CHAPTER IV
ELECTROMAGNETIC MASS DIFFERENCES OF HADRONS

The electromagnetic mass differences (EMMD) of hadrons have been studied within the framework of symmetry schemes\textsuperscript{56}, quark models\textsuperscript{58-70} and bag models\textsuperscript{114}. In several attempts\textsuperscript{58-61}, the naive quark model has been used to predict a number of relations among the EMMD's of hadrons. Using the assumptions that electromagnetic forces may have spin dependence, Rubinstein et al\textsuperscript{57}, have obtained the famous Coleman-Glashow relation\textsuperscript{71},

\[ (\Xi^0 - \Xi^{-}) = (n-p) + (\Sigma^+ - \Sigma^-), \]

\[ (-6.4 \pm 0.6 \text{ MeV.}) \quad (-6.68 \pm 0.08 \text{ MeV.}) \]

earlier derived using group theoretical consideration.

**Electromagnetic mass differences in quark model**

In quark model the electromagnetic mass splittings have been discussed\textsuperscript{58-61} assuming that electromagnetic mass shifts in hadrons arise from:

i) the one photon exchange; and

ii) the strong isospin violation.

The one photon exchange shift can be well understood in terms of electromagnetic interaction energy, where as the isospin breaking has been attributed to the difference in masses of up and down quarks, one may write the electromagnetic interaction potential as:

\[ V_{ij} = \alpha \frac{Q_i Q_j}{r_{ij}} - \frac{g \pi}{3} \bar{\mu}_i \bar{\mu}_j \delta^3(r_{ij}) \]  

(4.2)

where \( Q_i \) and \( \bar{\mu}_i \) are the charge, and magnetic moment of the \( i \)th quark, respectively. \( \alpha \) is the fine structure constant. The
expectation values $\langle \frac{1}{r_{ij}} \rangle$, and $\langle \delta^3(r_{ij}) \rangle$ are in general different for each isospin multiplets of hadrons, so no useful relation can be obtained. However, some inequalities among the EMMD's of hadrons can be obtained\textsuperscript{58-61}. If one assumes that above expectation values with respect to baryon wavefunctions depend only on the flavor and spin configurations of the $i$th and $j$th quarks and not on the third quark in the baryon, then one can obtain\textsuperscript{58-61} some mass relations which have earlier been derived in symmetry considerations. To estimate the electromagnetic mass differences, it was further assumed that the expectation values $\langle \frac{1}{r_{ij}} \rangle$, and $\langle \delta^3(r_{ij}) \rangle$ are independent of quark flavor and spin. With this approximation the mass differences are written in terms of three unknown parameters,

$$
\delta_m = (m_d - m_u), \\
a = \alpha \langle \frac{1}{r_{ij}} \rangle, \text{ and} \\
b = \frac{8\pi \alpha}{3} \langle \delta^3(r_{ij}) \rangle \quad (4.3)
$$

whose values are determined from the known EMMD's and rest of the splittings have been predicted. It is found by Itoh et al.\textsuperscript{60} that the predictions for mesons are considerably different from experimental data, whereas, in case of baryons, a reasonable agreement is observed. However, if one follows DGG\textsuperscript{38} and takes the quark magnetic moments to be proportional to their charges and inversely proportional to their effective masses, then the situation improves. Many authors\textsuperscript{69}, arguing that these discrepancies may be due to the use of non-relativistic quark model and/or the neglect of gluon contribution to EMMD's, have included such
contributions. It is noticed that the spin interaction term due to one gluon exchange on analogy with electromagnetic interaction gives some positive contributions to EMMD’s due to the mass difference \((m_d - m_u)\). Further it is also observed\(^{69}\) that if the distance between two quarks in symmetric state and in mixed state is taken same, the interaction hamiltonian does not give any impressive improvement over the results. However, if two distances are taken different which may be due to spin-spin interaction, a definite improvement is observed. But in the same formulism one obtained positive value of neutron charge radius in contradiction with experiment. However, this discrepancy is resolved when the general form of the quark-quark interaction hamiltonian (DGG) with QCD inspired is used and light quarks are taken to behave in a relativistic manner. Chan\(^{68}\), assuming that zero point energy and electromagnetic forces as flavor independent, has successfully studied the EMMD’s in DGG model. Capastick\(^{114}\), suggesting that the quark motion inside baryon tends to reduce the wavefunction of it and smaller states have larger electrostatic interactions, reproduced the good results of NRQM, for isospin splittings and the charge radii were found compatible with measurements. In particular \((\Sigma_c^0 - \Sigma_c^{++})\) was found in qualitative agreement with experiment.

