Chapter: 4

Multi-Objective Interval Transportation Problem (MOITP) and its Solution using Grey Situation Decision Making Theory

4.0 Introduction

In Mathematical Programming model the uncertainty is handled by Interval Programming which also possesses some interesting characteristics of model. The availability of information about a range of variation in some of the parameter is assumed by Interval Programming which allows specifying a model with interval coefficients [111]. In multi objective interval transportation problem (MOITP) the values of supply, demand and cost parameters are in interval form where, to minimize the interval objective function, the left, right, centre and half-width of interval defines the decision maker’s preference for interval profit [69]. This chapter explores the solution of MOITP in which, coefficients of objective functions and the value of supply and demand parameter are expressed into interval.

The problem involves interval data has been being investigated since few decades. In 1992 Urli and Nadeau [112] adopted MOLP models with interval coefficient, whose algorithm do not allow the decision-maker to take into account the worst-case and the best-case ‘scenarios’ in order to perceive the risk at stake. In 1995, Inuiguchi and Sakawa [113] gave a method to solve LPP having the coefficients of objective function in interval form. Das at al. (1999) [30] proposed a solution of MOITP in which the parameters are in interval form. He transformed the problem into MOTP by converting the interval source and destination parameter into deterministic form and then solved the problem using fuzzy approach. In the year of 2001 the linear interval number programming problems were examined by Sengupta, Pal, and Chakraborty [114] where, coefficients of the objective function and inequality constraints are all interval numbers. They also proposed the concept of ‘acceptability index’ to give solution of uncertain linear programming. A partial penalty function for each constraint was determined by Sengupta and Pal [114].
A two-level mathematical programming technique was found by Liu and Wang [115] (2007) to find the upper bound and lower bound of the range and to convert constant parameters into form of interval parameters. Wu and Huang (2007) [116] gave a hybrid of interval-parameter and fuzzy methodologies to form an integrated optimization system allowing uncertainties. In 2008 Jiang et al. [117] presented a non-linear interval number programming method to deal with uncertain optimization problems. As an optimization solver they also suggested the inter generation projection genetic algorithm with fine global convergence performance. Wu [118] (2009) derived Karush–Kuhn–Tucker conditions for MOPP with interval-valued objective functions and Pareto optimal solutions are proposed by considering two orderings on the class of all closed intervals. Chanas and Kuchta [14] generalized the known concept of the solution of the LPP with interval coefficients in the objective function based on preference relations between intervals.

In 2010, G. Natarajan [69] proposed separation method based on zero point method for solving the Integer Transportation Problem where, the values of cost, supply and demand were in the form of interval. To solve the fuzzy based transportation problem in 2010, Dutta and A. Satyanarayana Murthy [69] have given, Linear Functional Programming Method having interval costs. In 2011, P. Pandian and D. Anuradha [69] has given split and bound method to solve fuzzy based fully integer interval transportation problems having impurity constraints. Again in the same year, S.K. Roy and D. R. Mahapatra have given a multi-objective stochastic transportation problem (MOSTP) whose all parameters are interval form. In 2013, Arpita Panda and Chandan Bikash Das [119] have given the solution of Cost Varying Interval Transportation Problem (CVITP) under two vehicles in which, they determined the limits of transportation cost in interval. In 2014, Abdu Sala Mohamed Khaliah, E. E. have given Rough Interval Multi-Objective Transportation Problems (RITP) by using the concepts of rough interval based on upper interval (UUIT), upper lower interval (ULIT), lower upper interval (LUIT) and lower interval (LLIT) in which, the values of cost, supply and demand were in rough interval form. In 2015, Vincent F. Yu, Kuo-Jen Hu and An-Yuan Chang have given the compromised solution of MOTP, in which DM is able to express the coefficients of objective function, source and destination parameter into interval form. In the same year, Dalbinder Kaur, Sathi Mukherjee and KajlaBasu [120] have solved the multi-objective and multi-index real-life
transportation problem using exponential membership function to fuzzy programming technique and a non-dominated compromise solution was obtained.

In this chapter we try to find solution of MOITP with Grey situation making theory based approach as well as grey theory and fuzzy programming based approach with risk attitude parameter. The Chapter also discusses several numerical examples and their solutions with a comparison of the obtained solution with other approaches.

4.1 Multi-Objective Interval Transportation Problem
Let a Transportation Problem with ‘m’ supply points (factories) and ‘n’ demand points (customers) that minimize K interval-valued objective functions with interval source and interval destination parameters.

General form of interval transportation problem (MITP) with multiple objectives:
MOITP is the problem of minimizing K interval valued objective functions with interval source and interval destination parameters.

The mathematical model of MOITP when all the cost coefficient, supply and demand are interval-valued is given by:

**Model 4.1**

Minimize \( Z^k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ C_{L_i}^k, C_{R_i}^k \right] x_{ij} \)

where \( k = 1, 2, \ldots, K \),

Subject to constraints;

\[ \sum_{j=1}^{n} x_{ij} = \left[ a_{L_i}, a_{R_i} \right], \quad i = 1, 2, \ldots, m, \]

\[ \sum_{i=1}^{m} x_{ij} = \left[ b_{L_j}, b_{R_j} \right], \quad j = 1, 2, \ldots, n, \]

\( x_{ij} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, \)

With

\[ \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \quad \text{and} \quad \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \] (4.1)
Where, \( [C_{l_{ij}}, C_{r_{ij}}], (k = 1, 2, \ldots, K) \) is an interval of uncertain cost for the transportation problem; represents delivery time, quantity of goods delivered, under used capacity, etc. Also, the source parameter lies between left limit \( a_{L_i} \) and right limit \( a_{R_i} \). Similarly, destination parameter lies between left limit \( b_{L_j} \) and right limit \( b_{R_j} \) [121].

The general table for multi-objective interval transportation problem is given below:

<table>
<thead>
<tr>
<th>Destinations →</th>
<th>Resources ↓</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( \ldots )</th>
<th>( B_n )</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>\begin{bmatrix} c_{11}^{(1)}, c_{11}^{(2)} \end{bmatrix} &amp; \begin{bmatrix} c_{12}^{(1)}, c_{12}^{(2)} \end{bmatrix} &amp; \ldots &amp; \begin{bmatrix} c_{1n}^{(1)}, c_{1n}^{(2)} \end{bmatrix} &amp; \begin{bmatrix} a_{L_i}, a_{R_i} \end{bmatrix}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\ddots</td>
<td>\ddots</td>
<td>\ddots</td>
<td>\ddots</td>
<td>\ddots</td>
<td>\ddots</td>
<td>\ddots</td>
</tr>
<tr>
<td>( A_n )</td>
<td>\begin{bmatrix} c_{n1}^{(1)}, c_{n1}^{(2)} \end{bmatrix} &amp; \begin{bmatrix} c_{n2}^{(1)}, c_{n2}^{(2)} \end{bmatrix} &amp; \ldots &amp; \begin{bmatrix} c_{nn}^{(1)}, c_{nn}^{(2)} \end{bmatrix} &amp; \begin{bmatrix} a_{L_i}, a_{R_i} \end{bmatrix}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>\begin{bmatrix} b_{L_1}, b_{R_1} \end{bmatrix} &amp; \begin{bmatrix} b_{L_2}, b_{R_2} \end{bmatrix} &amp; \ldots &amp; \begin{bmatrix} b_{L_n}, b_{R_n} \end{bmatrix}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: General Multi-Objective Interval Transportation Table

Three major cases that may arise in a multi-objective interval transportation problem can be described as (Already discussed in Chapter-1):

1. The coefficients \( c_{ij}^k \) are in the form of interval, whereas source and destination parameters are deterministic.
2. The source and destination parameters i.e. \( a_i \) and \( b_j \), are in the form of interval but the objective functions coefficients \( c_{ij}^k \) are deterministic.
3. All the parameters i.e. Objective functions coefficients, the source (\( a_i \)) and destination (\( b_j \)) parameters are in form of interval.
4.2 Method for Finding Solution of Multi Objective Interval Transportation Problem using Grey Situation Decision Making Theory:

For analysis process in MOITP, DM introduces the different parameters/approaches like as risk attitude parameter, order relations between interval, fuzzy technique, substitution variable approach, Interval estimation etc., to convert the interval number into crisp number. In this section, risk attitude parameter is used to convert the interval numbers into crisp numbers and the sensitivity analysis is also discussed by changing the values of risk attitude parameters.

To convert the MOITP into the crisp MOTP model, the risk attitude parameter is utilized as follows:

Let $Z_k = [z_{ijk}]_{n \times n}$ is the cost (profit and loss) matrix with respect to the objective $k$, where $z_{ijk} = [z_{ijk}^L, z_{ijk}^U]$ is representing the cost to transport a product from $i^{th}$ origin to $j^{th}$ destination with respect to the objective $k$; $i, j = 1, 2, ..., n$, $k = 1, 2, ..., m$. This $Z_k = [z_{ijk}]_{n \times n}$ is converted into crisp values $Z_k^\varepsilon = [z_{ijk}^\varepsilon]_{n \times n}$ by using the equation

$$Z_k^\varepsilon = z_{ijk}^\varepsilon = \frac{(z_{ijk}^L + z_{ijk}^U)}{2} + \varepsilon(z_{ijk}^U - z_{ijk}^L), \ i, j = 1, 2, ..., n, \ k = 1, 2, ..., m$$  \hspace{1cm} (4.2)

Where, $\varepsilon$ is Decision maker’s (DM) risk attitude parameter for uncertain data on real world problem and it is $|\varepsilon| \leq 0.5$, $m(x) = \frac{x^U + x^L}{2}$ is the centre of $\tilde{x}$, $d(\tilde{x}) = x^U - x^L$ is the difference of an interval $\tilde{x}$. In the analysis process, the DM gives risk attitude parameter. The risk attitude parameter converts interval number in fixed form with their values $-0.5 \leq \varepsilon < 0, \varepsilon = 0, 0 < \varepsilon \leq 0.5$ respectively. To reflect the different scenario using risk attitude parameter, the multi-objective interval transportation problem is converted into crisp multi-objective Transportation problem as follows:
Model-4.1.1:

\[
\text{minimize } Z^\varepsilon_k = \sum_{i=1}^{n} \sum_{j=1}^{m} z^\varepsilon_{ijk},
\]

Subject to the constraints:

\[
\sum_{j=1}^{m} x_{ij} = \left[a_{L_i}, a_{R_i}\right], i = 1, 2, \ldots, m,
\]

\[
\sum_{i=1}^{n} x_{ij} = \left[b_{L_j}, b_{R_j}\right], j = 1, 2, \ldots, n,
\]

\[x_{ij} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n,\]

With

\[
\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{n} a_{R_i} = \sum_{j=1}^{m} b_{R_j}
\]

(4.3)

Once it is converted into MOTP there after the procedure discussed in chapter-3 is applied to find the solution of this MOTP. Change different value of the risk attitude parameter to cover the whole range of interval and accordingly find a different solution for MOITP.

