac{\partial}{\partial W_3} \beta_2^p(-1) + \frac{\partial}{\partial W_3} \beta_3^p(-2) = \beta_3^p(-1)
\end{align*}
\]

Using Cramer's rule, for solving (5B.2.7) we get the solution $W^*$, $W_2^*$ and $W_3^*$ as

\[
W^*_1 = \begin{vmatrix} \beta_1 & \beta_1^p(2) & \beta_4^p(-1) \\ \beta_1^p(2) & \beta_2^p(4) & \beta_2^p(1) \\ \beta_4^p(-1) & \beta_2^p(1) & \beta_5^p(-2) \end{vmatrix} / \Delta 
\]

(5B.2.8)

\[
W^*_2 = \begin{vmatrix} \beta_3 & \beta_1 & \beta_4^p(-1) \\ \beta_3^p(1) & \beta_2^p(2) & \beta_2^p(1) \\ \beta_4^p(-1) & \beta_2^p(1) & \beta_5^p(-2) \end{vmatrix} / \Delta 
\]

(5B.2.9)

\[
W^*_3 = \begin{vmatrix} \beta_3 & \beta_1^p(2) & \beta_1 \\ \beta_3^p(1) & \beta_2^p(4) & \beta_2^p(1) \\ \beta_4^p(-1) & \beta_2^p(1) & \beta_5^p(-1) \end{vmatrix} / \Delta 
\]

(5B.2.10)

where
Thus, the MMSE estimator in the family (5B.2.1) is

\[ Y^* = \tilde{w}_1 \hat{\beta}_1 \hat{\kappa}^2 + \tilde{w}_2 \hat{\kappa} \hat{S}^2 + \tilde{w}_3 \hat{\kappa} \hat{s}^2 / \hat{s} \]

(5B.2.12)

The corresponding Bias and MSE are

\[ \text{Bias}(Y^*) = -\frac{\text{MSE}(Y^*)}{\sigma^2} , \]

(5B.2.13)

and

\[ \text{MSE}(Y^*) = \left[ 1 - \tilde{w}_1 \hat{\beta}_1 - \tilde{w}_2 \hat{\beta}_2 (2) - \tilde{w}_3 \hat{\beta}_2 (1) \right] \sigma^4 \]

(5B.2.14)
SB.3 EFFICIENCY OF $Y^*$

In order to compare the proposed estimator $Y^*$ with that of Singh and Pandey (1975) estimator $\sigma^{*2}$, we define the relative efficiency as

$$RE(Y^*, \sigma^{*2}) = \frac{\text{MSE}(\sigma^{*2})}{\text{MSE}(Y^*)}$$

$$= \left(\lambda_1 \lambda_2 \right) \left[ \lambda_1 \lambda_2 \left( 1 - \frac{w^*_{1\theta}}{w^*_{2\theta}} - \frac{w^*_{1\theta}}{w^*_{3\theta}} (-1) \right) \right]^{-1}$$

(5B.3.1)

The relative efficiency of $Y^*$ is noted to be a function of $n$ and $\sigma^2$. Therefore it has been evaluated for some chosen sets of values of $n$ and $\sigma^2$, namely,

$$n = 5, 10, 20, \quad \sigma^2 = 0.2, 0.5, 1.0, 2.0, 5.0, 10.0.$$

The results have been summarised in Table 5B.4.1. From these results we observe that the proposed estimator $Y^*$ has smaller MSE as compared to Singh and Pandey (1975) estimator $\sigma^{*2}$ for all the chosen values of $n$ and $\sigma^2$. We further observe that the gain in efficiency is rapid for small $n$ when $\sigma^2$ increases. Thus, for given $\sigma^2$, particularly a larger value, use of the proposed estimator $Y^*$ in preference to Singh and Pandey (1975) estimator $\sigma^{*2}$ is advisable, more so if $n$ is small.
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