Here we wish to discuss the EMMD’s of hadrons (particularly of heavier hadrons) in three different approaches. In approach first, the contributions from \((m_d - m_u)\) mass difference and electromagnetic interaction are considered, whereas in approach second in addition
to them, the gluonic contribution within DGG potential is included to see the relative merit of the contributions. In approach third, one more additional contribution such as relativistic contribution along with the spatial dependence of the wavefunction is incorporated and EMMMD's are estimated. In the last we will compare present results obtained in three approaches with experiment which will definitely give a relative merit of the various contributions at different quark flavor level.

**Approach I:**

The sources which constitute EMMMD's are:

i) up and down quark mass difference; and

ii) the electromagnetic interaction.

The total electromagnetic mass difference is:

$$\text{(Am)}_{\text{total}} = \delta m + \text{(Am)}_{\text{electrostatic}} + \text{(Am)}_{\text{magnetic}} \quad (4.4)$$

The hadron mass can be written as:

$$M(h) = \mu(h) + \sum_{i} m_i + a \sum_{i > j} Q_i Q_j + b \sum_{i > j} \frac{Q_i Q_j}{m_i m_j}, \quad (4.5)$$

where $Q_i$ and $m_i$ are the charge and mass of the $i$th quark, respectively, and $a$ and $b$ are constants.

For baryons, the values of the parameters involved can be determined using known mass differences $(n-p), \left( \Xi^- - \Xi^0 \right), \text{and } (\Sigma^* - \Sigma^{*0})$ as input which come out to be:

$$\delta m = 2.39 \text{ MeV}, \quad a = 11.13 \text{ MeV}, \text{ and}$$

$$\frac{b}{\delta m^2} = K = -0.87 \text{ MeV}, \quad (4.6)$$
where we have used the quark masses:

\[ m_u = m_d = 335.7 \text{MeV}, \quad m_s = 512.5 \text{MeV}, \]
\[ m_c = 1674 \text{MeV}, \quad m_b = 5024 \text{MeV}, \quad \text{and} \]
\[ m_t = 60,000 \text{MeV}. \]  
\( (4.7) \)

In case of mesons with known \((\pi^+ - \pi^0); (K^- - K^0),\) and \((D^+_c - D^+_c)\) difference as input, the parameters are:

\[ \delta_m = 3.89 \text{MeV}, \quad a = 0.805 \text{MeV}, \quad \text{and} \]
\[ K = -0.027 \text{MeV}. \]  
\( (4.8) \)

Using these values of parameters, the electromagnetic mass differences for different isodoublet baryons and mesons are calculated and are given in table (4.1) and table (4.2), respectively.

**Approach - II**:

Here it is assumed that heavier hadron mass splittings are arising from:

i) the difference in the masses of up and down quark;

ii) the photon exchange; and

iii) the strong isospin violation due to intrinsic up and down quark mass difference (gluon exchange).

The total mass splitting of an isodoublet is

\[ (\Delta m)_{\text{total}} = \delta_m + (\Delta m)_{\text{electro.}} + (\Delta m)_{\text{mag.}} + (\Delta m)_{\text{gluon}}. \]  
\( (4.9) \)