4.3 Algorithm for Finding Solution MOITP using MGSD Theory:

This section discusses algorithm of grey situation decision making theory based approach to solve MOITP.

Step-1: Read MOITP.

Step-2: Convert each interval objective of transportation problem of MOITP in simple number objective by using formula

\[
\left[0.5(a + b) + \varepsilon \varepsilon (b - a)\right], \text{ where interval is given in the form of } [a, b], a < b \text{ with } -0.5 \leq \varepsilon \leq 0.5. \text{ So MOITP becomes MOTP.}
\]

Step-3: Find the lower effect measure \( r_{ij}^{(k)} \) or upper effect measure \( r_{ij}^{(k)} \) and accomplish the consistent matrix of effect measure \( R^{(k)} = [r_{ij}^{(k)}] \) in MOTP obtained from Step-1.

for k=1 to K do,

\[
r_{ij}^{(k)} = \min_{i} \min_{j} \left\{ u_{ij}^{(k)} \right\} / u_{ij}^{(k)},
\]

\[
r_{ij}^{(k)} = u_{ij}^{(k)} / \max_{i} \max_{j} \left\{ u_{ij}^{(k)} \right\},
\]
\[ R^{(k)} = \begin{bmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \cdots & r_{1m}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \cdots & r_{2m}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1}^{(k)} & r_{n2}^{(k)} & \cdots & r_{nm}^{(k)} \end{bmatrix} \]

**Step-4:** Get the comprehensive matrix of effect measure for situation \( s_j \) is \( R = [r_{ij}] \).

for \( k = 1 \) to \( s \) do

\[
R = [r_{ij}] = \sum_{k=1}^{s} r_{ij}^{(k)} \eta_k,
\]

end

**Step-5:** Convert comprehensive matrix \( R \) in minimization form.

**Step-6:** Find the solutions of converted single objective optimization problem by given demand and supply as like TP.

**Step-7:** Change different value of risk attitude parameter and repeat the step 3 to 6 for finding all other alternative solution that covers all ranges of MOITP.

### 4.4 Numerical Illustrations

#### Numerical Illustration-1 (Case-I):

To illustrate the efficiency of the proposed method we consider the following numerical example presented by S. K. Das [30]:

\[
\text{Min } Z_1^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} \left[ c_{L_y}^1, c_{R_y}^1 \right] x_{ij}; \quad \text{Min } Z_2^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} \left[ c_{L_y}^2, c_{R_y}^2 \right] x_{ij},
\]

Subject to the constraints

\[
\sum_{j=1}^{4} x_{1j} = 8, \quad \sum_{j=1}^{4} x_{2j} = 19, \quad \sum_{j=1}^{4} x_{3j} = 17, \quad \sum_{i=1}^{3} x_{1i} = 11, \quad \sum_{i=1}^{3} x_{2i} = 3, \quad \sum_{i=1}^{3} x_{3i} = 4, \quad \sum_{i=1}^{3} x_{4i} = 16,
\]

\( x_{ij} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \).

Where \( c^1 = \begin{bmatrix} 1,2 & 1,3 & 5,9 & 4,8 \end{bmatrix}, \quad c^2 = \begin{bmatrix} 3,5 & 2,6 & 2,4 & 1,5 \end{bmatrix} \)
Solution:

(1) Here, \(U^1 = \begin{bmatrix} 1,2 & 1,3 & 5,9 & 4,8 \\ 1,2 & 7,10 & 2,6 & 3,5 \\ 7,9 & 7,11 & 3,5 & 5,7 \end{bmatrix} \), \(U^2 = \begin{bmatrix} 3,5 & 2,6 & 2,4 & 1,5 \\ 4,6 & 7,9 & 7,10 & 9,11 \\ 4,8 & 1,3 & 3,6 & 1,2 \end{bmatrix} \).

(2) Convert each interval objective of transportation problem of MOITP in simple number objective by using formula \(0.5(a+b) + \varepsilon (b-a)\), where interval is given in the form of \([a, b], a < b\) with \(-0.5 \leq \varepsilon \leq 0.5\). So MOITP becomes MOTP.

For \(\varepsilon = 0\), \(U^{(1)}(1) = \begin{bmatrix} 1.5 & 2 & 7 & 6 \\ 1.5 & 8.5 & 4 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, U^{(2)}(1) = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 8.5 & 10 \\ 6 & 2 & 4.5 & 1.5 \end{bmatrix}\).

(3) For transporting a product, goals are less than its batter, hence lower effect measure is utilized at here. The lower effect measure for first data is

\[ r^{(1)}_{11} = \min_{u_{11}} (u_{11}) = \frac{1.5}{1.5} = 1 \]

Similarly, obtain lower effect measure for each data. Therefore, the consistent matrices of effect measure are given below.

\[ R^{(1)} = \begin{bmatrix} 1 & 0.75 & 0.21429 & 0.25 \\ 1 & 0.17647 & 0.375 & 0.375 \\ 0.1875 & 0.22222 & 1 & 0.66667 \\ 0.75 & 0.5 & 1 & 1 \end{bmatrix}, R^{(2)} = \begin{bmatrix} 0.8 & 0.25 & 0.35294 & 0.3 \\ 0.33333 & 1 & 0.44444 & 1 \end{bmatrix}\).

(4) Combine \(R^{(1)}\) and \(R^{(2)}\) with \(\eta_k = 0.5(k = 1, 2)\) we have comprehensive matrix

\[ R = \begin{bmatrix} 0.875 & 0.625 & 0.60714 & 0.375 \\ 0.9 & 0.21324 & 0.36397 & 0.2625 \\ 0.26042 & 0.61111 & 0.72222 & 0.83333 \\ 0.125 & 0.375 & 0.39286 & 0.625 \end{bmatrix}. \]

(5) Convert comprehensive matrix in minimization form by subtracting each data of comprehensive matrix \(R\) of effect measure from value 1 we get,

\[ R = \begin{bmatrix} 0.125 & 0.375 & 0.39286 & 0.625 \\ 0.1 & 0.78676 & 0.63603 & 0.7375 \\ 0.73958 & 0.38889 & 0.27778 & 0.16667 \end{bmatrix}. \]

(6) By using this comprehensive matrix \(R = [r_{ij}]\) of effect measure as well as demand and supply form single objective transportation problem and find its solution,
**Solution is.**

\[ x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 11, x_{34} = 16, \]
\[ Z^1 = 2 \times 3 + 7 \times 5 + 1.5 \times 11 + 4 \times 8 + 4 \times 1 + 6 \times 16 = 189.5, \]
\[ Z^2 = 4 \times 3 + 3 \times 5 + 5 \times 11 + 8.5 \times 8 + 4.5 \times 1 + 1.5 \times 16 = 178.5. \]

(7) If we change different value of the risk attitude parameter and repeat the above steps to find solution which will cover all interval range of MOITP and the solutions are as follows:

<table>
<thead>
<tr>
<th>Value of $\epsilon$</th>
<th>$Z^1$</th>
<th>$Z^2$</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>−0.5</td>
<td>138</td>
<td>137</td>
<td>(x_{14} = 8, x_{21} = 11, x_{23} = 8, x_{32} = 3, x_{33} = 6, x_{34} = 8)</td>
</tr>
<tr>
<td>−0.4</td>
<td>148.3</td>
<td>143.7</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>−0.3</td>
<td>158.6</td>
<td>152.4</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>−0.2</td>
<td>168.9</td>
<td>161.1</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>−0.1</td>
<td>179.2</td>
<td>169.8</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>0</td>
<td>189.5</td>
<td>178.5</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>0.1</td>
<td>199.8</td>
<td>187.2</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>0.2</td>
<td>210.1</td>
<td>195.9</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>0.3</td>
<td>220.4</td>
<td>204.6</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>0.4</td>
<td>230.7</td>
<td>213.3</td>
<td>(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16)</td>
</tr>
<tr>
<td>0.5</td>
<td>247</td>
<td>253</td>
<td>(x_{14} = 8, x_{21} = 11, x_{22} = 3, x_{23} = 5, x_{33} = 9, x_{34} = 8)</td>
</tr>
</tbody>
</table>

**Table 4.2: Solution of Numerical illustration-1 with different value of $\epsilon$**

Table 4.2 contains solution of multi-objective interval transportation problems with different values of $\epsilon$ where −0.5 ≤ $\epsilon$ ≤ 0.5. Chapter also includes different value of the risk attitude parameter to cover the whole range of interval and accordingly finds a different solution for MOITP.
Numerical Illustration-2 (Case-II): [30]

Minimize \( Z^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c^1_{ij} x_{ij} \), Minimize \( Z^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} c^2_{ij} x_{ij} \)

Subject to the constraints
\[
\sum_{j=1}^{4} x_{ij} = [7,9], \sum_{j=1}^{4} x_{2j} = [17,21], \sum_{j=1}^{4} x_{3j} = [16,18]
\]
\[
\sum_{i=1}^{3} x_{i1} = [10,12], \sum_{i=1}^{3} x_{i2} = [2,4], \sum_{i=1}^{3} x_{i3} = [13,15], \sum_{i=1}^{3} x_{i4} = [15,17]
\]
\( x_{ij} \geq 0, i = 1,2,3 \) and \( j = 1,2,3,4. \)

