Chapter-1

Introduction to Transportation Problem and Basics for Finding its Solution

1.0 Introduction

Operations Research (OR) may be represented as the application of scientific strategies to issues arising from operations involving integrated system of materials, machines, business, industries and peoples etc. In the previous few decades OR has been applied to many decision making problems. OR strategies are frequently utilized in mathematics, statistics, management, science, economics, engineering, etc., to find the most efficient action of a decision making problems under the certain constraints.

“Operation research as one of the quantitative aid to decision making offers to decision maker (DM) a method of evaluating every possible alternative by using various techniques to known potential outcomes” [63].

During the Second World War, the term OR was come into existence in the context of military in England. In the meantime, the authority of military department of England involved various interdisciplinary scientific teams to carry out their scientific research in the field of strategic in air and land military operation. In order to assist the military department, scientists provided specific plans for the optimal utilization of limited military resources and their implementation in effective manners. These plans were very useful in the World War-II as notable progress. As these plans were carried with the research work on military operation, it was called as Operation Research in U.K. After the effective utilization of the empowering results got by British OR panel, the various countries of the military departments were immediately roused to begin the similar activities in their countries.

After successfully implementation of OR subject in military operation, this term paying attention in the industrial administration to solve decision making problems. In this field, the massive numbers of method have been created to tackle the distinctive sorts of issues with joint endeavors and collaboration of interested individual in both academic associations and industries [63]. Moreover, the OR activities have been found in the
different division likes Transportation System, Agriculture, Economics, Libraries, Hospitals, Urban Planning, Financial Institutions, Health Systems, Government Services Organization etc. The application of OR covers the complete area of real life wherever there is a problem, there is OR because of its large applicability and its closed concern with our day to day practical life. In the environment of decision making, OR models are most suitable to formulate business decision problem, means it converts the verbal description and numerical data into mathematical formation which represents the relation amongst decision factors, objectives and constraints. The major concern of OR models is to provide best platform, to design and operate Person-Machine system in scientific way under the constraints of scare resource allocation. In many manufacturing industries even in financial and service organizations, OR has always been a decision making aid with its linear and non-linear optimization model which helps to make decision for a manager in the allocation of scare resources to different activities or projects.

1.1 Single-Objective Optimization Problem

In the Single-Objective Optimization Problem (SOP), an objective function is to be optimized with the consideration of given set of constraints, are satisfied by the solution. Mathematically, SOP can be written in the following form:

Model-1.1:

Objective function: \( \max (\text{or} \min) \ f(x), \)

Subject to the constraints:

\[ x \in \Omega, \quad (1.1) \]

Where, \( f \) is a given objective function from a general multidimensional space \( \mathbb{R}^n \) to the set of real \( \mathbb{R} \), and \( \Omega = \left\{ x \mid g_i(x) (\leq, =, \geq) b_i, i = 1, 2, \ldots, m \right\} \) is a subset of \( \mathbb{R}^n \) defined by various conditions called ‘Constraints’. One of the main goals of optimization is to find efficient algorithm to obtain an optimal solution that is, to find a vector \( x \in \Omega \) that optimizes (maximizes or minimizes) the objective function \( f \) among the set of all feasible solutions (or feasible regions). Depending on the nature of the function involved in problem of Model-1.1, can be divided into two categories, namely, Linear Programming Problem (LPP) and Non-Linear Programming Problem (NLPP).
(i) The mathematical programming problem Model-1.1 is called an LPP if the objective function \( f \) is linear and the set \( \Omega \) is described by using linear inequalities or equations.

(ii) The mathematical programming problem Model-1.1 is called an NLPP if the objective function \( f \) is nonlinear and/or the set \( \Omega \) is described by using at least one nonlinear inequality or equation.

Linear Programming (LP) is one of most useful technique of OR and most of real life decision making problems have been formulated successfully as a LP model. LP is mathematical modelling technique useful in various applications including Optimum Production, Allocation of Resources, Diet Planning, Transportation Problems including, minimum cost flows, maximum flows, shortest path problems and Transshipment Problems; Production Scheduling, Workforce Planning, which consists optimal assignment of jobs, scheduling of classes, and so on.

