CHAPTER II
Chapter II

SINGLE FILE FLOW OF BLOOD IN VERY NARROW CAPILLARIES

INTRODUCTION:

On microscopic scale, the movement of blood through very narrow capillaries must involve the passage of individual red cells, in single file along it, each separated from one by a bolus of viscous fluid (Prothero and Burton [9, 10]). It is expected that the cells are deformed elastically to enable them to pass through the tube which also suffer small elastic distension. Since experiments have revealed a close link between circulatory inefficiency, impaired red cell deformability and reduced red cell ATP levels. However, some recent work has suggested even closer tie between abnormal red cell deformability and vascular disease. A number of factors such as high blood pressure, obesity, diabetes and hyperuricaemia are considered to increase the risk of cardiovascular disease. In one trial, blood was taken from a number of subjects whose presence with one or more these risk factors and listed for red cell deformability. By
plotting risk factors against red cell deformability as measured by filtration, a direct co-relation was found between the number of risk factor and impairment of red cell filterability.

It has been shown that red cell deformability has a considerable influence on blood viscosity and indeed at the capillary level it is the most important single drawn factor affecting blood flow.

The restoration of red cell deformability and reduction of blood viscosity may thus become the quickest and most practical means of improving blood flow in patients with various form of peripheral and cerebral vascular disease. The purpose of the study is to get some qualitative and quantitative insight into the problems of flow in tubes under consideration where the concentration of lubrication film of plasma is present between each red cell and tube wall. This film is potentially important in region to mass transfer and to hydraulic resistance, as well as to the relative residence times of red cells and plasma in the capillary network. Prothero and Burton [9] pointed out that
the bolus of viscous plasma between two red cells must perform, relative to their motion, a toroidal circulation, toward on the tube axis and backward near walls. They estimated, from their model experiments, the pressure drop in the bolus of moving plasma between two red cells and deduced a contribution to overall capillary resistance less than that given by Poiseuille law, mainly because plasma with depleted RBC has a viscosity considerably lower than typical values measured for whole blood.

From the available experimental data it is observed that the flow resistance in narrow capillaries is greater than that of plasma and that it may depend on the hematocrit, flow rate and capillary diameter. **White more [16]** attempted the problem giving and axial train model. The model consists of a central core of cells (in single file) and plasma moving with a uniform speed, with a surrounding annulus of plasma in which the shearing occurs. The model is applicable in capillaries whose diameters are slightly greater than those of the RBC. Another model was developed by **Bloor [1]** and **Schweizer [12]** using a rigid pill box model for
the cell. Lighthill [4] developed an elastohydrodynamic lubrication model for the deformation of the RBC and the flow between the cell and the capillary wall. Fitz - Gerald [2], Vand and Fitz - Gerald [15] refined Lighthill's model and found an analytical solution for an idealized bolus flow model. The bolus flow models generally agree that the mean resistance across the bolus increases as the bolus length is decreased. Thus, the cell spacing plays an important role in the contribution to the resistance offered by the plasma in the blood. Tozeren and Skalak [14] have developed a mathematical model for the flow of closely fitting incompressible elastic spheres in a tube under zero drag condition (under this condition it is assumed that the resultant forces on the particle due to pressure and viscous stresses exerted by the fluid is equal to zero). This condition of zero drag on the neutrally buoyant particle is formulated by considering the equilibrium of a control volume bounded by the tube wall, particle surface and two planes tangential to the particle at the downstream and upstream ends.
Calculations of Fitz - lubrication model for the flow of plasma around the RBC and the deformation of cell predicted lower resistances. A choice of membrane deformation resistance between the value measured by Rand and Burton [11] and that predicted by Lingard [5,8] gave better agreement between resistance data obtained experimentally and those predicted by Fitz & Gerald [3] and Sugihara et al [13].

MATHEMATICAL ANALYSIS:

We consider the flow of elastic incompressible sphere in a rigid tube of uniform radius. Single file flow of RBC surrounded by an annulus of plasma is considered. In the case of movable buoyant particle, treated in the present study, the condition of zero – drag on the particle must be satisfied in addition to the Reynolds equation. In can be used to eliminate leak – back (which is equal to the discharge of the fluid observed relative to a reference frame fixed to the particle) leaving only pressure drop as an unknown.
The single RBC of biconcave – disk shape is deformed during the flow passage in very narrow capillary, as shown in Figure (2.1). It is assumed that the inertial terms are negligible, the equation of motion in cylindrical polar coordinate about the axis of symmetry is

\[ \frac{\partial p}{\partial z} = \eta \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w_z}{\partial r} \right) \]  \[ (2.1) \]