Using the equivalent deterministic transportation problem is written as:

Minimize \( Z^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c^1_{ij} x_{ij} \), Minimize \( Z^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} c^2_{ij} x_{ij} \)

Subject to the constraints;
\[
\sum_{j=1}^{4} x_{ij} \leq 9, \sum_{j=1}^{4} x_{ij} \geq 7, \sum_{j=1}^{4} x_{2j} \leq 21, \sum_{j=1}^{4} x_{3j} \geq 17, \sum_{j=1}^{4} x_{3j} \leq 18, \sum_{j=1}^{4} x_{3j} \geq 16,
\]
\[
\sum_{i=1}^{3} x_{i1} \leq 12, \sum_{i=1}^{3} x_{i1} \geq 10, \sum_{i=1}^{3} x_{i2} \leq 4, \sum_{i=1}^{3} x_{i2} \geq 2, \sum_{i=1}^{3} x_{i3} \leq 15, \sum_{i=1}^{3} x_{i3} \geq 13, \sum_{i=1}^{3} x_{i4} \leq 17, \sum_{i=1}^{3} x_{i4} \geq 15,
\]
\( x_{ij} \geq 0, i = 1,2,3 \) and \( j = 1,2,3,4. \)

Where \( c^{(1)} = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, c^{(2)} = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}. \)

Solution:

(1) Here,
\[
U^{(1)} = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, \quad U^{(2)} = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}.
\]

(2) For transporting a product, goals are less than its the batter, hence lower effect measure is utilized at here. The lower effect measure for first data is
\[
i^{(1)} = \frac{\min \min \{u_{11}\}}{u_{11}} \quad \text{where} \quad u_{11} = 1.
\]
Similarly, obtain lower effect measure for each data. Therefore, the consistent matrices of effect measure are given below
\[
R^{(1)} = \begin{bmatrix}
1 & 0.5 & 0.14286 & 0.14286 \\
1 & 0.11111 & 0.33333 & 0.25 \\
0.125 & 0.22222 & 0.75 & 0.66667
\end{bmatrix},
\]
\[
R^{(2)} = \begin{bmatrix}
0.75 & 0.5 & 1 & 0.25 \\
0.8 & 0.25 & 0.33333 & 0.1 \\
0.16667 & 0.5 & 0.2 & 1
\end{bmatrix}.
\]

(3) Combine \( R^{(1)} \) and \( R^{(2)} \) with \( \eta_k = 0.5 (k = 1, 2) \) we have comprehensive matrix
\[
R = \begin{bmatrix}
0.875 & 0.5 & 0.57143 & 0.19643 \\
0.9 & 0.18056 & 0.33333 & 0.175 \\
0.14583 & 0.36111 & 0.475 & 0.83333
\end{bmatrix}.
\]

(4) Convert comprehensive matrix in minimization form by subtracting each data of comprehensive matrix \( R \) of effect measure from value 1 we get,
\[
R = \begin{bmatrix}
0.125 & 0.5 & 0.42857 & 0.80357 \\
0.1 & 0.81944 & 0.66667 & 0.825 \\
0.85417 & 0.63889 & 0.525 & 0.16667
\end{bmatrix}.
\]

(5) By using this comprehensive matrix \( R = [r_{ij}] \) of effect measure as well as demand and supply form single objective transportation problem and find its solution,

**Solution is.**

\[
x_{12} = 2, x_{13} = 7, x_{21} = 12, x_{23} = 5, x_{33} = 1, x_{34} = 15,
\]
\[
U^{(1)} = 2 \times 2 + 7 \times 7 + 1 \times 12 + 3 \times 5 + 4 \times 1 + 6 \times 15 = 174.
\]
\[
U^{(2)} = 4 \times 2 + 3 \times 7 + 5 \times 12 + 9 \times 5 + 5 \times 1 + 1 \times 15 = 154.
\]

**Numerical Illustration-3 (Case-III): [30]**

Minimize \( Z^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^1 x_{ij} \), Minimize \( Z^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^2 x_{ij} \)

Subject to the constraints
\[
\sum_{j=1}^{4} x_{ij} = [7, 9], \sum_{j=1}^{4} x_{j2} = [17, 21], \sum_{j=1}^{4} x_{j3} = [16, 18],
\]
\[
\sum_{i=1}^{3} x_{i1} = [10, 12], \sum_{i=1}^{3} x_{i2} = [2, 4], \sum_{i=1}^{3} x_{i3} = [13, 15], \sum_{i=1}^{3} x_{i4} = [15, 17],
\]
\[
x_{ij} \geq 0, i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4.
\]

Equivalent multi-objective deterministic transportation problem may be written as:
Minimize $Z_R^1 = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{R_{ij}}^1 x_{ij}$, Minimize $Z_R^2 = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{R_{ij}}^2 x_{ij}$,

Minimize $Z_C^1 = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{C_{ij}}^1 x_{ij}$, Minimize $Z_C^2 = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{C_{ij}}^2 x_{ij}$,

Subject to the constraints

\[ \sum_{j=1}^{4} x_{ij} \leq 9, \sum_{j=1}^{4} x_{ij} \geq 7, \sum_{j=1}^{4} x_{ij} \leq 21, \sum_{j=1}^{4} x_{ij} \geq 17, \sum_{j=1}^{4} x_{ij} \leq 18, \sum_{j=1}^{4} x_{ij} \geq 16, \]

\[ \sum_{i=1}^{3} x_{ij} \leq 12, \sum_{i=1}^{3} x_{ij} \geq 10, \sum_{i=1}^{3} x_{ij} \leq 4, \sum_{i=1}^{3} x_{ij} \geq 2, \sum_{i=1}^{3} x_{ij} \leq 15, \sum_{i=1}^{3} x_{ij} \geq 13, \sum_{i=1}^{3} x_{ij} \leq 17, \sum_{i=1}^{3} x_{ij} \geq 15, \]

$x_{ij} \geq 0, i=1,2,3$ and $j=1,2,3,4$.

$c_{i,1}^1 = \begin{bmatrix} 1 & 1 & 5 & 4 \\ 1 & 7 & 2 & 3 \\ 7 & 7 & 3 & 5 \end{bmatrix}$, $c_{i,2}^1 = \begin{bmatrix} 3 & 2 & 2 & 1 \\ 4 & 7 & 7 & 9 \\ 4 & 1 & 3 & 1 \end{bmatrix}$

Where $c_{i,1}^{r_1} = \begin{bmatrix} 2 & 3 & 9 & 8 \\ 2 & 10 & 6 & 5 \\ 9 & 11 & 5 & 7 \end{bmatrix}$, $c_{i,2}^{r_1} = \begin{bmatrix} 5 & 6 & 4 & 5 \\ 6 & 9 & 10 & 11 \\ 8 & 3 & 6 & 2 \end{bmatrix}$

$c_{i,1}^{c_1} = \begin{bmatrix} 1.5 & 2 & 7 & 6 \\ 1.5 & 8.5 & 4 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}$, $c_{i,2}^{c_1} = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 8.5 & 10 \\ 6 & 2 & 4.5 & 1.5 \end{bmatrix}$

Solution:

(1) Here,


(2) Convert each interval objective of transportation problem of MOITP in simple number objective by using formula $0.5(a+b)+\epsilon \cdot (b-a)$, where interval is given in the form of $[a,b]$, $a < b$ with $-0.5 \leq \epsilon \leq 0.5$. So MOITP becomes MOTP.

For $\epsilon = 0$, $U^{(1)} = \begin{bmatrix} 1.5 & 2 & 7 & 6 \\ 1.5 & 8.5 & 4 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}$, $U^{(2)} = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 8.5 & 10 \\ 6 & 2 & 4.5 & 1.5 \end{bmatrix}$

(3) For transporting a product, goals are less than its better, hence lower effect measure is utilized at here. The lower effect measure for first data is
Similarly, obtain lower effect measure for each data. Therefore, the consistent matrices of effect measure are given below.

\[
R^{(1)} = \begin{bmatrix}
1 & 0.75 & 0.21429 & 0.25 \\
1 & 0.17647 & 0.375 & 0.375 \\
0.1875 & 0.22222 & 1 & 0.66667 \\
\end{bmatrix}
\]

\[
R^{(2)} = \begin{bmatrix}
0.75 & 0.5 & 1 & 1 \\
0.8 & 0.25 & 0.35294 & 0.3 \\
0.33333 & 1 & 0.44444 & 1 \\
\end{bmatrix}
\]

(4) Combine \( R^{(1)} \) and \( R^{(2)} \) with \( \eta_k = 0.5 (k=1,2) \) we have comprehensive matrix

\[
R = \begin{bmatrix}
0.875 & 0.625 & 0.60714 & 0.375 \\
0.9 & 0.21324 & 0.36397 & 0.2625 \\
0.26042 & 0.61111 & 0.72222 & 0.83333 \\
\end{bmatrix}
\]

(5) Convert comprehensive matrix in minimization form by subtracting each data of comprehensive matrix \( R \) of effect measure from value 1 we get,

\[
R = \begin{bmatrix}
0.125 & 0.375 & 0.39286 & 0.625 \\
0.1 & 0.78676 & 0.63603 & 0.7375 \\
0.73958 & 0.38889 & 0.27778 & 0.16667 \\
\end{bmatrix}
\]

(6) By using this comprehensive matrix \( R = [r_{ij}] \) of effect measure as well as demand and supply form single objective transportation problem and find its solution,

**Solution is:**

\[
\begin{align*}
x_{12} &= 2, x_{13} = 5, x_{21} = 12, x_{23} = 5, x_{33} = 3, x_{34} = 15, \\
Z^1 &= 2 \times 2 + 7 \times 5 + 1.5 \times 12 + 4 \times 8 + 4 \times 5 + 6 \times 15 = 179, \\
Z^2 &= 4 \times 2 + 3 \times 5 + 5 \times 12 + 8.5 \times 5 + 4.5 \times 3 + 1.5 \times 15 = 161.5.
\end{align*}
\]

(7) If we change different value of the risk attitude parameter and repeat the above steps to find solution which will covers all interval range of MOITP then the solutions are as follows:
Table 4.3: Solution of Numerical illustration-3 with different value of $\varepsilon$

Table 4.3 contains solution of multi-objective interval transportation problems with different values of $\varepsilon$ where $-0.5 \leq \varepsilon \leq 0.5$. Chapter also include different value of the risk attitude parameter to cover the whole range of interval and accordingly finds a different solution for MOITP.