Initially LP was developed by Leonid Kantorovich [1] in 1939, for proper planning of expenditures and returns for reducing total cost of army and increasing losing for the enemy during the Second World War. After getting tremendous success in this war, many industries realized the usefulness of LP in their daily planning. LP refers to mathematical programming, is one of the most widely used method to allocate resources in optimal way. It is a successful quantitative analysis to resolve business issues related to physical distribution of products, commonly considered as transportation problems. Basically, the aim of the transportation problem is to minimize cost of shipping goods from origin to destination rather one to another location to satisfy its requirement. Various LP methods are available to solve transportation problem out of them one of the widely used methods is simplex. However, the special structure of transportation problem is solved efficiently and faster way than the simplex method.

The foundation of the mathematical theory of nonlinear programming can be departed back to the work done by Kuhn and Tucker [2] in 1951. Their work became out of a project on game theory after World War II by the Officer of Naval Research in the United States. NLPPs come in a wide range of structures and shapes. Disparate the simplex method for LPPs, no single algorithm can tackle all different types of NLPPs and algorithms have been developed for such NLPPs.
1.2 Multi-Objective Optimization Problem

Multi-Objective Optimization Problem (MOOP) having more than one objective functions which are to be optimized simultaneously. It is a field of multi criteria decision making which is applied in Science, Engineering, Economics and logistic. In which, the optimum decision has to be taken in the situation of trade-off between two or more conflicting objectives. Mathematically, MOP can be written as follows:

Model-1.2:
Objective functions: \( \max(\text{or min}) \phi(x) = (\phi_1(x), \phi_2(x), \ldots, \phi_p(x)) \),

Subject to constraints:
\[ x \in \Omega. \quad (1.2) \]

Where, \( \phi : \Omega \rightarrow \mathbb{R}^p \) is a given vector function consisting of \( p \) objective functions to be maximized or minimized.

- Objective Space: Space in which the objective vector belongs.
- Decision Space: Image of the feasible set.

Model-1.2 shows that MOP is different than the SOP as it has more than one objective functions to be optimized. In SOP, a single objective having a single optimum solution while in MOP, multiple objectives do not have only single optimum solution which can satisfy all the conflicting objectives at the same time. Here, a set of solutions called Pareto optimum solution is obtained where no improvement is possible in any objective function without deteriorating at least one of the other objective functions. When, setting up and solving MOP, the prime objective is to obtain this solution and quantify the trade-off to satisfy the multiple objectives.

**Pareto optimal solution or efficient solution:**
A point \( x^* \in \Omega \) is said to be a Pareto optimal solution for the multi-objective problem if and only if there is no \( x \in \Omega \) such that \( \phi(x) \leq \phi(x^*) \) and \( \phi(x) < \phi(x^*) \); for at least one function.

**Weakly Pareto optimal solution or weakly efficient solution:**
A point \( x^* \in \Omega \) is said to be a Pareto optimal solution for the multi-objective problem if and only if there is no \( x \in \Omega \) such that \( \phi(x) < \phi(x^*) \)
**Strict Pareto optimal solution or strict efficient solution:**
A point $x^* \in \Omega$ is said to be a Pareto optimal solution for the multi-objective problem if and only if there is no $x \in \Omega$ such that $\phi(x) \leq \phi(x^*)$.

**Pareto front:**
Image of all the efficient set, is called Pareto front or Pareto curve or surface. Its shape indicates the nature of the trade-off between the different objective functions.

**Preferred Solution:**
A particular efficient solution is finally selected by a Decision Maker (DM) after preference in decision-making, under the consideration of conflicting objectives.