The hadron masses are written as

\[ M(h) = \mu(h) + \sum_i m_i + a_h^h \Sigma h D_h m_u^2 \cdot 4 \Sigma \frac{\hat{S}_i \cdot \hat{S}_j}{m_i m_j} + a_C \Sigma_{i > j} Q_i Q_j \]
\[ - a_D m_u^2 \cdot 4 \Sigma_{i > j} Q_i Q_j m_i m_j \]  
\( (4.10) \)
where the index $h$ takes on two values $B$ for baryon and $M$ for meson. $m_1$, $Q_1$ and $S_1$ are the effective constituent quark mass, charge, and spin of $i$th quark, respectively. According to $SU(3)$ QCD, $K_M = \frac{4}{3}$ and $K_B = \frac{2}{3}$. The $\alpha_s^h$ term is the color spin-spin interaction, and the last two terms are the coulomb and magnetic interactions. $\alpha$ and $\alpha_s^h$ are the fine structure constant and the effective QCD coupling constant, respectively. The zero point energy $\mu_h$, $C_h = \langle \frac{1}{r} \rangle_h$; and $D_h = (\frac{2\pi}{3m_2^2}) \langle \delta^3(r) \rangle_h$ are assumed to be constant which is a fair approximation for the mass matrix.

The masses of mesons are given by

$$M_{5}(m_1, m_2) = \mu_M + m_1 + m_2 + aC_{5}Q_1Q_2$$

$$+ \left( \frac{4}{3} \alpha_s^M - aQ_1Q_2 \right)D_M (\frac{m_1^2}{m_2^2})[2S(S+1)-3]$$

(4.11)

where $S$ is the total spin. The baryon masses are given by

$$B_{S,S_{12}}^{B,Q}(m_1, m_2, m_3) = \mu_B + m_1 + m_2 + m_3$$

$$+ \frac{1}{2} aC_{B}[(Q-Q_1)(Q-Q_2)+2Q_1Q_2]$$

$$+ \frac{2}{3} \alpha_s^B m_u^2 \left\{ \frac{1}{m_1m_2} [2S_{12}(S_{12}+1)-3] \right\}$$

$$+ \left( \frac{m_1}{m_2} + \frac{m_2}{m_3} \right) \left[ S(S+1)-S_{12}(S_{12}+1)-\frac{3}{4} \right]$$

$$- aD_{B} m_u^2 \left\{ \frac{Q_1}{m_1m_2} [2S_{12}(S_{12}+1)-3] \right\}$$

$$+ \frac{Q_2}{m_3} \left( \frac{Q_1}{m_1} + \frac{Q_2}{m_2} \right) \left[ S(S+1)-S_{12}(S_{12}+1)-\frac{3}{4} \right]$$

(4.12)
where \( Q = Q_1 + Q_2 + Q_3 \), \( \vec{S}_{12} = 1 \) for \( \vec{S} = 3/2 \), and \( \vec{S}_{12} = 0,1 \) for \( \vec{S} = 1/2 \). The off-diagonal matrix element which mixes the \( \vec{S}_{12} = 0 \), and \( \vec{S}_{12} = 1 \) state is
\[
\frac{2}{3} \sqrt{3} \alpha_s B^B \left( \frac{m_1 - m_2}{m_2} \right) \frac{m_1}{m_3} .
\]

Using the assumptions:

i) the only flavor symmetry breaking mechanism is the quark mass;

ii) the flavor symmetry breaking due to the sea quarks or quark loop is negligible;

iii) the quark masses satisfy,
\[
(m_d - m_u) \ll (m_s - m_d) \ll (m_c - m_s) \ll (m_b - m_c) \ll (m_t - m_b);
\]

iv) the corrections to the mass eigenvalues are negligibly small, one can write the expressions for different isospin for baryons and mesons. By neglecting the isospin violation one determines the parameters which are

\[
\begin{align*}
\alpha_{s B}^B &= 73.3 \text{ MeV}, \\
\alpha_{D B} &= 1.18 \pm 0.08 \text{ MeV}, \\
\alpha_C &= 2.96 \text{ MeV}, \\
\alpha_{s D}^M &= 113.9 \text{ MeV}, \\
\alpha_s^B &= 0.45 \pm 0.05 \text{ MeV}, \\
\mu_M &= -53.7 \text{ MeV}, \\
\alpha_D^M &= 0.8 \pm 0.3 \text{ MeV}, \\
\alpha_C^M &= 1.5 \pm 0.8 \text{ MeV}, \\
\omega_s^M &= 1.1 \pm 0.5 \text{ MeV},
\end{align*}
\]

where \( \delta_m = 2.66 \text{ MeV} \). (4.13)

Once the parameters are known the electromagnetic mass differences of various isospin doublets are estimated and are given in table (4.1) and table (4.2), respectively.

