CHAPTER - III
INTRODUCTION

In the recent times, a new area of study in the Bio-Mechanics is emerged as the study of behaviour of the fluid dynamics of various blood like fluids in the presence of magnetic field. The blood pressure can be controlled by using this magnetic field in the clinical cases of hypertension and other diseases. In the human physiology, there are mainly two types of circulations macro circulation and micro circulation. The macro-circulation system consists of large arteries and vessels while micro circulation system comprise of smallest arteries, veins and capillaries. There is vast difference between the patterns of circulation of blood in very small capillaries with comparison to the large arteries. For describing the mechanics of red blood cell motion in narrow capillaries, we distinguished two situations according to the convenience with which the cells fit into the vessels. In the first case when the capillary has diameter more than that of cell, the cell can be fit easily into
the tube without any distortion. This flow situation is called positive clearance. When the diameter of the red blood cell is larger than that of the capillary as such the cell will be deformed in order to fit into the capillary. In this case pressure must be generated in thin layer of fluid round the edge of the cell in order to deform it and depends on elastic properties of the cell. When the red cell is severely deformed then in blood flow the red cell seems to plug the capillary of blood vessel and the motion of the plasma in capillary between successive red cells is called bolus flow. The significance of bolus flow was first pointed out by PROTHERO, et al [1]. Pressure within capillaries show periodic fluctuations; such variations can result both from local functions and from changes in central arterial and venous pressure. Systematic measurement of the pressure distribution in small blood vessels was usually done by WIEDERTIELM, et at [2]. LIGHTHILL [3] discussed the behaviour of tightly fitting solid pellets, which may be deformed and may be forced by pressure difference to move. Lighthill model was extended by FITZ-GERALD [4] to examine the effects of axisymmetry and tube porosity. ZWWEIFACH, et al [5] made extensive measurements of the pressure distribution in micro vessels. CARO et al [6] described that the measurement of representative flow rates and velocity profile in the very narrow and small vessels has proved even more difficult than the measurement of the pressure. This has been mainly due to the
difficulty of observing the movement of individual red cell in vessels larger than capillaries. But in addition any one who has observed a micro vascular bed in vivo with a microscope would have noted the variability of flow in a given vessel; flow can be steady for a period of time and then suddenly slow down or stop altogether. Such changes in flow rate have not been closely correlated to vascular pressure; this is perhaps to be expected because in such a network pressures and flow rates in adjacent vessels are related. In capillaries, where individual red cell can be observed in the flow, flow rates can be estimated on the basis of red cell velocity.

TOZERN, et al [7, 8] analyzed the steady flow of elastic spheres in a circular cylindrical tube, using a series expansion for the practical displacement and lubrication theory for the fluid motion. The results revealed that the apparent viscosity depends on the shear modulus of elasticity of the particles. OZKAYAN [9] formulated a mathematical model for the flow of red blood cells in barrow capillaries and used lubrication theory to describe the plasma flow in narrow gap between red cells and tube wall. Relative apparent blood viscosity as a function of tube diameter and hematocrit was described by PRIES, et al [10]. BISHOP, et al [11] recognized that the rheological behaviour of the blood flow needed to understand the function of circulatory system. SHARAN, et al [12] developed a two phase model for the flow of blood in narrow tubes and

Various studies have been performed on rheological behaviour of blood flow in narrow capillaries to establish relationship among resistance, viscosity, clearance (both positive and negative) of cell and other parameters. From earlier works, we observe that the flow resistance for plasma in narrow capillaries is greater than in large capillaries. The study of flow under the influence of a magnetic field is also important so far as flow and resistance are concerned. In the present work, we study the motion of the blood through a very narrow capillary under the action of transverse magnetic field. Our
study is confined to the micro-circulation i.e. single file flow of red blood cell through narrow capillary.

**OBJECTIVE OF THIS RESEARCH WORK:**

In this chapter our aim is to find the impact on velocity flow of lubricating zone relative to the tube wall in very fine capillaries. We are also interested to find the relation between leak-back flow rate in relation with traverse magnetic field intensity. We will also discuss the viscosity effect in the lubricating zone. We are also interested to find the skin friction at the surface of RBC.