A MOP can be solved by different approaches, handling as a single-objective optimization problem viz. Weighted Sum Approach [49,50,53] in which, multiple objectives are weighted and summed together to make a composite objective function whose optimistic results in the optimization of individual objective function that depends upon the weights chosen. $\varepsilon$-constraint method [51] chooses one of the objective functions form MOP and treat remaining as constraints by limiting each of them within certain pre-defined limits. This approach converts MOP into a single objective problem and its outcome results depend upon the constraint values chosen. Weighted Metric Method [50] minimizing an $L_p$-Metric which is created from each objective. Goal Programming Method [54, 55, 56, 57, 58, 59, 60] minimizing a weighted sum of deviations of objectives from user-specified targets. Value-function method [50, 51, 52] is also one of the methods to solve MOP.

**1.3 Transportation problem**
Transportation problem is one of the class of LP, related to daily activities of life, mainly deals with logistics helps to make distribution rather transportation of goods.

It essential to minimize the transportation cost while varieties of distribution channels are between manufacturer and consumers. It requires a well-organized assistant to determine number of units to be transported from each supply source to demand destinations with specific route [63]. In this process, one side we have a set of sources with specific capacity and other side a set of destinations with specific requirement. Transportation problem deals with a product manufactured at different sources transport to different
destinations for satisfying their demand with the minimum possible transportation cost. To fulfill the objective, the quantity of available supplies and the quantities demanded should be known. Also, the transportation cost of a unit from source to destination should be known. The model is useful for selecting optimum transportation routes based on capacity and demand of units in such a manner such that the cost of shipping or transportation is minimum. The transportation model also helps in locating new facility for manufacturing plant when two or more number of locations is under consideration so that, the total transportation cost and production costs are to be minimized.

In 1941, F.I. Hitchcock [4] developed the basic transportation problem but in the 1951 B. Dantzig [5] solved it by using LP concepts. The necessary condition for transportation problem is, total supply available at the all sources must exactly be matched to the total demand at various destinations such that neither excess supply nor excess demand of products would be there, known as “balanced transportation problem”. Generally, in the cost matrix (per unit) of transportation problem, supply is written in the rows, while demand is expressed in columns. If a transportation problem has m rows and n columns, then the problem is solvable if there are exactly \((m + n - 1)\) basic variables. While the supply and demand are not equal a transportation problem is known as unbalanced TP. If total supply is lower than total demand, then a dummy supply variable is introduced in the equation to make it balanced. Similarly, if total demand is lower than supply then a dummy demand variable is introduced [3]. The subsection given below discusses different kind of transportation problems and their mathematical formulations which are useful to understand subsequent chapters.

### 1.3.1 Single Objective Transportation Problem

Transportation model works on minimizing transportation cost of products to be produced at different factories and to be sent to various warehouses. There are basically three inputs for Transportation Problem (i) Availability of units at source-factories (ii) Requirement of units at Destinations-Warehouses (iii) Transportation cost per unit from source to destination. The required mathematical model of transportation problem is stated as follows [4]:
Mathematical Model of Transportation Problem:

Transportation problem is a special type of LPP, where Objective Function: To be minimized the cost of transportation, Subject to Constraints: Supply as per availability of Sources and Requirement of Destinations. Let, there are \( m \) sources and \( n \) destinations. Let \( a_i \) is the availability of commodity at the \( i^{th} \) source and \( b_j \) be the requirement of commodity at the \( j^{th} \) destination. \( C_{ij} \) be the transportation cost per unit of commodity from \( i^{th} \) source to \( j^{th} \) destination. \( x_{ij} \) be the quantity of the commodity transported from \( i^{th} \) source to the \( j^{th} \) destination. \((i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)\). Thus, the LPP of Transportation Problem is represented as;

Model-1.3:

\[
\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij},
\]

subject to the constraints

\[
\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, \ldots, m.
\]

\[
\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, \ldots, n.
\]

and \( X_{ij} \geq 0, \forall i, j. \) \hfill (1.3)

The objective of the LPP is to determine the values of \( X_{ij} \) such that total transportation cost would be minimum. Let us assume that the total quantity available at sources and total requirement of destinations is equal. i.e. \( \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \) which is the case of balanced transportation problem. In case of unbalanced transportation problem, means inequality of availability and requirement, add a dummy source or a dummy destination having supply “\( m \)” or demand “\( d \)” such that it is converted into balanced transportation problem.