**Approach - III:**

In approaches I and II, the effect of relativistic motion of quarks inside hadrons and the effect of spatial dependence of
spin-spin interaction are neglected. Following Itoh et al.,
and using Fermi-Breit type interaction potential between the
quarks in the hadrons where spin-spin interactions are repulsive
between spin triplet quarks and attractive between spin singlet
quarks, the effect of the differences in the relative distances
between quarks in a hadron is introduced into the wavefunctions.
The spatial wavefunctions of the baryon and meson are

\[ \psi(r_{123}) = N \exp\left[-ar_{12}^2 - br_{23}^2 - cr_{31}^2 \right] \]  

(4.15)

and

\[ \psi(r_{12}) = N' \exp[-a'r_{12}^2], \]  

(4.16)

respectively, where \( N \) and \( N' \) are normalisation factors. The
parameters \( a, b, c \) and \( a' \) are related to the distances between
quarks or quark-antiquark.

**Electromagnetic Mass Differences**

The various contributions to electromagnetic mass difference
between two members of an isomultiplet are due to

i) coulombic interaction of electromagnetism and chromomagnetism

\[ [\text{Coul} = \sum_{i>j} (aQ_iQ_j + K\alpha_s) \frac{1}{r_{ij}}] \]

ii) Darwin-Breit interaction of electromagnetism and chromomagnetism

\[ [\text{Rel 1} = -\sum_{i>j} (aQ_iQ_j + K\alpha_s) \cdot \frac{1}{2m_im_j} \frac{\hat{p_i} \cdot \hat{p_j}}{|r_{ij}|} + \frac{\hat{r}_{ij} \cdot (\hat{r}_{ij} \cdot \hat{p}_i) \hat{p}_j}{|r_{ij}|^3}] \]

iii) Zetterbewegung interaction of electrodynamics and chromodynamics

\[ [\text{Rel 2} = -\frac{\pi}{2} \sum_{i>j} (aQ_iQ_j + K\alpha_s) \cdot \frac{1}{m_i^2} \frac{1}{m_j^2} \cdot (\frac{1}{m_i^2} + \frac{1}{m_j^2}) <\delta^3(r_{ij})>] \]
iv) the change induced by the \((m_d - m_u)\) in the electric charge and color hyperfine interactions

\[
\begin{align*}
\text{Mag} = - \frac{n}{2} \sum_{i>j} (\alpha Q_i Q_j + K\alpha_s) \cdot \frac{16}{3 m_i m_j} < \hat{S}_i \cdot \hat{S}_j \delta^3(r_{ij}) > 
\end{align*}
\]

v) the change in free energy of the system

\[
E_f = \sum \left( m_i^2 + p_i^2 \right)
\]

Thus one writes the total electromagnetic mass difference as

\[
(\Delta m)_{\text{total}} = (\Delta m)_{\text{elec.}} + (\Delta m)_{\text{mag.}} + (\Delta m)_{\text{glu.}} + (\Delta m)_{\text{kin.}}
\]

(4.17)

**Baryons**

\[ J^P = \frac{1}{2}^+ \text{ baryons} \]

The expectation value of any relevant function \(f(r_1 r_2 r_3)\) is given by

\[
< f > = \int \psi^*(r_1 r_2 r_3)f(r_1 r_2 r_3)\psi(r_1 r_2 r_3)dr_1 dr_2 dr_3
\]

(4.18)

Using this expression and eqn. (4.15), one can easily deduce the following results

\[
\begin{align*}
< \frac{1}{r_{12}} > &= 2 \left[ \frac{2(ab + bc + ca)}{\pi(b + c)} \right]^{1/2} = \frac{1}{r_{12}} \\
< \frac{r_{12}^2}{r_{12}} &= \frac{6}{\pi} \frac{r_{12}^2}{r_{12}} \\
< \delta^3(r_{12}) &= \frac{1}{8r_{12}^3} \\
\frac{\hat{p}_1 \cdot \hat{p}_2}{r_{12}^4} + \frac{r_{12}(\hat{r}_{12} \cdot \hat{p}_1 \cdot \hat{p}_2)}{r_{12}^3} &= - \frac{4a}{r_{12}}
\end{align*}
\]