The equation of continuity valid in the fluid is

\[ \frac{\partial}{\partial r} \left( r w_r \right) + \frac{\partial}{\partial z} \left( r w_z \right) = 0 \]  \[ (2.2) \]

Where \( w_r, w_z \) are radial and axial velocities, respectively, \( \eta \) the dynamic viscosity of fluid, \( p \) the pressure not varying with \( r \).
Boundary conditions are:

When \( r = R_0 \);
\[ w_z = -w_0; \quad w_r = 0 \quad \text{(2.3)} \]

When \( r = R_c \) (z),
\[ w_z = 0; \quad w_r = 0 \quad \text{(2.4)} \]

\[ P(-b) - P(b) = \Delta P_0; \quad h = R_0 - R_c(z) \quad \text{(2.5)} \]

From incompressible continuity condition (2.2), we have

\[ -\int \frac{\partial}{\partial r} (r w_r) dr = \int \frac{\partial}{\partial z} (r w_z) dr \]

\[ \frac{\partial}{\partial z} \int_{R_c}^{R_0} r w_z dr = -\int_{R_c}^{R_0} \frac{\partial}{\partial r} (r w_r) dr \]

or

\[ - \left[ (r w_r) \right]_{R_c}^{R_0} = -[0 - R_c \cdot 0] \]

\[ = 0 \]

Further

\[ \int_{R_c}^{R_0} r w_z dr = C = -Q_0 \quad \text{(2.6)} \]

Where \( Q_0 \) is the leak-back, given by

\[ 2\pi R_0 Q_0 = \pi R_0^2 U_0 - \pi R_0^2 w_0 \quad \text{(2.7)} \]

\( w_0 \) is the average velocity of the fluid in lubricating zone.

From equation (2.7),
Integrate equation (2.1) and get,

\[ w_z = \frac{1}{4}\frac{dp}{dz} r^2 + A \log r + B \]  \hspace{0.5cm} \text{--- (2.9)}

Where, A and B are constants to be determined with boundary conditions (2.3) and (2.4), as

\[ A = \left[ -U_0 - \frac{1}{4}\frac{dp}{dz} (R_0^2 - R_c^2) \right] \log \frac{R_0}{R_c} \]

And,

\[ B = \frac{1}{4}\frac{dp}{dz} R_c^2 - \left[ -U_0 - \frac{1}{4}\frac{dp}{dz} (R_0^2 - R_c^2) \right] \log R_c \log \frac{R_0}{R_c} \]

Put the value of A and B in equation (2.9) and get

\[ w_z = \frac{1}{4}\frac{R_0^2}{8\eta} \frac{dp}{dz} \left[ \left( \frac{r}{R_0} \right)^2 - \frac{R_c^2}{R_0^2} + \frac{1 - (R_c/R_0)^2}{\log(1-H)} \log \frac{r}{R_0} \right] + U_0 \frac{R_c/R_0}{\log(1-H)} \]  \hspace{0.5cm} \text{--- (2.10)}

If we put \( R_0 = R_c + h \), then equation (2.10) becomes
\[ \bar{w}_z = \left[ x^2 - (1 - H)^2 + \frac{1 - (1 - H)^2}{(1 - H) \log(1 - H)} \right] \log \frac{x}{(1 - H)} + \frac{U_0 \log x}{(1 - H) \log(1 - H)} \quad \text{(2.11)} \]

From equation (2.6) and (2.10), we find

\[ \frac{dp}{dz} = \frac{16 \eta}{R_0^2 - R_c^2(z)} \left[ Q_0 - U_0 \left( \frac{R_0^2 + R_c^2(z)}{2} + \frac{R_0^2 - R_c^2(z)}{4 \log \frac{R_c(z)}{R_0}} \right) \right] \quad \text{(2.12)} \]

Under zero-drag condition, we have, pressure force acting on the particle. Viscosity stresses experienced by the particle = 0.

\[ \pi \int_{b}^{b} R_c^2(z) \frac{dp}{dz} \, dz - 2 \pi \eta \int_{b}^{b} R_c(z) \left( \frac{\partial w_z}{\partial r} \right)_{r=R_c(z)} \, dz = 0 \quad \text{(2.13)} \]

if \( \Omega \) is fluid volume, then from equation (2.1) we have

\[ \int_{\Omega} \frac{dp}{dz} \, d\Omega = \eta \int_{\Omega} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w_z}{\partial r} \right) \, d\Omega \quad \text{(2.14)} \]

\[ \frac{1}{\eta} \left\{ \pi R_0^2 \left[ P(b) - P(-b) \right] - \pi \int_{-b}^{b} R_c^2(z) \frac{dp}{dz} \, dz \right\} \]
In view of equation (2.13), equation (2.15) can be written as

\[ \pi R_0^2 \Delta P_o = -2 \pi \eta R_0 \int_{-b}^{b} \left( \frac{\partial \omega_z}{\partial r} \right)_{r=R_0} dz \quad (2.16) \]