**Numerical Illustration-4[122]:**

Minimize $Z^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$, Minimize $Z^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^2 x_{ij}$

Subject to the constraints

$$\sum_{j=1}^{4} x_{ij} = [14,18], \sum_{j=1}^{4} x_{2j} = [16,20], \sum_{j=1}^{4} x_{3j} = [12,16],$$

$$\sum_{i=1}^{3} x_{ij} = [10,14], \sum_{i=1}^{3} x_{12} = [15,19], \sum_{i=1}^{3} x_{13} = [17,21],$$

$x_{ij} \geq 0, i=1,2,3$ and $j=1,2,3,4$.

Equivalent multi-objective deterministic transportation problem may be written as:

Minimize $Z_R^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{R_{ij}} x_{ij}$, Minimize $Z_R^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{R_{ij}}^2 x_{ij}$,

Minimize $Z_C^1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{C_{ij}} x_{ij}$, Minimize $Z_C^2 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{C_{ij}}^2 x_{ij}$,
Subject to the constraints

\[
\begin{align*}
\sum_{j=1}^{3} x_{ij} &\leq 18, \\
\sum_{j=1}^{3} x_{ij} &\geq 14, \\
\sum_{j=1}^{3} x_{ij} &\leq 20, \\
\sum_{j=1}^{3} x_{ij} &\geq 16, \\
\sum_{j=1}^{3} x_{ij} &\leq 16, \\
\sum_{j=1}^{3} x_{ij} &\geq 12,
\end{align*}
\]

\[
\begin{align*}
\sum_{i=1}^{3} x_{ii} &\leq 14, \\
\sum_{i=1}^{3} x_{ii} &\geq 10, \\
\sum_{i=1}^{3} x_{ii} &\leq 19, \\
\sum_{i=1}^{3} x_{ii} &\geq 15, \\
\sum_{i=1}^{3} x_{ii} &\leq 21, \\
\sum_{i=1}^{3} x_{ii} &\geq 17,
\end{align*}
\]

\[x_{ij} \geq 0, i = 1, 2, 3 \text{ and } j = 1, 2, 3.\]

Where \(c^1 = \begin{bmatrix} 3 & 9 & 8 \\ 10 & 6 & 5 \\ 11 & 5 & 7 \end{bmatrix}, \quad c^2 = \begin{bmatrix} 5 & 4 & 5 \\ 6 & 10 & 11 \\ 8 & 6 & 2 \end{bmatrix}\)

\[c^1 = \begin{bmatrix} 2 & 7 & 6 \\ 8.5 & 4 & 4 \\ 9 & 4 & 6 \end{bmatrix}, \quad c^2 = \begin{bmatrix} 4 & 3 & 3 \\ 5 & 8.5 & 10 \\ 6 & 4.5 & 1.5 \end{bmatrix}\]

**Solution:**


(2) Convert each interval objective of transportation problem of MOITP in simple number objective by using formula \(0.5(a+b) + \epsilon(b-a)\), where interval is given in the form of \([a,b]\), \(a < b\) with \(-0.5 \leq \epsilon \leq 0.5\). So MOITP becomes MOTP.

\[\text{for } \epsilon = 0, \quad U^{(1)} = \begin{bmatrix} 2 & 7 & 6 \\ 8.5 & 4 & 4 \\ 8.5 & 4 & 6 \end{bmatrix}, \quad U^{(2)} = \begin{bmatrix} 4 & 3 & 3 \\ 5 & 8.5 & 10 \\ 6 & 4.5 & 1.5 \end{bmatrix}\]

(3) For transporting a product, goals are less than its better, hence lower effect measure is utilized at here. The lower effect measure for first data is

\[\mu^{(1)} = \min \min \{u_{11}\} \cdot \frac{2}{u_{11}} = \frac{2}{2}\]

Similarly, obtain lower effect measure for each data. Therefore, the consistent matrices of effect measure are given below.

\[R^{(1)} = \begin{bmatrix} 1 & 0.2857 & 0.3333 \\ 0.2353 & 1 & 1 \\ 0.2222 & 1 & 0.6667 \end{bmatrix}, \quad R^{(2)} = \begin{bmatrix} 0.75 & 1 & 0.5 \\ 0.8 & 0.3529 & 0.15 \\ 0.25 & 0.3333 & 1 \end{bmatrix}\]
(4) Combine \( R^{(1)} \) and \( R^{(2)} \) with \( \eta_k = 0.5 \) (\( k=1,2 \)) we have comprehensive matrix, 
\[
R = \begin{bmatrix}
0.875 & 0.6429 & 0.4167 \\
0.5177 & 0.6765 & 0.575 \\
0.2361 & 0.6667 & 0.8333
\end{bmatrix}.
\]

(5) Convert comprehensive matrix in minimization form by subtracting each 
Data of comprehensive matrix \( R \) of effect measure from value 1 we get, 
\[
R = \begin{bmatrix}
0.125 & 0.3571 & 0.5833 \\
0.4823 & 0.3235 & 0.425 \\
0.7639 & 0.3333 & 0.1667
\end{bmatrix}.
\]

(6) By using this comprehensive matrix \( R = [r_{ij}] \) of effect measure as well as demand 
and supply form single objective transportation problem and find its solution, 

**Solution is:**

\[
x_{11} = 14, x_{22} = 15, x_{33} = 1, x_{33} = 16,
\]

\[
Z^1 = 2 \times 14 + 4 \times 15 + 4 \times 1 + 6 \times 16 = 188.
\]

\[
Z^2 = 4 \times 14 + 8.5 \times 15 + 10 \times 1 + 1.5 \times 16 = 217.5.
\]

(7) If we change different value of the risk attitude parameter and repeat the above 
steps to find solution which will covers all interval range of MOITP then the 
solutions are as follows:

<table>
<thead>
<tr>
<th>Value of ( \varepsilon )</th>
<th>( Z^1 )</th>
<th>( Z^2 )</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>123</td>
<td>159</td>
<td>( x_{11} = 10, x_{13} = 4, x_{22} = 16, x_{33} = 13 )</td>
</tr>
<tr>
<td>-0.4</td>
<td>135.6</td>
<td>168.7</td>
<td>( x_{11} = 10, x_{13} = 4, x_{22} = 16, x_{33} = 13 )</td>
</tr>
<tr>
<td>-0.3</td>
<td>151.4</td>
<td>190.2</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
<tr>
<td>-0.2</td>
<td>163.6</td>
<td>199.3</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
<tr>
<td>-0.1</td>
<td>175.8</td>
<td>208.4</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
<tr>
<td>0</td>
<td>188</td>
<td>217.5</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
<tr>
<td>0.1</td>
<td>202.2</td>
<td>226.6</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
<tr>
<td>0.2</td>
<td>212.4</td>
<td>235.7</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
<tr>
<td>0.3</td>
<td>224.6</td>
<td>244.8</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
<tr>
<td>0.4</td>
<td>236.8</td>
<td>253.9</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
<tr>
<td>0.5</td>
<td>249</td>
<td>263</td>
<td>( x_{11} = 14, x_{23} = 1, x_{22} = 15, x_{33} = 16 )</td>
</tr>
</tbody>
</table>

Table 4.4: Solution of Numerical illustration-4 with different value of \( \varepsilon \)
Table 4.4 contains solution of multi-objective interval transportation problems with different values of $\varepsilon$ where $-0.5 \leq \varepsilon \leq 0.5$. Chapter also include different value of the risk attitude parameter to cover the whole range of interval and accordingly finds a different solution for MOITP.

### 4.5 Comparison of Grey Situation Decision Making Theory Approach with Other Approaches

This section discusses comparison of grey situation decision making theory based approach with other developed approaches. The comparison Table 4.5 shows that the grey situation decision making theory based approach provides very efficient alternative approach to find solution of multi-objective interval transportation problem.

<table>
<thead>
<tr>
<th>Sr.no.</th>
<th>Multi-objective Grey situation decision(MGSD) making theory approach</th>
<th>Other approach</th>
</tr>
</thead>
</table>
| Numerical Illustration: 1 | At $\varepsilon = 0$
$Z^1 = 189.5$, $Z^2 = 178.5$. | By S.K.Das et.al. [30]:
| Numerical Illustration: 2 | At $\varepsilon = 0$
$Z^1 = 174$, $Z^2 = 154$. | By S.K.Das et.al. [30]:
$Z_1 = 149.6$, $Z_2 = 174$. |
| Numerical Illustration: 3 | At $\varepsilon = 0$
$Z^1 = 179$, $Z^2 = 161.5$. | By S.K.Das et.al. [30]:
$Z_1 = [113, 204.9]$, $Z_2 = [129.89, 227.86]$. |
| Numerical Illustration: 4 | At $\varepsilon = 0$
$Z^1 = 188$, $Z^2 = 217.5$. | By Samayan et.al. [122]:

**Table 4.5: Comparison of numerical illustrations with other approach:**

Table 4.5 contains comparison of multi-objective interval transportation problems with different approaches. In this table MGSD theory based approach having value of $\varepsilon = 0$. As per the Table-4.2, Table-4.3 and Table-4.4, some change in value of $\varepsilon$, there is some change in objective value. This value of $\varepsilon$ is decided by decision makers and according to their requirements, solutions are achieved.
4.6  Grey Situation Decision Making Theory and Fuzzy Programming Technique based Approach to Solve MOITP:

In this approach first MOITP is converted into first in MOTP by using risk attitude parameter as follows:

Let $Z_k = [z_{ijk}]_{n \times m}$ is the cost (profit and loss) matrix with respect to the objective $k$,

where $z_{ijk} = [z_{jk}^{L}, z_{jk}^{U}]$ is representing the cost with respect to the objective $k$; $i, j = 1, 2, ..., n$, $k = 1, 2, ..., m$. This $Z_k = [z_{ijk}]_{n \times m}$ is converted into crisp values $Z_k^e = [z_{ijk}^e]_{n \times m}$ by using the equation (4.2). So the problem MOITP becomes MOTP which can be easily solved by the approach discussed in Chapter -3.