(4.19)
and analogous relations for the permutation of the three quarks. The relative mean square distance of each pair of quarks can be written as

\[ < r_{12}^2 > = \frac{3}{4} \left( \frac{b+c}{ab+bc+ca} \right), \]
\[ < r_{23}^2 > = \frac{3}{4} \left( \frac{a+c}{ab+bc+ca} \right), \]
\[ < r_{31}^2 > = \frac{3}{4} \left( \frac{a+b}{ab+bc+ca} \right). \]  

(4.20)

In case of \( \frac{1}{2}^+ \) baryons, it is assumed that one pair of quarks (same quarks or two lighter quarks) is in spin triplet state while the other two pairs are in mixed states of spin-singlet and spin-triplet with the same relative probabilities. So if \( r_t = r_{12} \) then \( r_{23} = r_{31} = r_m \). Using the expectation values given in (4.19) and spin and unitary spin wavefunctions for \( \frac{1}{2}^+ \) baryons (app.V) one can easily calculate the various contributions to EMMD's.

\( J^P = \frac{3}{2}^+ \) baryons

In case of \( \frac{3}{2}^+ \) baryons, where all the three quarks have their spin-up, the distance between the different pairs of quarks remains the same i.e.

\[ r_{12} = r_{23} = r_{31} = r_D, \]

or \( a = b = c = a_D \)

(4.21)

Hence

\[ < \frac{1}{r_{1j}} > = \frac{1}{r_D}, \]
\[ < r_{1j}^2 > = \frac{6}{4} \frac{r_D^2}{D}, \]
Using these expressions and the spin and unitary spin wavefunctions for \( \frac{3}{2} \) baryons (app. VI) various contributions to EMM's are calculated.

The uncertainty principle requires several hundred MeV. of quark momenta in hadrons and therefore, its effect cannot be ignored. The expectation value of free energy term \( (m^2 + p^2)^{1/2} \), which can be conveniently carried out, in the momentum space, comes out to be

\[
< (m_1^2 + p_1^2)^{1/2} > = \frac{4}{\sqrt{\pi D}} \int_0^{\infty} (m_1^2 + p^2)^{1/2} \exp(-p^2) p^2 dp,
\]

where \( D = [2(a+c)]^{-1} \) and the analogous relations for

\[
< (m_2^2 + p_2^2)^{1/2} > \quad \text{and} \quad < (m_3^2 + p_3^2)^{1/2} >
\]

can be written. The differences of free energy can be expanded in terms of \( \delta m/m_u \) i.e.

\[
E_f = [ < (m_d^2 + p^2)^{1/2} > - < (m_u^2 + p^2)^{1/2} > ] = \delta m I(z) + \delta m/m_u^2
\]

\[
I(z) = \frac{4\sqrt{3}}{\pi} z \int_0^{\infty} (p^2 + \frac{3}{2} z^2)^{-1/2} e^{-p^2} p^2 dp,
\]

where \( z = (\frac{\pi}{2} m_u D)^{1/2} \).
Mesons

Pseudoscalar mesons

The expectation value of a function \( f(r_1, r_2) \) is given by

\[
\langle f(r_1, r_2) \rangle = \int \varphi^*(r_1, r_2) f(r_1, r_2) \varphi(r_1, r_2) \, dr_1 \, dr_2
\]

(4.26)

Using the expression of (4.16) one can obtain

\[
\langle \frac{1}{|r_{12}|} \rangle = 2\left(\frac{2a_p}{\pi}\right)^{1/2} = \frac{1}{r_p},
\]

\[
\langle r_{12}^2 \rangle = \frac{6}{\pi} r_p^2,
\]

\[
\langle \delta^3(r_{12}) \rangle = \frac{1}{8r_p^3},
\]

\[
\langle \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{|r_{12}|^2} + \frac{\mathbf{r}_{12} \cdot (\mathbf{r}_{12} \cdot \mathbf{p}_1) \mathbf{p}_2}{|r_{12}|^3} \rangle = \frac{\pi}{2r_p^3}
\]

(4.27)

Using these expressions and wave functions (app. VII), various contributions to EMMD's are calculated.