**MATHEMATICAL MODEL AND GOVERNING EQUATIONS:**

In this model we consider the axially symmetric and Newtonian flow of blood in a tube of uniform radius. The blood is assumed to be homogeneous fluid, while the red blood cells are assumed to be elastic and incompressible. The single cell is fitted in the tube so as to generate a single file flow. In this investigation, we study fluid flow in lubricating zone i.e. fluid flow between red blood cell and tube wall. The effect of transverse magnetic field on the flow of narrow capillary is taken into account. The induced magnetic field has been neglected. The viscous force is predominant in the flow of such tubes. The inertial terms are supposed to be negligible. During passing down single red cell in narrow capillary, it deforms due
to its elastic property. The shape of red cell is bi-concave disk. The axial velocity is taken zero at the surface of red blood cell and (-W) at the tube wall. To obtain the axial velocity of the fluid relative to the tube, we add 'w' velocity in the direction of the flow of fluid (cf. 5)

Let us assume the coordinate z in the direction of the axis of the tube. 'r' is transverse distance from the highest point of the surface of the RBC (Figure 3.1).
FIGURE 3.1

(Schematic diagram of fluid flow in lubricating zone)
Following notations are being used for Mathematical formulation are as follows:

- **R**  Tube radius
- **h** thickness of gap between the red cell and tube wall
- **z** axial distance
- **v(w)** Radial (axial) velocity of fluid
- **w** velocity relative to the tube wall
- **w_m** mean velocity flow
- **W** velocity of red blood cell
- **P** pressure
- **C_f** skin friction coefficient
- **i** imaginary number
- **n** frequency of pulse
- **ρ** Density of fluid
- **μ** viscosity of fluid
- **μ_e** Magnetic permeability
- **σ** coefficient of conductivity
- **τ_w** shear stress at tube wall
- **B_0** Transverse magnetic field intensity
- **Q_{lb}** leak back flow rate
- **MHD** Magneto hydrodynamic
The governing equations of the motion of the fluid flow are:

\[
\frac{\partial (rv)}{\partial r} + \frac{\partial (rw)}{\partial z} = 0
\]  
(3.1)

\[
\rho \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) - \sigma \mu^2 B_0^2 w
\]  
(3.2)

Boundary conditions are:

\[
\begin{align*}
w &= -We^{-\text{int}}, & v &= 0, & \text{at } r = h & t > 0 \\
w &= 0, & v &= 0, & \text{at } r = 0 & t > 0 
\end{align*}
\]  
(3.3)

From the equation (1) and (3), we find:

\[
\int_0^h rw \, dr = -Q_b \quad \text{(a constant)}
\]  
(3.4)

The pressure gradient of blood flow can be taken as:

\[
\frac{\partial p}{\partial z} = -Pe^{-\text{int}}
\]

Also we may take the velocity as \( w(r, t) = f(r)e^{-\text{int}} \) without the loss of generality.

From the equation (3.2) using these values of \( \frac{\partial p}{\partial z} \) and \( w(r, t) \), we get:

\[
f''(r) + \frac{1}{r} f'(r) - \frac{A}{\mu} f(r) = -\frac{P}{\mu}
\]  
(3.5)

where, \( A = -in\rho + \sigma \mu^2 B_0^2 \)

The solution of (5) is given by,

\[
f(r) = A_0 J_0 \left( ir \sqrt{A/\mu} \right) + B_0 Y_0 \left( ir \sqrt{A/\mu} \right) + \frac{P}{A}
\]  
(3.6)
Where \( J_0 \) & \( Y_0 \) are Bessel's functions of first kind and second kind respectively. Now \( B_0 = 0 \) as otherwise at \( r = 0 \), \( f(r) \) is not finite.

Then the solution of (3.6) becomes:

\[
f(r) = A_0 J_0 \left( ir \sqrt{A/\mu} \right) + \frac{P}{A}
\]

(3.7)

Now transformed boundary conditions are:

\[
\begin{align*}
  f(r) &= 0, \quad \text{at } r = 0, \quad t > 0 \\
  f(r) &= -W, \quad \text{at } r = h, \quad t > 0
\end{align*}
\]

(3.8)

Now using second condition of (3.8) in the equation (3.7), we get:

\[
A_0 = -\frac{P}{A}
\]

Therefore now (3.7) becomes:

\[
f(r) = \left[ 1 - J_0 \left( ir \sqrt{A/\mu} \right) \right] \frac{P}{A}
\]

(3.9)

By using first condition of (3.8) in equation (3.7), we get

\[
P = -\frac{AW}{\left( 1 - J_0 \left( ih \sqrt{A/\mu} \right) \right)}
\]

In this situation now the equation (3.7) gives:

\[
f(r) = -W \frac{\left[ 1 - J_0 \left( ir \sqrt{A/\mu} \right) \right]}{\left( 1 - J_0 \left( ih \sqrt{A/\mu} \right) \right)} e^{-i\omega t}
\]