Here, in this developed approach we first utilize Grey situation decision making theory to find the lower effect measure $r_{ij}^{(k)}$ or upper effect measure $r_{ij}^{(k)}$ and accomplish the consistent matrix of effect measure $R_k^{(k)} = [r_{ij}^{(k)}]$ for each objective $k$ at different vale of $e$. These matrices of each objective are utilized as a cost matrix of each objective in fuzzy programming technique and solution are obtained. So here Grey situation decision making theory is utilized for normalization of data.

The solution of MOITP can be obtained by the following steps:

**Step-1:** Read MOITP with an interval.

**Step-2:** Convert each interval objective of transportation problem of MOITP in simple number objective by using formula $[0.5(a + b)] + \epsilon(b - a)$, where interval is given in the form of $[a, b], a < b$ with $-0.5 \leq \epsilon \leq 0.5$. So MOITP becomes MOTP.

**Step-3:** Find the lower effect measure $r_{ij}^{(k)}$ or upper effect measure $r_{ij}^{(k)}$ and accomplish the consistent matrix of effect measure $R_k^{(k)} = [r_{ij}^{(k)}]$ in MOTP obtained from Step-2.

for $k = 1$ to $s$ do

$$r_{ij}^{(k)} = \min_j \min_i \left\{u_{ij}^{(k)} \right\}/u_{ij}^{(k)}$$

$$r_{ij}^{(k)} = u_{ij}^{(k)} \max_i \max_j \left\{u_{ij}^{(k)} \right\}$$


\[ R^{(k)} = \begin{bmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \cdots & r_{1m}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \cdots & r_{2m}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1}^{(k)} & r_{n2}^{(k)} & \cdots & r_{nm}^{(k)} \end{bmatrix} \]

end

**Step-4:** From the results achieve in Step-3, determine the corresponding values for every objective at each solution derived. According to each solution and value for every objective, we can find a pay-off matrix as follows:

<table>
<thead>
<tr>
<th></th>
<th>( z_1(x) )</th>
<th>( z_2(x) )</th>
<th>( \cdots )</th>
<th>( z_K(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X^{(1)} )</td>
<td>( z_{11} )</td>
<td>( z_{12} )</td>
<td>( \cdots )</td>
<td>( z_{1K} )</td>
</tr>
<tr>
<td>( X^{(2)} )</td>
<td>( z_{21} )</td>
<td>( z_{22} )</td>
<td>( \cdots )</td>
<td>( z_{2K} )</td>
</tr>
<tr>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( X^{(k)} )</td>
<td>( z_{k1} )</td>
<td>( z_{k2} )</td>
<td>( \cdots )</td>
<td>( z_{kK} )</td>
</tr>
</tbody>
</table>

**Table 4.6: Pay-off matrix MOITP**

Where \( X^{(1)}, X^{(2)}, \ldots, X^{(k)} \) are the isolated optimal solutions of the \( K \) different transportation problems for \( K \) different objective function.

**Step-5:** Define a membership function \( \mu(z_k) \) for the \( k^{th} \) objective function.

**Step-6:** Develop single objective optimization problem using fuzzy linear membership function and hyperbolic function as follows;

Maximize \( \lambda \),

\[ \lambda \leq \mu(z_k^e) \]

Subject to the constraints

\[ \sum_{j=1}^{m} x_{ij} = a_i, \quad i = 1, 2, \ldots, m. \]

\[ \sum_{i=1}^{n} x_{ij} = b_j, \quad j = 1, 2, \ldots, n. \]

\( x_{ij} \geq 0, \forall i, j. \)

\( \lambda \geq 0. \)
If we use a linear membership function, the crisp model can be simplified as:

Maximize $\lambda$,

$$z^e_k + \lambda(U_k - L_k) \leq U_k, k = 1, 2, \ldots, K.$$ 

Subject to the constraints

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \ldots, m.$$ 

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \ldots, n.$$ 

$$x_{ij} \geq 0, \forall i, j.$$ 

$$\lambda \geq 0.$$ 

If we will use the hyperbolic membership function, then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize $\lambda$,

$$\lambda \leq \frac{1}{2} \frac{e^{\left(\frac{U_k + L_k}{2} - z^e_k(x)\right) a_i} - e^{\left(\frac{U_k + L_k}{2} - z^e_k(x)\right) a_i}}{e^{\left(\frac{U_k + L_k}{2} - z^e_k(x)\right) a_i} + e^{\left(\frac{U_k + L_k}{2} - z^e_k(x)\right) a_i}} \frac{1}{2, i \Gamma k = 1, 2, \ldots, K}$$

where $a_k = \frac{6}{U_k - L_k}$

Subject to the constraints

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \ldots, m.$$ 

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \ldots, n.$$ 

$$x_{ij} \geq 0, \forall i, j.$$ 

$$\lambda \geq 0.$$ 

**Step-7:** Solve model developed in step-6 and find compromise solution

**Step-8:** Change different value of the risk attitude parameter and repeat the step 3 to 7 in finding all other alternative solution that covers all ranges of MOITP.
4.7 Algorithm for Finding Solution of MOITP with Membership Function using MGSD Theory:

**Input**
MOITP

**Output**
Solution of MOITP

Compute the efficient solution of MOITP using the transportation model of objective weight.

**Begin**

**Step-1:** Write your MOITP with an interval.

**Step-2:** Convert each interval objective of transportation problem of MOITP in simple number objective by using formula $\left[0.5(a+b)+\varepsilon(b-a)\right]$, where interval is given in the form of $[a,b]$, $a < b$ with $-0.5 \leq \varepsilon \leq 0.5$. So MOITP becomes MOTP.

**Step-3:** Read: problem obtain in Step-2
while problem = MOTP do,
  for k=1 to s do,
    enter effect measure matrix $U^{(k)}$,
  end

**Step-4:** Find the lower effect measure $r_{ij}^{(k)}$ or upper effect measure $R^{(k)}$ and accomplish the consistent matrix of effect measure $R^{(k)} = \left[r_{ij}^{(k)} \right]$ in MOTP obtained from Step-3.

  for k=1 to s do,
    $r_{ij}^{(k)} = \min \min \left\{ u_{ij}^{(k)} \right\} / u_{ij}^{(k)}$,
    $r_{ij}^{(k)} = \max \max \left\{ u_{ij}^{(k)} \right\}$,
  end

$R^{(k)} = \left[r_{ij}^{(k)} \right]$,
**Step-5:** Convert comprehensive matrix $R^{(k)}$ of $k^{th}$ objective in minimization form for all objectives by subtracting each data of comprehensive matrix $R$ of effect measure from 1.

**Step-6:** Find optimal solution to each objective by using simplex method.

**Step-7:** Find pay-off matrix by using each objective solution.

**Step-8:** Define linear as well as a hyperbolic membership function using pay-off matrix.

**Step-9:** Develop single objective transportation problem using fuzzy linear membership function and hyperbolic membership function.

**Step-10:** Solve model developed in step-6 and find a compromise solution.

**Step-11:** Change different value of the risk attitude parameter and repeat the step 2 to 10 for finding all other alternative solution that covers all ranges of MOITP.

### 4.8 Numerical Illustrations

**Numerical Illustration-1(Case-I):**

(Same as Numerical illustration-1 in section 4.4), Now by applying MGSD theory with membership function to this MOITP

1. Effect measure matrices under goals are given below: for $\epsilon = 0$

   \[
   U^{(1)} = \begin{bmatrix} 1.5 & 2 & 7 & 6 \\ 1.5 & 8.5 & 4 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, \quad U^{(2)} = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 8.5 & 10 \\ 6 & 2 & 4.5 & 1.5 \end{bmatrix}.
   \]

2. For transporting a product, goals are less than its the batter, hence lower effect measure is utilized at here. The lower effect measure for first data is

   \[
   r_{11}^{(1)} = \min \min \{ u_{11} \} = \frac{1.5}{1.5} = 1.
   \]

   Similarly, obtain lower effect measure for each data and convert consistent matrix of effect measure $R^{(k)}$ of $k^{th}$ objective in minimization form for all objectives by subtracting each data of comprehensive matrix $R$ of effect measure from 1 we get,

   \[
   R^{(1)} = \begin{bmatrix} 0 & 0.25 & 0.7857 & 0.75 \\ 0 & 0.8235 & 0.625 & 0.625 \\ 0.8125 & 0.7778 & 0 & 0.3333 \end{bmatrix}, \quad R^{(2)} = \begin{bmatrix} 0.25 & 0.5 & 0 & 0.5 \\ 0.2 & 0.75 & 0.6471 & 0.85 \\ 0.75 & 0.25 & 0.6667 & 0 \end{bmatrix}.
   \]

3. Find each objective, solution for multi-objective transportation problem by using consistent matrix of effect measure with simplex method.