When the total of row allocations is same as the total availabilities and total of column allocations is same as the total requirements then the solution is called as Feasible Solution. Whereas, the solution with \( m + n - 1 \) allocations is called as Basic Solution. While, solving the transportation problem, the number of all possible routes should be
Moreover, if it is equal to zero, the solution is an Optimum Solution and it is unique [4].

1.3.2 Multi-Objective Transportation Problem

In real situations many transportation problems have more than one objectives which are characterized by multiple objective functions are considered as Multi-Objective Transportation Problems (MOTP). It is a special type of LPP in which constraints are of equality type and all the objectives are conflicting with each other [6]. Same as typical transportation problem, here in MOTP, commodity of units is to be transported from \( m \) sources having capacities \( a_1, a_2, \ldots, a_m \) to \( n \) destinations having demands \( b_1, b_2, \ldots, b_n \) respectively. \( C_{ij} \) is the cost associated with transportation of a unit from \( i^{th} \) source to \( j^{th} \) destination. This cost may include transportation charge, delivery time or safety of delivery etc. A variable \( x_{ij} \) represents the quantity of products to be shipped from \( i^{th} \) source to \( j^{th} \) destination. An LPP form of MOTP with \( r \) objectives, having \( m \) sources and \( n \) destinations is written as follows [6]:

\[
\text{Model-1.4:}
\]

\[
\text{Minimize } z_r = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^r x_{ij},
\]

subject to the constraints

\[
\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m.
\]

\[
\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n.
\]

\[
\text{and } x_{ij} \geq 0, \forall i, j.
\]

(1.4)

The subscript on \( Z_r \) and superscript on \( C_{ij}^r \) are related to the \( r^{th} \) penalty criterion. Without loss of generality, it may be assumed that \( a_i \geq 0 \) and \( b_j \geq 0 \) the equilibrium condition \( \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \) is satisfied.
A linear MOTP can be divided into two categories (i) Generate set of all efficient solutions and (ii) Identification of best compromise solution from the obtained solutions set. Practically the knowledge of the solution set is always not required but the procedure is needed to determine a compromise solution. As a result, different approaches are developed in the context of MOTP to find the compromise solution. The various approaches to solve MOTP are: (i) Fuzzy Programming Technique (ii) Fuzzy Goal Programming Approach (iii) Interactive Procedure (iv) Spanning Tree based Genetic Algorithm (v) Geometric Programming Approach (vi) Genetic Algorithm (vii) Interactive Fuzzy Multi-Objective Linear Programming.

1.3.3 Multi-Objective Interval Transportation Problem

From the existing literature, it is observed that several researchers developed the different methodologies for solving transportation problem. Among all this work, to the best of our knowledge, deterministic real numbers have been used in effective matrices of the concerned transportation problem. However, in real life situations, the elements of the effective matrices should be an imprecise number instead of deterministic because of people’s limited knowledge related to the problematic areas, the lack of data, inaccurate estimation etc. This inaccurate information on decision parameter represented by interval numbers or fuzzy number [64]. Thus, these axes of evaluation are normally operationalized by the structure of Interval Transportation Problem (ITP). When, transportation problem with interval numbers involve with multiple criteria, then it becomes Multi-Objective Interval Transportation Problem (MOITP). In MOITP, K interval valued objective functions are to be minimized with interval source and interval destination parameters is stated as follows [18]:
Model-1.5:

\[
\text{Minimize } Z^k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ c_{ij}^k, c_{ij}^k \right] x_{ij}, \text{ where } k = 1, 2, \ldots, K
\]

Subject to constraints:

\[
\sum_{j=1}^{n} x_{ij} = \left[ a_L, a_R \right], i = 1, 2, \ldots, m.
\]

\[
\sum_{i=1}^{m} x_{ij} = \left[ b_L, b_R \right], j = 1, 2, \ldots, n.
\]

\[
x_{ij} \geq 0, i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n.
\]

With \( \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_{ij} \) and \( \sum_{i=1}^{m} a_L = \sum_{j=1}^{n} b_{ij} \).