(3.10)

Therefore,

\[
w = -W \frac{\left[ 1 - J_0 \left( ir \sqrt{A/\mu} \right) \right]}{\left( 1 - J_0 \left( ih \sqrt{A/\mu} \right) \right)} e^{-i\omega t}
\]
Expanding the Bessel function in a series and retaining only up to second degree terms, we get

\[ w = -W \frac{r^2(16\mu + Ar^2)}{h^2(16\mu + Ah^2)}e^{-int} \]  

---------- (3.11)

The real part of \( w \) is given by:

\[ \text{Re}(w) = -W \frac{r^2 \left[ \left(16\mu + \sigma \mu_e^2 B_0^2 r^2\right)(16\mu + \sigma \mu_e^2 B_0^2 h^2) + (nh_\rho r)^2 \right] \cos nt - \left[ n\rho h^2 \left(16\mu + \sigma \mu_e^2 B_0^2 r^2\right) - n\rho r^2 \left(16\mu + \sigma \mu_e^2 B_0^2 h^2\right) \right] \sin nt \]  

\[ \frac{h^2 \left( \left(16\mu + \sigma \mu_e^2 B_0^2 h^2\right)^2 + n^2 \rho^2 h^4 \right)}{h^2 \left(16\mu + \sigma \mu_e^2 B_0^2 h^2\right)^2 + n^2 \rho^2 h^4} \]  

---------- (3.12)

The velocity relative to tube wall is given by:

\[ w_1 = w + W \cos{nt} \]  

---------- (3.13)

THE LEAK-BACK FLOW RATE AND SKIN FRACTION

With the help of equation number (3.4), the leak back flow rate \( Q_{lb} \) is given by:

\[ Q_{lb} = - \int_0^h rwdr = \]  

\[ \frac{h^2 \left[ \left(64\mu + \frac{20\sigma \mu_e^2 B_0^2 h^2}{3}\right) + \frac{h^4}{6} \left(\sigma \mu_e^4 B_0^4 + n^2 \rho^2\right) \right] \cos nt - \frac{4}{3} n\rho \mu h^2 \sin nt \]  

\[ \left[ \left(16\mu + \sigma \mu_e^2 B_0^2 h^2\right)^2 + n^2 \rho^2 h^4 \right] \]  

---------- (3.14)

The skin friction at the surface of RBC is given by:

\[ \tau_{RBC} = -\mu \left( \frac{1}{r} \frac{\partial w}{\partial r} \right)_{r=0} = \frac{32\mu^2 w \left[ \left(16\mu + \sigma \mu_e^2 B_0^2 h^2\right) \cos nt - n\rho h^2 \sin nt \right]}{h^2 \left(16\mu + \sigma \mu_e^2 B_0^2 h^2\right)^2 + n^2 \rho^2 h^4} \]  

---------- (3.15)
NUMERICAL RESULTS AND DISCUSSION

In this section, we validate the analytical results for blood flow in narrow capillaries by setting the default parameters as \( h = 0.5, \mu = 3, \sigma = 2, \mu_e = 1, \rho = 1.05, n = 8, w = 1.1, r = 0.05, \mu = 2.5 \) and \( B_0 = 3 \).

Figures 3.2, 3.3 and 3.4 depict the relative velocity profiles for different values of \( B_0, \mu \& r \) in the lubricating zone.
Figure 3.2

(Velocity profile for different values of $B_0$)
Figure 3.3

(Velocity profile for different values of $\mu$)
Figure 3.4

(Velocity profile for different values of r)

It is noticed that the relative flow velocity along the capillary between RBC and tube wall is almost uniform and the effect of magnetic intensity of the flow velocity is negligible (figure 3.2). It is also clear from these figures that the flow is pulsatile where velocity changes periodically. It is also observed in the figure 3.3 that the effect of viscosity ($\mu$) on the flow velocity relative to the tube wall is almost negligible. From the figure 3.4, we find that the velocity relative to the tube (tube
velocity relative to RBC is \( -w \) increases as transverse distance from highest point of RBC surface decreases and at the wall it is zero. From this we conclude that the velocity relative to the tube wall declines towards wall and the velocity along the transverse distance is independent of viscosity.

Figures 3.5, 3.6 and 3.7 display leak-back flow rate for different values of \( \mu \) and \( B_0 \).