   For first objective: the optimal allocations are

   \[
   x_{11} = 5, x_{12} = 3, x_{21} = 6, x_{24} = 13, x_{33} = 14, x_{34} = 3.
   \]
Apply these allocations to first, second objective therefore we have 

\[ z_1(X^1) = 9.87499, \quad z_2(X^1) = 24.33338 \]

For second objective: the optimal allocations are 

\[ x_{13} = 8, \quad x_{21} = 11, \quad x_{23} = 2, \quad x_{32} = 6, \quad x_{34} = 1, \quad x_{34} = 16 \]

Apply these allocations to first, second objective therefore we have 

\[ z_1(X^2) = 17.7938, \quad z_2(X^2) = 7.83236 \]

Hence, pay-off matrix:

\[
\begin{bmatrix}
Z_i(X^i) & Z_j(X^i) \\
Z_i(X^2) & Z_j(X^2)
\end{bmatrix} = \begin{bmatrix}
9.87499 & 24.33338 \\
17.7938 & 7.83236
\end{bmatrix},
\]

Now,

\[
U_i = \max (9.87499, 17.7938) = 17.7938, \\
L_i = \min (9.87499, 17.7938) = 9.87499, \\
U_1 - L_1 = 7.91881, \quad U_1 + L_1 = 27.66879, \\
U_2 = \max (7.83236, 24.33338) = 24.33338, \\
L_2 = \min (7.83236, 24.33338) = 7.83236, \\
U_2 - L_2 = 16.50102, \quad U_2 + L_2 = 32.16574
\]

Applying fuzzy linear membership function, we get the following model 

Maximize \( \lambda \),

\[
0x_{11} + 0.25x_{12} + 0.7857x_{13} + 0.75x_{14} + 0x_{21} + 0.8235x_{22} + 0.625x_{23} + 0.625x_{24} + 0.8125x_{31} + 0.7778x_{32} + 0x_{33} + 0.3333x_{34} + \lambda (7.91881) \leq 17.7983, \\
0.25x_{11} + 0.5x_{12} + 0x_{13} + 0.5x_{14} + 0.2x_{21} + 0.75x_{22} + 0.6471x_{23} + 0.85x_{24} + 0.75x_{31} + 0.25x_{32} + 0.6667x_{33} + 0x_{34} + \lambda (16.50102) \leq 24.33338,
\]

Subject to the constraints:

\[
\sum_{j=1}^{4} x_{ij} = 8, \quad \sum_{j=1}^{4} x_{2j} = 19, \quad \sum_{j=1}^{4} x_{3j} = 17, \quad \sum_{i=1}^{3} x_{i1} = 11, \quad \sum_{i=1}^{3} x_{i2} = 3, \quad \sum_{i=1}^{3} x_{i3} = 14, \quad \sum_{i=1}^{3} x_{i4} = 16, \\
x_{ij} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, \lambda \geq 0.
\]

**Solution of this model is:**

The optimal allocations are

\[
x_{12} = 3, \quad x_{13} = 5, \quad x_{21} = 11, \quad x_{23} = 2.250569, \quad x_{24} = 5.749431, \\
x_{33} = 6.749431, \quad x_{34} = 10.25057.
\]

Using these allocations we have \( Z_1 = 178.001144, Z_2 = 204.372441 \)

With degree of satisfaction \( \lambda = 0.593325 \).
Applying fuzzy hyperbolic membership function, we get the following model

Maximize \( \lambda \),

\[
\lambda \leq \frac{1}{2} e^T \begin{pmatrix}
\frac{27.66879}{2} - (0.25 x_{11} + 0.75 x_{14}) \\
+ 0.25 x_{12} + 0.75 x_{13} + 0.5 x_{14}
\end{pmatrix}
\]

Subject to the constraints:

\[
\sum_{j=1}^{4} x_{ij} = 8, \quad \sum_{j=1}^{4} x_{2j} = 19, \quad \sum_{j=1}^{4} x_{3j} = 17, \quad \sum_{i=1}^{3} x_{i1} = 11, \quad \sum_{i=1}^{3} x_{i2} = 3, \quad \sum_{i=1}^{3} x_{i3} = 14
\]

\[
\sum_{i=1}^{3} x_{i4} = 16,
\]

\( x_{ij} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, \lambda \geq 0. \)

Solution of this model is:

The optimal allocations are

\( x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 2.250569, x_{24} = 5.749431, \)

\( x_{31} = 6.749431, x_{34} = 10.25057. \)

Using these allocations we have \( Z_1 = 178.001144, Z_2 = 204.372441 \)

With degree of satisfaction \( \lambda = 0.75397. \)
Table 4.7: Solution of Numerical illustration-1 with different value of $\varepsilon$

Table 4.7 contains solution of multi-objective interval transportation problems with different values of $\varepsilon$ where $-0.5 \leq \varepsilon \leq 0.5$ using membership function. Change different value of the risk attitude parameter to cover the whole range of interval and accordingly finds a different solution for MOITP.

Numerical Illustration-2(Case-II):

(Same as Numerical illustration-2 in section 4.4) Now by applying MGSD theory with membership function to this MOITP

(1) Effect measure matrices under goals are given below:

$$U^{(1)} = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, \quad U^{(2)} = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}$$

(2) For transporting a product, goals are less than its the batter, hence lower effect measure is utilized at here. The lower effect measure for first data is

$$\eta_{11}^{(1)} = \min \left\{ \frac{\eta_{11}}{u_{11}} \right\} = \frac{1}{1}$$

Similarly, obtain lower effect measure for each data and convert consistent matrix of effect measure $R^{(k)}$ of $k^{th}$ objective in minimization form for all objectives by
subtracting each data of comprehensive matrix \( R \) of effect measure from 1 we get,
\[
R^{(1)} = \begin{bmatrix}
0 & 0.5 & 0.8571 & 0.8571 \\
0 & 0.8889 & 0.6667 & 0.75 \\
0.875 & 0.7778 & 0.25 & 0.3333
\end{bmatrix},
R^{(2)} = \begin{bmatrix}
0.25 & 0.5 & 0 & 0.6667 \\
0.2 & 0.75 & 0.6667 & 0.9 \\
0.8333 & 0.5 & 0.8 & 0
\end{bmatrix}
\]

(3) Find each objective, solution for multi-objective transportation problem by using consistent matrix of effect measure with simplex method.

For first objective we get: The optimal allocations are
\[
x_{11} = 5, x_{12} = 2, x_{21} = 7, x_{23} = 10, x_{33} = 3, x_{34} = 15
\]
Apply these allocations to first, second objective therefore we have
\[
Z_1\left(X^1\right) = 13.4165, Z_2\left(X^1\right) = 12.7170
\]
For second objective: The optimal allocations are
\[
x_{13} = 9, x_{21} = 12, x_{22} = 1, x_{23} = 4, x_{32} = 1, x_{34} = 15
\]
Apply these allocations to first, second objective therefore we have
\[
Z_1\left(X^2\right) = 17.0469, Z_2\left(X^2\right) = 6.3168
\]
Pay-off matrix=
\[
\begin{bmatrix}
z_1(X^1) & z_2(X^1) \\
z_1(X^2) & z_2(X^2)
\end{bmatrix} = \begin{bmatrix}
13.4165 & 12.7170 \\
17.0469 & 6.3168
\end{bmatrix}
\]
Now,
\[
U_1 = \max (13.4165, 17.0469) = 17.0469,
L_1 = \min (13.4165, 17.0469) = 13.4165,
U_1 - L_1 = 3.6304, U_1 + L_1 = 30.4634,
U_2 = \max (12.7170, 6.3168) = 12.7170,
L_2 = \min (12.7170, 6.3168) = 6.3168,
U_2 - L_2 = 6.4002, U_2 + L_2 = 19.0338
\]
Applying fuzzy linear membership function, we get the following model
Maximize \( \lambda \),
\[
0x_{11} + 0.5x_{12} + 0.8571x_{13} + 0.8571x_{14} + 0x_{21} + 0.8889x_{22} + 0.6667x_{23} + 0.75x_{24} + 0.875x_{31} + 0.7778x_{32} + 0.25x_{33} + 0.3333x_{34} + \lambda (3.6304) \leq 17.0469,
0.25x_{11} + 0.5x_{12} + 0x_{13} + 0.6667x_{14} + 0.2x_{21} + 0.75x_{22} + 0.6667x_{23} + 0.9x_{24} + 0.8333x_{31} + 0.5x_{32} + 0.8x_{33} + 0x_{34} + \lambda (6.4002) \leq 12.7170,
\]
Subject to the constraints:

\[
\sum_{j=1}^{4} x_{ij} \leq 9, \sum_{j=1}^{4} x_{ij} \geq 7, \sum_{j=1}^{4} x_{ij} \leq 21, \sum_{j=1}^{4} x_{ij} \geq 17, \\
\sum_{j=1}^{4} x_{ij} \leq 18, \sum_{j=1}^{4} x_{ij} \geq 16, \sum_{j=1}^{4} x_{ij} \leq 12, \sum_{j=1}^{4} x_{ij} \geq 10, \sum_{j=1}^{4} x_{ij} \leq 4, \\
\sum_{i=1}^{3} x_{ij} \geq 2, \sum_{i=1}^{3} x_{ij} \leq 15, \sum_{i=1}^{3} x_{ij} \geq 13, \sum_{i=1}^{3} x_{ij} \leq 17, \sum_{i=1}^{3} x_{ij} \geq 15, \\
x_{ij} \geq 0, i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4, \lambda \geq 0.
\]

Solution of this model is.

The optimal allocations are

\[ x_{i2} = 2, x_{i3} = 5.608664, x_{21} = 12, x_{23} = 5, x_{33} = 2.391336, x_{34} = 15. \]

Using these allocations we have \( Z_2 = 169.826, Z_2 = 156.7827. \)

With degree of satisfaction \( \lambda = 0.635985. \)

Applying fuzzy hyperbolic membership function, we get the following model

Maximize \( \lambda, \)

\[
\lambda \leq \frac{1}{2} \left( e^{\frac{1}{2}} - e^{-\frac{1}{2}} \right) + \frac{1}{2} e^{-\frac{1}{2}}. 
\]

Subject to the constraints

\[
\sum_{j=1}^{4} x_{ij} \leq 9, \sum_{j=1}^{4} x_{ij} \geq 7, \sum_{j=1}^{4} x_{ij} \leq 21, \sum_{j=1}^{4} x_{ij} \geq 17, \\
\sum_{i=1}^{3} x_{ij} \leq 12, \sum_{i=1}^{3} x_{ij} \geq 10, \sum_{i=1}^{3} x_{ij} \leq 4, \sum_{i=1}^{3} x_{ij} \geq 2, \\
x_{ij} \geq 0, i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4, \lambda \geq 0.
\]
Solution of this model is:

The optimal allocations are

\[ x_{12} = 2, x_{13} = 5.608664, x_{21} = 12, x_{23} = 5, x_{33} = 2.391336, x_{34} = 15. \]

Using these allocations we have \( Z_1 = 169.826, Z_2 = 156.7827. \)

With degree of satisfaction \( \lambda = 0.836419 \).

