**Preface**

**Topic:** “A study on some aspect of certain spaces and algebras of analytic functions represented by Dirichlet series”.

We know that the spaces and algebras of holomorphic functions has been a subject of study of a number of mathematicians in the last part of last century. They have made important and significant contributions to give an apex position to this study by providing a number of applications in Science and Engineering see [Chapter - 1]. In the present thesis we confined ourselves to only these aspects of the theory of Holomorphic functions, represented in the form of Dirichlet series, which as a class, becomes well behaved spaces or algebras under suitable algebraic operations and norm. We have studied the properties and some applications of these algebras in this work. Thus keeping in view of the contents of our contributions, the present thesis is divided into six chapters including the first one which is an introduction giving the details of history of previous work done and motivations and methodologies etc. The details about contents of the chapters wise are given as follows;

In *chapter 2*, we study a class of Dirichlet series (D.S.) $\Omega_u$ as a Banach space. Infact $\Omega_u$ consists of all those function $f, f(s) = \sum_{k=1}^{\infty} a_k e^{\lambda_k s}$ being a
Dirichlet series such that \( \left\{ \frac{a_k}{\alpha_k} \right\} \) is bounded where \( u(s) = \sum_{k=1}^{\infty} \alpha_k e^{s\lambda_k} \) is a given fixed Dirichlet series with none of the \( \alpha_k \) being zero with given abscissa of absolute convergence \( \sigma_a = -\limsup \frac{\log |\alpha_k|}{\lambda_k} = \alpha \). In theorem 2.3.1 & 2.3.2, \( \Omega_u \) has been shown a non-uniformely convex Banach space which is non-separable also. Further it can not be converted into a Hilbert space, has been proved in section theorem 2.4.2. Another class \( \Omega_{u(1)} \) of function \( f, f(s) = \sum_{k=1}^{\infty} a_k e^{s\lambda_k} \) such that \( \sum |a_k b_k| < \infty \) has also been introduced and it was found that \( \Omega_u^* \) (dual of \( \Omega_u \)) contain a proper subspace which is linearly isometric to \( \Omega_{u(1)} \) (theorem 2.5.1). In order to obtain all the continuous linear transformation on the space \( \Omega_u \), the matrix transformation on \( \Omega_u \) has been defined and bounded linear transformation on \( \Omega_u \) to itself has been characterized in theorem 2.6.1. All the bounded linear transformation from the space \( \Omega_u \) to \( \Omega_o \) has also been obtained in theorem 2.6.2. Finally in the last section an application of solving an operator equation for an unique solution in the space \( \Omega_u \) has been done. Contents of this chapter has been published in JPAS Vol (14) 2008, 106-113.

In chapter 3, a class \( \Omega_u^2 \) of functions represented by Dirichlet series
\[
f(s) = \sum_{k=1}^{\infty} a_k e^{s\lambda_k} \quad \text{with} \quad \sum \left| \frac{a_k}{\alpha_k} \right|^2 < \infty
\] has been studied as an Hilbert space. In
theorem 3.1.1, the class $\Omega^2_u$ has been shown to be an Hilbert space under a suitable inner product. In section 3.2, an orthonormal basis in $\Omega^2_u$ and a direct sum of $\Omega^2_u$ were obtained. Isometric image of $\Omega^2_u$ has been obtained in theorem 3.3.1. Bounded linear compact operators on $\Omega^2_u$ has been characterized in theorem 3.4.1. Multiplier on $\Omega^2_u$ has been studied in section 3.5 and multiplier class on $\Omega^2_u$ i.e. $(\Omega^2_u, \Omega^2_u)$ was characterized in theorem 3.5.1.

Chapter 4 deals with some special operators on the Hilbert space $\Omega^2_u$. A diagonal operator $T$ has been introduced and its conjugate $T^*$ has been obtained. The norm of the diagonal operator $T$ has been calculated in theorem 4.2.1. Theorem 4.2.2 gives respectively the conditions under which this operator becomes self-adjoint, positive and unitary. Condition of being compact operator, Hilbert Schmidt operator, and Nuclear operator has also been obtained in Theorem 4.2.3. Uniform and Strong stability has been studied in theorem 4.3.1. Spectrum of the operator has been studied in article 4.4. Orthogonal projections and resolution of Identity were obtained in section 4.5. Some application for solving certain operator equations in $\Omega^2_u$ has been made in section 4.6. Contents of this chapter has been accepted for publication in BPAM Vol (5) Nov/Dec 2011.
In chapter 5, we study $\Omega_u$ as a Banach algebra under a suitable product. In Lemma 5.2.1, $\Omega_u$ has been shown to be a Banach algebra with identity ‘$u$’ 

$u(s) = \sum_{k=1}^{\infty} \alpha_k e^{s\lambda_k}$ being the fixed Dirichlet series. Characterization of regular/singular elements and topological zero divisor (TZD) has been obtained in theorem 5.3.1 and 5.3.2 respectively. Spectrum $\sigma(f)$ of an element $f$ belonging to the Banach-algebra $\Omega_u$, has been obtained in theorem 5.3.3. Spectral radius $r(f) = \|f\|$ has been established in theorem 5.3.4. $\Omega_u$ is not division algebra has been proved in theorem 5.4.1. Two ideals $I_1$ and $I_2$ of $\Omega_u$ were obtained such that $\Omega_u = I_1 \oplus I_2$ in theorem 5.5.1. Theorem 5.6.1 study $\Omega_u$ as a $B^*$-algebra. Characterization of self-adjoint, unitary and normal element in $\Omega_u$ were made in theorem 5.6.2. Theorem 5.7.1 deals with the $*$- homomorphism on $\Omega_u$. In theorem 5.8.1, $\Omega_u$ has been studied as a Banach Lattice under an order relation ($<$). Further $(\Omega_u, <)$ has been shown to be an l-ring in theorem 5.8.2. Theorem 5.8.4 shows that $(\Omega_u, <)$ is an AM-space while theorem 5.8.5 proves that it is an Abstract Lebesgue-space (AL-space).

Chapter 6 is a brief study of certain sub-algebras $\Omega_o$ and $\Omega_1$ of $\Omega_u$. For these sub-algebras which are without identity characterization of quasi-regular / singular elements has been obtained in theorem 6.3.2. Every element of the two sub-algebras $\Omega_o$ and $\Omega_1$ has been shown to be topological zero divisor (TZD) in theorem 6.3.4. In theorem 6.4.1, $\Omega_o$ has been shown to be a
B*-algebra under an involution (*). While $\Omega_1$ is not a B*-algebra under this involution has been given in theorem 6.4.2. In theorem 6.4.4, $\Omega_0$ and $\Omega_1$ has been shown to be symmetric subset of $\Omega_u$. Theorem 6.5.1 study $\Omega_1$ as a two norm space. Concluding last two theorems 6.5.4 and 6.5.5 shows that two norm space $\Omega_1$ is a Sack space.

Best efforts has been made to explore the literature (research papers and books) on the subject matters of the present thesis and related topics. Thus more than 200 references has been consulted, though listed only 133, in the preparation of this thesis starting from the year 1909 to 2012. Thus the present thesis is submitted in partial fulfillment of the requirements for Doctor of Philosophy in mathematics to C.S.J.M. University, Kanpur.

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