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3.5}
\caption{Figure 3.5}
\end{figure}

\textit{(Leak-Back flow rate for different values of 'Bo' and \( \mu = 2.5 \))}
Figure 3.6

(Leak-Back flow rate for different values of 'Bo' and $\mu = 4.5$)
Figure 3.7
(Leak-Back flow rate for different values of 'Bo,' and $\mu = 6$)

It is noted that the leak-back flow rate increases as $B_0$ increases. For the values of $B_0$ from 10 to 100, the flow rate increases slightly but beyond these values, the flow rate is independent of $B_0$. But on decreasing the value of $B_0$ (< 10), the leak-back flow rate suddenly declines. For the increasing viscosity the leak-back flow rate retards significantly.
Figures 3.8, 3.9 and 3.10 display the effect of thickness (gap between RBC and tube wall) of fluid on leak-back for different values of $\mu$.

![Graph showing leak-back flow rate for different values of 'h' and $\mu = 2.5$](image)

**Figure 3.8**

*(Leak-Back flow rate for different values of 'h' and $\mu = 2.5$)*
Figure 3.9

(Leak-Back flow rate for different values of 'h' and $\mu = 4.5$)
Figure 3.10
(Leak-Back flow rate for different values of ‘h’ and μ = 6)
The leak-back flow rate increases as 'h' increases whereas on increasing viscosity, the flow rate decreases. This demonstrates that the effect of thickness and viscosity on flow rate is remarkable.

**Figures 3.11, 3.12 and 3.13** shows skin friction vs. time for different values of $B_0$ and $\mu$ the skin friction decreases (increases) as $B_0(\mu)$.

![Graph showing skin friction vs time for different values of B0 and for $\mu = 2.5$](image)

**Figure 3.11**

(Skin friction vs time for different values of $B_0$ and for $\mu = 2.5$)
Figure 3.12
(Skin friction vs time for different values of $B_0$ and for $\mu = 4.5$)
Figure 3.13
(Skin friction vs time for different values of $B_o$ and for $\mu = 6$)
At \( B_0 = 100 \) and beyond this value the skin friction is zero. Thus the effect of \( B_0 \) and viscosity on skin friction can't be ignored.

**Figures 3.14, 3.15 and 3.16** shows the profiles for the skin friction for different values of 'h' and '\( \mu \)'.

![Graph showing skin friction vs time for different values of h and for \( \mu = 2.5 \)](image)

**Figure 3.14**

*(Skin friction vs time for different values of h and for \( \mu = 2.5 \))"
Figure 3.15
(Skin friction vs time for different values of h and for $\mu = 4.5$)
Figure 3.16
(Skin friction vs time for different values of $h$ and for $\mu = 6$)
The skin friction at the surface of RBC decreases as the thickness (gap between RBC and tube wall) of the lubricating zone increases. However the increasing viscosity enhances the skin friction.

From all these numerical and graphical analysis we conclude that:

(i) The velocity flow of the lubricating zone relative to the tube wall in narrow capillaries, is periodic and almost independent of time, viscosity and $B_0$.

(ii) The velocity flow relative to the tube wall increases towards RBC surface.

(iii) The effect of leak back flow rate is worth mentioning as higher values for $B_0$ enhance the leak-back flow rate.

(iv) The viscosity effect within the lubricating zone is worth nothing with respect to the increasing thickness of lubricating zone.

(v) The skin friction at RBC surface decreases as $B_0$ and 'h' increase.
CONCLUDING REMARKS AND SUGGESTIONS

In this study, we have developed a transient Mathematical model for blood flow in very narrow capillaries. The velocity and volumetric flow rate are calculated and validated by numerical results. We also make the sensitivity analysis to examine the effect of various parameters. It is found that the magnetic field has no effect on velocity flow in lubricating zone relative to the tube wall and it is also independent viscosity. The effect of magnetic field on leak-back flow rate and skin friction seems to be significant. The viscosity effect is very much dependent on the thickness of the lubricating zone (thickness between RBC and tube wall). The magnetic field and thickness of lubricating zone affect the skin friction at RBC surface remarkably.

COMPARISON WITH EARLIER STUDIES:

In our entire study we remained focused on the basis objectives that we have given in the beginning of this chapter. Our results are matching with some other works that was done by some eminent researchers. We have found that our many results are in accordance with the works of PRIES and SECOMB [14], HUO and KASSAB [18], ZWEIFACH and LIPOWSKY [5]. Many parts of our study was qualifying the results of AVILES, EBNES and RITTER [19]. We are very fortunate that our work is also matching with some recent work done by JAIN, SHARMA and SINGH [20].
REFERENCES


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