<table>
<thead>
<tr>
<th>( Z^1 )</th>
<th>( Z^2 )</th>
<th>linear ( \lambda )</th>
<th>Hyperbolic ( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>169.826</td>
<td>156.7827</td>
<td>( \lambda = 0.635985 )</td>
<td>( \lambda = 0.836419 )</td>
</tr>
</tbody>
</table>

Table 4.8: Solution of Numerical illustration-2

Numerical Illustration-3(Case-III):

(Same as Numerical illustration-3 in section 4.4), Now by applying MGSD theory with membership function to this MOTP

(1) Effect measure matrices under goals are given below: for \( \epsilon = 0 \)

\[
U^{(1)} = \begin{bmatrix}
1.5 & 2 & 7 & 6 \\
1.5 & 8.5 & 4 & 4 \\
8 & 9 & 4 & 6 \\
\end{bmatrix} \quad \text{and} \quad U^{(2)} = \begin{bmatrix}
4 & 4 & 3 & 3 \\
5 & 8 & 8.5 & 10 \\
6 & 2 & 4.5 & 1.5 \\
\end{bmatrix}
\]

(2) For transporting a product, goals are less than its the batter, hence lower effect measure is utilized at here. The lower effect measure for first data is

\[
\hat{r}^{(1)}_{11} = \frac{\min \{u_{11}\}}{u_{11}} = \frac{1.5}{1.5}
\]

Similarly, obtain lower effect measure for each data and convert consistent matrix of effect measure \( R^{(k)} \) of \( k^{th} \) objective in minimization form for all objectives by subtracting each data of comprehensive matrix \( R \) of effect measure from 1 we get,

\[
R^{(1)} = \begin{bmatrix}
0 & 0.25 & 0.7857 & 0.75 \\
0 & 0.8235 & 0.625 & 0.625 \\
0.8125 & 0.7778 & 0 & 0.3333 \\
\end{bmatrix} \quad \text{,} \quad R^{(2)} = \begin{bmatrix}
0.25 & 0.5 & 0 & 0.5 \\
0.2 & 0.75 & 0.6471 & 0.85 \\
0.75 & 0.25 & 0.6667 & 0 \\
\end{bmatrix}
\]

(3) Find each objective solutions for multi-objective transportation problem by using consistent matrix of effect measure with simplex method.

For first objective: The optimal allocations are

\[ x_{11} = 5, x_{12} = 2, x_{21} = 7, x_{24} = 10, x_{33} = 13, x_{34} = 5. \]

Apply these allocations to first and second objective therefore we have

\[ Z_1(\lambda^1) = 8.4165, Z_2(\lambda^1) = 20.8167. \]
For second objective: The optimal allocations are 
\[ x_{13} = 9, x_{21} = 12, x_{22} = 1, x_{23} = 4, x_{32} = 1, x_{34} = 15. \]

Apply these allocations to first and second objective therefore we have 
\[ Z_1(X^2) = 16.1727, Z_2(X^2) = 5.9884. \]

Pay off matrix=
\[
\begin{bmatrix}
Z_1(X^1) & Z_2(X^1) \\
Z_1(X^2) & Z_2(X^2)
\end{bmatrix} =
\begin{bmatrix}
8.4165 & 20.8167 \\
16.1727 & 5.9884
\end{bmatrix}
\]

Now,
\[ U_1 = \max (16.1727, 8.4165) = 16.1727, \]
\[ L_4 = \min (16.1727, 8.4165) = 8.4165, \]
\[ U_1 - L_4 = 7.7562, U_1 + L_4 = 24.5892, \]
\[ U_2 = \max (20.8167, 5.9884) = 20.8167, \]
\[ L_3 = \min (20.8167, 5.9884) = 5.9884, \]
\[ U_2 - L_3 = 14.8283, U_2 + L_3 = 26.8051 \]

Applying fuzzy linear membership function, we get the following mode.
Maximize \( \lambda \),
\[ 0x_{11} + 0.25x_{12} + 0.7857x_{13} + 0.75x_{14} + 0x_{21} + 0.8235x_{22} + 0.625x_{23} + 0.625x_{34} + 0.8125x_{31} + 0.7778x_{32} + 0x_{33} + 0.3333x_{34} + \lambda (7.91881) \leq 17.7983, \]
\[ 0.25x_{11} + 0.5x_{12} + 0x_{13} + 0.5x_{14} + 0.2x_{21} + 0.75x_{22} + 0.6471x_{23} + 0.85x_{34} + 0.75x_{31} + 0.25x_{32} + 0.6667x_{33} + 0x_{34} + \lambda (16.50102) \leq 24.33338, \]

Subject to the constraints;
\[ \sum_{j=1}^{4} x_{ij} = [7, 9], \sum_{j=1}^{4} x_{2j} = [17, 21], \sum_{j=1}^{4} x_{3j} = [16, 18], \]
\[ \sum_{i=1}^{4} x_{i1} = [10, 12], \sum_{i=1}^{4} x_{i2} = [2, 4], \sum_{i=1}^{4} x_{i3} = [13, 15], \sum_{i=1}^{4} x_{i4} = [15, 17], \]
\[ x_{ij} \geq 0, i = 1, 2, 3 and j = 1, 2, 3, 4, \lambda \geq 0. \]

**Solution of this model is:**
The optimal allocations are
\[ x_{12} = 2, x_{13} = 5, x_{21} = 12, x_{23} = 1.508529, x_{24} = 3.491471, x_{33} = 6.491471, x_{34} = 11.50853. \]

Using these allocations we have \( Z_1 = 172.01706, Z_2 = 177.21162. \)
With degree of satisfaction \( \lambda = 0.6167205. \)
Applying fuzzy hyperbolic membership function, we get the following model

Maximize $\lambda$,

$$\sum_{i=1}^{4} \sum_{j=1}^{4} \left[ \frac{27.66879}{2} \left( a_{ij} \right) + \left( 0.25 x_{ij} + 0.7857 x_{ij} + 0.75 x_{ij} \right) \right]$$

subject to the constraints

$$\sum_{j=1}^{4} x_{ij} = [7, 9], \sum_{j=1}^{4} x_{ij} = [17, 21], \sum_{j=1}^{4} x_{ij} = [16, 18],$$

$$\sum_{j=1}^{4} x_{ij} = [10, 12], \sum_{j=1}^{4} x_{ij} = [2, 4], \sum_{j=1}^{4} x_{ij} = [13, 15], \sum_{j=1}^{4} x_{ij} = [15, 17],$$

$x_{ij} \geq 0, i = 1, 2, 3$ and $j = 1, 2, 3, 4, \dot{\lambda} \geq 0$.

Solution of this model is:

The optimal allocations are $x_{12} = 2, x_{13} = 5, x_{21} = 12, x_{23} = 1.508529, x_{32} = 3.491471, x_{33} = 6.491471, x_{42} = 11.50853$.

Using these allocations we have $Z_1 = 172.01706, Z_2 = 177.21162$.

With degree of satisfaction $\lambda = 0.8022863$.

<table>
<thead>
<tr>
<th>Value of $\varepsilon$</th>
<th>$Z^1$</th>
<th>$Z^1$</th>
<th>linear $\lambda$</th>
<th>Hyperbolic $\dot{\lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.5$</td>
<td>132.45763</td>
<td>121.18079</td>
<td>0.66114</td>
<td>0.87365</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>136.71389</td>
<td>136.59770</td>
<td>0.78199</td>
<td>0.96720</td>
</tr>
<tr>
<td>$-0.3$</td>
<td>142.52198</td>
<td>142.52681</td>
<td>0.71083</td>
<td>0.92622</td>
</tr>
<tr>
<td>$-0.2$</td>
<td>151.77475</td>
<td>151.77185</td>
<td>0.65688</td>
<td>0.86790</td>
</tr>
<tr>
<td>$-0.1$</td>
<td>163.55100</td>
<td>168.67542</td>
<td>0.62877</td>
<td>0.82423</td>
</tr>
<tr>
<td>$0$</td>
<td>172.01706</td>
<td>177.21162</td>
<td>0.61672</td>
<td>0.80229</td>
</tr>
<tr>
<td>$0.1$</td>
<td>180.52051</td>
<td>185.65714</td>
<td>0.61556</td>
<td>0.80007</td>
</tr>
<tr>
<td>$0.2$</td>
<td>189.03503</td>
<td>194.08145</td>
<td>0.61466</td>
<td>0.79833</td>
</tr>
<tr>
<td>$0.3$</td>
<td>197.60793</td>
<td>202.40076</td>
<td>0.61505</td>
<td>0.79908</td>
</tr>
<tr>
<td>$0.4$</td>
<td>206.08681</td>
<td>210.89812</td>
<td>0.61323</td>
<td>0.79555</td>
</tr>
<tr>
<td>$0.5$</td>
<td>214.62195</td>
<td>219.29674</td>
<td>0.61266</td>
<td>0.79443</td>
</tr>
</tbody>
</table>

Table 4.9: Solution of Numerical illustration-3 with different value of $\varepsilon$
Table.4.9 contains solution of multi-objective interval transportation problems with different values of $\epsilon$ where $-0.5 \leq \epsilon \leq 0.5$ using membership function. Change different value of the risk attitude parameter to cover the whole range of interval and accordingly finds a different solution for MOITP.

4.9 Comparison of Grey Situation Decision Making Theory Approach using Membership Function with other approach

This section discussed comparison of grey situation decision making theory based approach with other developed approaches. The comparison table shows that the grey situation decision making theory based approach provides very efficient alternative approach to find solution of multi-objective interval transportation problem.