Using the quantity \( \eta U / a \) in the pressure and stress terms, the non-dimensional quantities becomes

\[ \bar{b} = \frac{pa}{\eta U}, \bar{\tau} = \frac{\tau_a}{\eta U}, \bar{U}_0 = \frac{U_0}{1 \frac{dp}{4\eta \frac{dz}{dz}} R_0^2}, \bar{w}_z = \frac{w_z}{1 \frac{dp}{4\eta \frac{dz}{dz}} R_0^2}, \bar{z} = \frac{z}{R_0}, \bar{r} = \frac{r}{a} \]

\[ x = \frac{r}{R_0}, \bar{R}_c(z) = \frac{R_c(z)}{R_0}, H = \frac{h}{R_0}, \alpha = \frac{a}{b}, \beta = \frac{a}{R_0}, C_0 = \frac{2Q_0}{UR_0} \quad (2.17) \]

Velocity field \( \bar{w} \) and pressure gradient \( dp/dz \) given by equations (2.11) and (2.12) take the forms,

\[ \bar{w}_z (\bar{r}, \bar{z}) = \frac{1}{4\beta} \frac{dp}{dz} \left[ \beta^2 \bar{r}^2 - \bar{R}_c^2(z) + \frac{1 - \bar{R}_c^2(z)}{\log \bar{R}_c(z)} \log \frac{\beta \bar{r}}{\bar{R}_c(z)} \right] \]

\[ + \bar{U}_0 \frac{\log \frac{\beta \bar{r}}{\bar{R}_c(z)}}{\log \bar{R}_c(z)} \quad (2.18) \]
\[
\frac{dp}{dz} = 8\beta \frac{C_0 - U_0 \left(1 + \frac{1 - R_c^2(z)}{2 \log e R_c(z)}\right)}{\left[1 - R_c^2(z)\right] \left[1 + R_c^2(z)\right] \frac{1 - R_c^2(z)}{\log e R_c(z)}} \quad --- (2.19)
\]

For the sake of convenience we omit bars in proceeding expressions. Equation (2.18) gives

\[
\frac{\partial \omega}{\partial r} = \frac{1}{4\beta} \frac{dp}{dz} \left[2\beta^2 r + \frac{1 - R_c^2(z)}{\log e R_c(z)}\right] + \frac{U_0}{\log e R_c(z)} \quad --- (2.20)
\]

With the help of equations (2.20) and (2.16) we obtain

\[
\Delta P_0 = 4\beta \int_{-\beta / \alpha}^{\beta / \alpha} \left\{2 + \frac{1 - R_c^2(z)}{\log e R_c(z)}\right\} \left\{C_0 - U_0 \left(1 + \frac{1 - R_c^2(z)}{2 \log e R_c(z)}\right)\right\}
\]
\[
\times \left\{1 - R_c^2(z)\right\} \left[1 + R_c^2(z) + \frac{1 - R_c^2(z)}{\log e R_c(z)}\right] + \frac{U_0}{2 \log e R_c(z)} \right\} dz \quad --- (2.21)
\]

As,

\[
\Delta P_0 = P\left(-\frac{\beta}{\alpha}\right) - P\left(\frac{\beta}{\alpha}\right) \quad --- (2.22)
\]

Then,

\[
\Delta P_0 = 8\beta \int_{-\beta / \alpha}^{\beta / \alpha} \left\{C_0 - U_0 \left(1 + \frac{1 - R_c^2(z)}{2 \log e R_c(z)}\right)\right\}
\]
\[
\times \left\{\left[1 - R_c^2(z)\right] \left[1 + R_c^2(z) + \frac{1 - R_c^2(z)}{\log e R_c(z)}\right]\right\} dz \quad --- (2.23)
\]