Vector mesons

The expectation values of various terms can simply be obtained by replacing \( r_p \) and \( a_p \) by \( r_v \) and \( a_v \), respectively in (4.27). The different contributions are calculated using expectation values and wavefunctions (app. VIII).

The free energy of quarks in meson is given by

\[
\langle (m^2 + p^2)^{1/2} \rangle = \frac{4}{\sqrt{\pi D'}} \int_0^\infty (m^2 + p^2)^{1/2} e^{-p^2} p^2 \, dp,
\]

(4.28)

where \( D' = (2d')^{-1} \). The change in free energy will be
\[ E_F = \left[ \langle m_d^2 + p^2 \rangle^{1/2} - \langle m_u^2 + p^2 \rangle^{1/2} \right] = \delta_m I(z') + \Theta \cdot \left( \frac{\delta m_2}{m_u} \right) \]

(4.29)

where \( z' = (\frac{\pi}{3} m_u^2 D')^{1/2} \).

**Estimation of EM mass differences**

Various contributions to EM mass splittings are evaluated in terms of

\[ C_t = \frac{\alpha}{3r_t}, \quad C_m = \frac{\alpha}{3r_m}, \quad C_D = \frac{\alpha}{3r_D}, \quad C_p = \frac{\alpha}{3r_p}, \quad C_v = \frac{\alpha}{3r_v} \]

\[ M_t = \frac{\pi a^2}{144r_t^3}, \quad M_m = \frac{\pi a^2}{144r_m^3}, \quad M_D = \frac{\pi a^2}{144r_D^3}, \quad M_p = \frac{\pi a^2}{48r_p^3}, \quad M_v = \frac{\pi a^2}{48r_v^3} \]

\[ G_m = \frac{\pi a^2 \delta m}{36r_m^3}, \quad G_D = \frac{\pi a^2 \delta m}{36r_D^3}, \quad G_p = \frac{\pi a^2 \delta m}{6r_p^3}, \quad G_v = \frac{\pi a^2 \delta m}{6r_v^3} \]

\[ f = \frac{3\omega}{1+2\omega}, \quad g = \frac{3(1+\omega)}{2(1+2\omega)}, \quad z_0 = m_u r_t (\frac{1+2\omega}{3})^{1/2}, \]

\[ z_t = m_u r_t (\frac{1}{g})^{1/2}, \quad z_D = m_u r_D, \quad z_p = 2\frac{m_u r_p}{3}, \quad z_v = 2\frac{m_u r_v}{3} \]

\[ X = \frac{m_u}{m_d} = 1, \quad Y = \frac{m_u}{m_s} = 0.62, \quad Z = \frac{m_u}{m_c} = 0.22, \quad K = \frac{m_u}{m_b} = 0.08 \]

(4.30)

The subscripts \( t, m, D, p \) and \( v \) denote spin-triplet, mixed state spin-triplet and singlet, decuplet, pseudoscalar, and vector mesons, respectively.
Taking the following experimental values (in MeV.)

\[
\begin{align*}
(n-p) & = 1.29, 
(\Sigma^- - \Sigma^0) & = 4.88, 
(\Sigma^0 - \Sigma^+) & = 3.10, 
(p^+ - p^0) & = 4.60, 
(\rho^+ - \rho^0) & = 1.4, 
(K^- - K^0) & = -4.0, 
(\Delta^0 - \Delta^{++}) & = 2.6,
\end{align*}
\]

as input one can obtain the numerical values of the various parameters as

\[
[\alpha_s]_{\text{strange}} = 0.60, 
\omega = a/b = 0.473,
\]

\[
\begin{align*}
rt & = 3.00 \text{ GeV}^{-1} = 0.59 f, 
rm & = 2.58 \text{ GeV}^{-1} = 0.51 f, 
p & = 2.33 \text{ GeV}^{-1} = 0.46 f, 
v & = 2.9 \text{ GeV}^{-1} = 0.57 f, 
D & = 3.3 \text{ GeV}^{-1} = 0.65 f
\end{align*}
\]

\[
\delta_m = (m_d - m_u) = 3.82 \text{ MeV.}
\]