<table>
<thead>
<tr>
<th>Sr.no.</th>
<th>Multi-objective Grey situation decision(MGSD) theory using membership function approach</th>
<th>Other approach</th>
</tr>
</thead>
</table>
| Numerical Illustration:1 | At $\epsilon = 0$
$Z_1 = 178.001144, Z_2 = 204.372441.$ | By S.K.Das et.al. [30]:
$Z_1 = [119.14, 214.42], Z_2 = [180.64, 241.1].$ |
| Numerical Illustration:2 | At $\epsilon = 0$
$Z_1 = 169.826, Z_2 = 156.7827.$ | By S.K.Das et.al. [30]:
$Z_1 = 149.6, Z_2 = 174.$ |
| Numerical Illustration:3 | At $\epsilon = 0$
$Z_1 = 172.01706, Z_2 = 177.21162.$ | By S.K.Das et.al. [30]:
$Z_1 = [113.204.9], Z_2 = [129.89, 227.86].$ |

Table 4.10: Comparison of numerical illustrations with other approach

Table.4.10 contains comparison of multi-objective interval transportation problems with different approaches. In this table MGSD theory based approach having value of $\epsilon = 0$. As per the Table-4.7 and Table-4.9, some change in value of $\epsilon$, there is some change in objective value. This value of $\epsilon$ is decided by decision makers and according to their requirements, solutions are achieved.
Case Study: [1]

Numerical Illustration-5: [111]

The case company is a leading producer of 3C (Computers/Communications/Consumer) products in Taiwan. It produces electronic products to satisfy demand for four distribution centres (DCs) in North Taiwan, Central Taiwan, South Taiwan and East Taiwan with production based at three factories in Taoyuan, Hsinchu and Taichung. The company’s logistics centre seeks the right transportation plan for allocating consumer products from factories to DCs. Each factory has a total available supply to distribute to various destinations, and each destination has a forecast demand that is received from various factories. The case company attempts to determine the optimal volumes to be transported from each factory to each destination in order to simultaneously minimize total production and transportation costs, number of defective items and delivery time. The information related to this case are describe in Table 4.11 and Table 4.12.

<table>
<thead>
<tr>
<th>Factory</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>Available supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$T_{ij}$</td>
<td>[0.5, 1.1]</td>
<td>[0.8, 1.2]</td>
<td>[1.3, 1.7]</td>
<td>[1.8, 2.2]</td>
</tr>
<tr>
<td></td>
<td>$D_{ij}$</td>
<td>[4, 6]</td>
<td>[5, 8]</td>
<td>[7, 9]</td>
<td>[8, 12]</td>
</tr>
<tr>
<td>B</td>
<td>$T_{ij}$</td>
<td>[1.2, 1.6]</td>
<td>[1.3, 1.5]</td>
<td>[1.5, 1.9]</td>
<td>[2.0, 2.4]</td>
</tr>
<tr>
<td></td>
<td>$D_{ij}$</td>
<td>[6, 8]</td>
<td>[7, 13]</td>
<td>[10, 14]</td>
<td>[15, 19]</td>
</tr>
<tr>
<td>C</td>
<td>$T_{ij}$</td>
<td>[1.6, 2.0]</td>
<td>[1.8, 2.2]</td>
<td>[2.2, 1.4]</td>
<td>[1.2, 1.6]</td>
</tr>
<tr>
<td></td>
<td>$D_{ij}$</td>
<td>[8, 12]</td>
<td>[10, 14]</td>
<td>[16, 20]</td>
<td>[8, 10]</td>
</tr>
<tr>
<td></td>
<td>Demand lower bound</td>
<td>8000</td>
<td>10,000</td>
<td>11,000</td>
<td>14,000</td>
</tr>
<tr>
<td></td>
<td>Demand upper bound</td>
<td>9000</td>
<td>11,000</td>
<td>12,000</td>
<td>16,000</td>
</tr>
</tbody>
</table>

Table: 4.11 Transportation information of the case company
Table 4.12 presents production data of all factories for shipping

The following notation are utilized to convert this transportation problem in Mathematical form: i Index for factory, for all \( i = 1, 2, \ldots, m \) and \( j \) Index for DC, for \( j = 1, 2, \ldots, n \).

**Decision variable**

\( x_{ij} \) Quantity transported from factory \( i \) to DC \( j \)

**Parameter**

- \( PC_{ij} \) : Production cost per unit delivered from factory \( i \) to DC \( j \),
- \( TC_{ij}^{\pm} \) : Transportation cost/per unit transported from factory \( i \) to DC \( j \),
- \( DR_{ij}^{\pm} \) : Defective rate delivered from factory \( i \) to DC \( j \),
- \( DT_{ij}^{\pm} \) : Delivery time from factory \( i \) to DC \( j \),
- \( SL_{ij}^{\pm} \) : Total supply available for each factory \( i \) to DC \( j \),
- \( DL_{ij}^{\pm} \) : Total demand for each DC \( j \),
- \( LH_{ij}^{\pm} \) : Labour hours per unit for each factory \( i \) input,
- \( MH_{ij}^{\pm} \) : Machine hours per unit produced by each factory \( i \),
- \( B_{ij}^{\pm} \) : Budget allocated to each factory \( i \),
- \( W_{i_{\text{max}}}^{\pm} \) : Maximum labour level of each factory \( i \),
- \( MC_{i_{\text{max}}}^{\pm} \) : Maximum machine capacity of each factory \( i \),
- \( QF_{ij}^{\pm} \) : Quota flexibility for each factory \( i \) to DC \( j \),
- \( F_{i_{\text{min}}}^{\pm} \) : Lower bound of quota flexibility for each factory \( i \),
The mathematical programming model for this problem is as follows;

**Objective functions**

\[
\begin{align*}
\text{Min } Z_1 \text{ (total transportation cost)} &= \sum_{j=1}^{m} \sum_{i=1}^{n} (PC_{ij} + TC_{ij}^z) x_{ij} \\
\text{Min } Z_2 \text{ (total number of defective items)} &= \sum_{j=1}^{m} \sum_{i=1}^{n} DR_{ij}^z x_{ij} \\
\text{Min } Z_3 \text{ (total delivery time)} &= \sum_{j=1}^{m} \sum_{i=1}^{n} DT_{ij}^z x_{ij}
\end{align*}
\]  
(4.4) (4.5) (4.6)

**Subject to the Constraints**

\[
\begin{align*}
\sum_{j=1}^{n} x_{ij} &\leq S_i^z, \quad \forall i, \\
\sum_{i=1}^{m} x_{ij} &= D_j^z, \quad \forall j, \\
\sum_{j=1}^{n} (PC_{ij} + TC_{ij}^z) x_{ij} &\leq B_i^z, \quad \forall i, \\
\sum_{j=1}^{n} LH_{ij}^z x_{ij} &\leq w_{i_{\text{max}}}^z, \quad \forall i, \\
\sum_{j=1}^{n} MH_{ij}^z x_{ij} &\leq MC_{i_{\text{max}}}^z, \quad \forall i, \\
\sum_{j=1}^{n} QF_{ij}^z x_{ij} &\geq F_{i_{\text{min}}}^z, \quad \forall i, \\
x_{ij} &\geq 0, \quad \forall i, j
\end{align*}
\]  
(4.7) (4.8) (4.9) (4.10) (4.11) (4.12) (4.13)

Where, the objective functions in Formulae (4.4)–(4.6) are to minimize transportation cost, defective rate and delivery time. Constraint (4.7) makes sure that maximum available supply is no more than all factories’ combined capacity. Constraint (4.8) guarantees that the available quantities of transported products from each factory to DC can meet total demand. Constraint (4.9) ensures that production–transportation costs of all factories do not exceed total budget. Constraints (4.10) and (4.11) ensure that labour hours and machine hours are not over maximal level. Constraint (4.12) makes sure that quota flexibility for each factory is more than the lower bound of flexibility. Constraint (4.13) ensures that all decision variables are non-negative. Table 4.11 shows basic transportation information of the case company. Table 4.12 presents production data of all factories for shipping.
Solution:
The solution of this problem by using Grey situation decision making theory and fuzzy programming based approach is as follows:

<table>
<thead>
<tr>
<th>ε</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>237730.8701</td>
<td>850.000082</td>
<td>327230.524</td>
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<tr>
<td>0</td>
<td>246053.9203</td>
<td>1310.000124</td>
<td>396615.2814</td>
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<tr>
<td>0.5</td>
<td>254376.9706</td>
<td>1770.0000166</td>
<td>466000.0388</td>
</tr>
</tbody>
</table>

Table 4.13 Solution by using linear membership function

The optimal allocations are:

$x_{12} = 1230.494, x_{13} = 10769.51, x_{14} = 5000,$
$x_{21} = 9000, x_{22} = 8769.506, x_{23} = 230.4942, x_{34} = 11000,$

With degree of satisfaction $\lambda = 0.9659285$

<table>
<thead>
<tr>
<th>ε</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>232430.8701</td>
<td>850.000082</td>
<td>323230.524</td>
</tr>
<tr>
<td>0</td>
<td>244553.9</td>
<td>1310</td>
<td>391115.3</td>
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<tr>
<td>0.5</td>
<td>252876.9706</td>
<td>1770.0000166</td>
<td>459000.0388</td>
</tr>
</tbody>
</table>

Table 4.14 Solution by using the hyperbolic membership function

The optimal allocations are:

$x_{12} = 1230.494, x_{13} = 10769.51, x_{14} = 5000,$
$x_{21} = 9000, x_{22} = 8769.506, x_{23} = 230.4942, x_{34} = 11000,$

$\lambda = 0.996283$

Conclusion:
Chapter has discussed Grey situation decision making theory and Fuzzy Programming Technique based solution to find the solution of Multi-Objective Interval Transportation Problem (MOITP) and provide an alternative approach to find the solution of Multi-Objective Transportation Problem (MOTP) as well as other multi objective problems.