(123)
If we put,

\[
D_{11} = 4\beta \int_{-\beta/\alpha}^{\beta/\alpha} \frac{2 + \frac{1-R_c^2(z)}{\log R_c(z)}}{(1-R_c^2(z))\{1+R_c^2(C) + \frac{1-R_c^2(z)}{\log R_c(z)}\}} \, dz \tag{2.24}
\]

\[
D_{12} = 4\beta \int_{-\beta/\alpha}^{\beta/\alpha} \frac{\left\{2 + \frac{1-R_c^2(z)}{\log R_c(z)}\right\}\left\{1+\frac{1-R_c^2(z)}{2\log R_c(z)}\right\}}{(1-R_c^2(z))\{1+R_c^2(C) + \frac{1-R_c^2(z)}{\log R_c(z)}\}} \, dz \tag{2.25}
\]

\[
D_{21} = 8\beta \int_{-\beta/\alpha}^{\beta/\alpha} \frac{dz}{(1-R_c^2(z))\{1+R_c^2(C) + \frac{1-R_c^2(z)}{\log R_c(z)}\}} \tag{2.26}
\]

\[
D_{22} = 8\beta \int_{-\beta/\alpha}^{\beta/\alpha} \frac{\left\{1+\frac{1-R_c^2(z)}{2\log R_c(z)}\right\}}{(1-R_c^2(z))\{1+R_c^2(C) + \frac{1-R_c^2(z)}{\log R_c(z)}\}} \, dz \tag{2.27}
\]

Then equation (2.21) takes the form

\[
D_{11} C_0 + \Delta P_0 = D_{12} U_0 \tag{2.28}
\]

\[
D_{21} C_0 + \Delta P_0 = D_{22} U_0 \tag{2.29}
\]

For \( U_0 = 1 \), above equation give

\[
C_0 = \frac{D_{12} - D_{22}}{D_{11} - D_{21}}, \quad \Delta P_0 = D_{12} - D_{11} C_0 \tag{2.30}
\]
\[
\frac{U_0}{w_0} = (1 - C_0)^{-1}; \text{ effective viscosity } \eta_e = \frac{\alpha U_0 \Delta P_0}{16 w_0}
\] --- (2.31)

We have calculated the value of \( \frac{U_0}{w_0} \) and \( \eta_e \) and compared the calculated value from the results obtained.
RESULT AND DISCUSSION:

The variations of fluid velocity are shown in Tables and Graphs. The fluid velocity decrease as the tube radius increase for constant value of gap thickness.

Also the fluid velocity decreases with gap thickness increases for constant value of tube radius.

The variation of velocity at the axis is shown in Tables – 2.1 and 2.2 we observe that velocity of the fluid decreases with respect to tube radius and gap thickness.

The graph of the Tables – 2.1 and 2.2 are shown in such a way that velocity is plotted on y-axis while tube radius on x-axis and also in second Table the gap thick are shown on x-axis.

The variations of the pressure gradient and gap thickness are shown by Table 2.3 and Graph 2.3. The pressure gradient increases as the gap thickness increases for constant value of tube radius.
### Table 2.1

Variation of fluid velocity with respect to tube radius for constant value of gap thickness

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\overline{w}_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H = 0.1$</td>
</tr>
<tr>
<td>0.2</td>
<td>9.0707</td>
</tr>
<tr>
<td>0.4</td>
<td>4.6559</td>
</tr>
<tr>
<td>0.6</td>
<td>2.2029</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6163</td>
</tr>
</tbody>
</table>
Graph 2.1: Variation in tube radius for constant value of gap thickness
Table - 2.2

Variation of fluid velocity with respect to gap thickness for constant value of tube radius

<table>
<thead>
<tr>
<th>H</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x = 0.1$</td>
</tr>
<tr>
<td>0.2</td>
<td>7.3759</td>
</tr>
<tr>
<td>0.4</td>
<td>3.6473</td>
</tr>
<tr>
<td>0.6</td>
<td>1.8767</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5986</td>
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</table>
Graph 2.2: Variation in fluid velocity for constant value of gap thickness.
Table – 2.3
Variation of pressure gradient with respect to gap thickness

<table>
<thead>
<tr>
<th>H</th>
<th>$\frac{dp}{dz}$ for constant Value of $K = 5$</th>
<th>$\frac{dp}{dz}$ for constant value of $K = 10$</th>
<th>$\frac{dp}{dz}$ for constant value of $K = 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>-7.6552</td>
<td>-15.8295</td>
<td>-24.0038</td>
</tr>
<tr>
<td>0.4</td>
<td>-2.7713</td>
<td>-6.6471</td>
<td>-10.1560</td>
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<td>0.6</td>
<td>-2.5548</td>
<td>-5.4619</td>
<td>-8.3748</td>
</tr>
<tr>
<td>0.8</td>
<td>-3.1143</td>
<td>-6.6603</td>
<td>-10.2063</td>
</tr>
</tbody>
</table>
Graph 2.3: Variation of pressure gradient relative to the gap thickness
REFERENCES


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