The values of the gluon coupling constant \( \alpha_s \) goes on decreasing as we go to the heavier quark sectors. Its value for the c-quark and b-quark sectors can be calculated from the expression (Isgur 1978)

\[
\alpha_s(q^2) = \frac{12\pi}{(33-2N_f)\log \frac{q^2}{\Lambda^2}},
\]

where \( N_f \) is the number of flavors and \( \Lambda \) is QCD parameter. These come out to be

\[
[\alpha_s]_{\text{charm}} = 0.58 \quad (4.34)
\]

\[
[\alpha_s]_{\text{beauty}} = 0.30 \quad (4.35)
\]
Table 4.1. **Electromagnetic Mass Differences of Baryons (in MeV.)**

<table>
<thead>
<tr>
<th>Mass-Difference</th>
<th>Present Analysis</th>
<th>Experimental Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>$(n-p)$</td>
<td>$1.29^+$</td>
<td>$1.29^+$</td>
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<tr>
<td>$(\Sigma^0-\Sigma^+)$</td>
<td>2.18</td>
<td>3.09</td>
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<td>$(\Sigma^-\Sigma^0)$</td>
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<td>1.29</td>
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<td>Present Analysis</td>
<td>Experimental Values</td>
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<tr>
<td></td>
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<td>B</td>
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<td>$(\pi^+ - \pi^0)$</td>
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<td>5.4$^{+0.8}_{-0.8}$</td>
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<td>$(K^- - K^0)$</td>
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<td>-4.01$^{+0.13}_{-0.13}$</td>
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<td>4.72$^+$</td>
<td>4.7$^{+0.4}_{-0.4}$</td>
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<tr>
<td>$(D_b^- - D_b^0)$</td>
<td>-3.62</td>
<td>-2.3$^{+0.3}_{-0.3}$</td>
</tr>
<tr>
<td>$(D_t^+ - D_t^0)$</td>
<td>4.42</td>
<td>-2.17$^{+0.27}_{-0.27}$</td>
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<td>4.59$^+$</td>
<td>1.22</td>
</tr>
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<td>$(K^*^- - K^{*0})$</td>
<td>-4.0</td>
<td>-1.56$^{+0.17}_{-0.17}$</td>
</tr>
<tr>
<td>$(D_{c'}^+ - D_{c'}^0)$</td>
<td>4.72</td>
<td>3.3$^{+0.4}_{-0.4}$</td>
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<td>-2.0$^{+0.3}_{-0.3}$</td>
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<td>$(D_{t'}^+ - D_{t'}^0)$</td>
<td>4.42</td>
<td>-2.16$^{+0.27}_{-0.27}$</td>
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</table>

Table 4.2 ELECTROMAGNETIC MASS DIFFERENCES OF MESONS (IN MeV.)
Using these numerical values of the parameters, the various contributions to EM mass differences are estimated and given in table (4.1) for baryons and table (4.2) for mesons.

Conclusions

Many authors have calculated EMMD's of hadrons within the framework of constituent quark model giving importance to some or the other contributions. We have estimated their values in three schemes in which the number of contributions is increased going from one scheme to the other. From our analysis it is noticed that the assumption that EMMS come only through quark mass difference, and the coulomb and magnetic interaction are not enough to explain the most of the experimental values. It is found that if we include gluonic contribution the results improve significantly. Most of the differences come very close to the recent data. It is also noticed that if spin-spin interaction affected interquark separation is taken into account with simultaneous inclusion of the relativistic form \( \sqrt{m^2+p^2} \) for the free energy of the quark, then data on EMMD's favorably improves. Since the available data in b and t quark sector is very meager, a thorough comparative study is not possible at present. The future results from recent sofesticated experiments will give the real test of the above model and involved assumptions.