PUBLISHED PAPERS
A New Subclass of Goodman-Ronning Type Harmonic Multivalent Functions

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ABSTRACT

In the present paper, we introduce and study a new class of Goodman-Ronning type harmonic multivalent functions by using Salagean derivative. We determine coefficient estimates, extreme points, distortion bounds, convolution properties and convex combination for the above class of harmonic functions.

Keywords: Harmonic, Univalent, Starlike, Multivalent, Convex functions, Salagean derivative

1. Introduction

A continuous function $f = u + iv$ is defined in a domain $D \subseteq C$ is harmonic in $D$ if $u$ and $v$ are real harmonic in $D$. In any simply connected subdomain of $D$, we can write $f = h + \bar{g}$, where $h$ and $g$ are analytic and we call $h$ the analytic part of $f$ and $g$ the co-analytic part of $f$. The Jacobian of $f$ is given by

$$J_f(z) = |h'(z)|^2 - |g'(z)|^2.$$  

The mapping $z \rightarrow f(z)$ is locally one-one if $J_f(z) \neq 0$ in $D$. A result of Lewy [5] shows that the converse is true for harmonic mapping, and therefore $f$ is locally one-one and sense-preserving if, and only if,

$$|g'(z)| < |h'(z)|.$$  

We call such mappings locally univalent, and we say $f$ is univalent in $D$ if $z \rightarrow f(z)$ is one-one and sense-preserving in $D$.

Denote by $S_H$, the class of functions $f = h + \bar{g}$ that are harmonic univalent and sense-preserving in the unit disc $U = \{z : |z| < 1\}$ for which $h(0) = f(0) = f_z(0) - 1 = 0$. Then for $f = h + \bar{g} \in S_H$, we may express the analytic functions $h$ and $g$ as

$$h(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad g(z) = \sum_{n=1}^{\infty} b_n z^n, \quad |b_1| < 1.$$  

Note that $S_H$ reduces to the class of normalized analytic univalent functions if the co-analytic part of its members is zero.

In 1984, Clunie and Sheil-Small [2] investigated the class $S_H$ as well its geometric subclasses and obtained some coefficient bounds. Since then, there have been several related papers on $S_H$ and its sub-classes as Jahangiri et al. [4], Silverman [7], Silverman and Silvia [8] etc.

Recently, Rosy et al. [6], defined the subclass $G_H(\gamma) \subseteq S_H$ consisting of harmonic univalent functions $f(z)$ satisfying the following condition

$$\text{Re}\left\{ (1 + e^{i\alpha}) \frac{zf'(z)}{f(z)} - e^{i\alpha} \right\} \geq \gamma, \quad 0 \leq \gamma < 1, \quad \alpha \in R.$$  

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ON A NEW SUBCLASS OF HARMONIC UNIVALENT FUNCTIONS DEFINED BY GENERALIZED SALAGANE OPERATOR

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ABSTRACT

The purpose of the present paper is to study some results involving coefficient conditions, extreme points, distortion bounds, convolution conditions and convex combination for a new class of generalized Salagean-Type harmonic univalent functions in the open unit disc. Relevant connections of the results presented here with various known results are briefly indicated.

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Keywords and Phrases: Harmonic, Univalent functions, Convex and Starlike functions.

1. INTRODUCTION

A continuous complex-valued function $f = u + iv$ defined in a simply connected domain $D$ is said to be harmonic in $D$ if both $u$ and $v$ are real harmonic in $D$. In any simply connected domain we can write $f = h + g$, where $h$ and $g$ are analytic in $D$. We call $h$ the analytic part and $g$ the co-analytic part of $f$. A necessary and sufficient condition for $f$ to be locally univalent and sense preserving in $D$ is that

$$|h'(z)| > |g'(z)|, z \in D.$$ 

Let $S_H$ denote the class of functions $f = h + g$ which are harmonic univalent and sense preserving in the open unit disc $U = \{z : |z| < 1\}$ for which $f(0) = f_z(0) = 0$. Then for $f = h + g \in S_H$ we may express the analytic functions $h$ and $g$ as

$$h(z) = z + \sum_{k=2}^{\infty} a_k z^k, g(z) = \sum_{k=1}^{\infty} b_k z^k, |b_k| < 1.$$  \hbox{(1.1)}

Clunie and Sheil-Small [3] investigated the class $S_H$ as well as its geometric subclasses and established some coefficient bounds. Since then, there have been several related papers on $S_H$ and its subclasses.

For $f = h + g$ given by (1.1), we defined the modified generalized Salagean operator of $f$ as

$$D^\lambda_\mu f(z) = D^\lambda_\mu h(z) + (-1)^{1-\mu} D^\lambda_\mu g(z), \quad (m \in N_0, N_0 \equiv N \cup \{0\}, 0 \leq \lambda \leq 1)$$  \hbox{(1.2)}

where

$$D^\lambda_\mu h(z) = z + \sum_{k=2}^{\infty} [(k-1)\lambda+1]^\mu a_k z^k$$

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