(1.5)

Where \( \left[ c_{ij}^k, c_{ij}^k \right], k = 1, 2, \ldots, K \) is an interval represents the uncertain cost of given transportation problem such as delivery time, quantity of goods delivered, under used capacity etc. The source parameter lies between left limit \( a_L \) and right limit \( a_R \), and the destination parameter lies between left limit \( b_L \) and right limit \( b_R \). In MOITP three major cases are raised (1) Coefficients \( c_{ij}^k \) of objective functions are in the form of interval, whereas source and destination parameters are deterministic. (2) The source and destination parameters respectively \( a_i \) and \( b_j \) are in the form of intervals but the objective functions coefficients \( c_{ij}^k \) are deterministic. (3) Both are in the form of interval.

Case: I

The objective functions coefficients \( c_{ij}^k \) are in the interval form i.e. \( c_{ij}^k = \left[ c_{ij}^k, c_{ij}^k \right] \), and the constraints parameters \( a_i \) and \( b_j \) are deterministic then the multi-objective interval transportation problem is described as follows [18]:
Model-1.6:

\[ \text{Minimize } Z^k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ c_{i,j}^k, c_{i,j}^k \right] x_{i,j}, \text{ where } k = 1, 2, \ldots, K \]

Subject to the constraints

\[ \sum_{j=1}^{n} x_{i,j} = a_i, i = 1, 2, \ldots, m. \]

\[ \sum_{i=1}^{m} x_{i,j} = b_j, j = 1, 2, \ldots, n. \]

\[ x_{i,j} \geq 0, i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n. \]

With \( \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \).

(1.6)

Case-II

The objective functions coefficients \( c_{i,j}^k \) are deterministic and the constraints parameters \( a_i \) and \( b_j \) are in the form of interval then the multi objective transportation problem is stated as follows [18]:

Model-1.7:

\[ \text{Minimize } Z^k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{i,j}^k x_{i,j}, \]

Subject to the constraints

\[ \sum_{j=1}^{n} x_{i,j} = \left[ a_{i,L}, a_{i,R} \right], \sum_{i=1}^{m} x_{i,j} = \left[ b_{j,L}, b_{j,R} \right], \]

\[ x_{i,j} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \]

with \( \sum_{i=1}^{m} a_{i,L} = \sum_{j=1}^{n} b_{j,L} \) and \( \sum_{i=1}^{m} a_{i,R} = \sum_{j=1}^{n} b_{j,R} \).

(1.7)

Case-III

Both the objective functions coefficients \( c_{i,j}^k \) and the constraints parameters \( a_i \) and \( b_j \) are in the form of interval then the MOITP is formulated as follows [18]:
Model-1.8:

Minimize $Z^k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ c_{t_i}^k, c_{R_j}^k \right] x_{ij}, (k = 1, 2, ..., K)$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = \left[ a_{L_i}, a_{R_i} \right], \sum_{i=1}^{m} x_{ij} = \left[ b_{L_j}, b_{R_j} \right],$$

$$x_{ij} \geq 0, i = 1, 2, ..., m, j = 1, 2, ..., n,$$

With $\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j},$

$$\sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}. \quad (1.8)$$

1.3.4 Fixed Charge Transportation Problem

The Fixed Charge Transportation Problem (FCTP) was given by Hirsch and Danzig [37] in which the fixed charge of each source is incurred. The transportation by Road, Rail and Trucks having freight rates that includes both fixed cost and variable cost are modelled into FCTPs. The fixed cost can be referred as cost of vehicle rent, landing fees at airport, setup cost of machinery etc. Let, the number of sources is $m$ and the number of destinations is $n$. Also, the fixed cost and variable cost of transportation from source $i$ having capacity $a_i (i = 1, 2, ..., m)$ to destination $j$, having demands $b_j (j = 1, 2, ..., n)$ are denoted by $\eta_{ij}$ and $\xi_{ij}$ respectively. Let $x_{ij}$ be the quantity to be transported from source $i$ to destination $j$ and $y_{ij}$ be a 0-1 variable to describe the transportation activity from source $i$ to destination $j$ and is defined as $y_{ij} = 0$ if $x_{ij} = 0$, $y_{ij} = 1$ if $x_{ij} > 0$. Then the FCTP can be formulated as follows [37]:
Model-1.9:

Minimize \( \sum_{i=1}^{m} \sum_{j=1}^{n} (\xi_{ij} x_{ij} + \eta_{ij} y_{ij}) \),

subject to the constraints

\[ \sum_{j=1}^{n} x_{ij} \leq a_i, i = 1, 2, \ldots, m. \]

\[ \sum_{i=1}^{m} x_{ij} \geq b_j, j = 1, 2, \ldots, n. \]

\[ x_{ij} \geq 0, \forall i, j \quad (1.9) \]

Where \( y_{ij} = 0 \) if \( x_{ij} = 0 \), \( y_{ij} = 1 \) if \( x_{ij} > 0 \).

Here, the first constraint indicates that, total commodity transported from source \( i \) is no more than \( a_i \) and the second constraint indicates that the total commodity transported from \( i \) sources should satisfy the demand of destination \( b_j \) with non-negative constraints of quantities.

1.3.5 Multi-Objective Fixed Charge Transportation Problem (MOFCTP)

Suppose there are \( m \) origins and \( n \) destinations, the quantities of a uniform product available at the origins and required at the destinations are given. The total quantity available at the sources is precisely the same as the total quantity required at the destinations and it is possible to transport to any destination from any origin. In this case the \( k \) objectives FCTP problem can be formulated as follows [159]:

Model-1.10:

Minimize \( \sum_{i=1}^{m} \sum_{j=1}^{n} (\xi_{ij}^l x_{ij} + \eta_{ij}^l y_{ij}); l = 1, 2, \ldots, k \),

subject to the constraints:

\[ \sum_{j=1}^{n} x_{ij} \leq a_i, i = 1, 2, \ldots, m, \]

\[ \sum_{i=1}^{m} x_{ij} \geq b_j, j = 1, 2, \ldots, n, \]

\[ x_{ij} \geq 0, \forall i, j \quad (1.10) \]

where \( y_{ij} = 0 \) if \( x_{ij} = 0 \) and \( y_{ij} = 1 \) if \( x_{ij} > 0 \).

\( \xi_{ij}^l \) = the units of cost of transportation of one unit of the product from origin \( i \) to destination \( j \) corresponding to \( k^{th} \) objectives i.e. \( l = 1, 2, \ldots, k \), \( \eta_{ij}^l \) = fixed cost of
transportation of one unit of the product from origin i to destination j corresponding to k^{th} objectives i.e. \( l = 1, 2, \ldots, k \), \( a_i \) = the units of the product available at origin \( i \), \( b_j \) = the units of the product required at destination \( j \), \( x_{ij} \) = the number of units of the product transported from origin \( i \) to destination \( j \).

### 1.3.6 Solid Transportation Problem

The Solid Transportation Problem (STP) is a generalization of the well-known classical TP. The necessity of considering this special type of TP arises when there are different types of products to be transported using heterogeneous transportation modes called ‘Conveyances’. Thus, three item properties are taken into account in the set of constraints STP instead of two. The STP was stated by Shell [7] who discussed four different cases, based on the given data of the item properties such as three planar sums, two planar sums, one planar and one axial sum, and three axial sums. The STP in typical form is defined as follows.

Let, from each \( m \) sources a homogeneous product is transported to \( n \) destinations. The said source points having production facilities with available capacities \( a_i \), \( i = 1, 2, 3, \ldots, m \). The destination points consume products with required levels of demand \( b_j \), \( j = 1, 2, 3, \ldots, n \). Let \( e_k \), be the number of units transported by \( k^{th} \) type of the conveyance \( k = 1, 2, 3, \ldots, l \) from sources to destinations. The conveyances may be trucks, air freight, freight trains and ships. A penalty or cost \( c_{ijk} \geq 0 \) and a cost function \( f_{ijk} \) are associated with transportation of a unit of the product from source \( i \) to the destination \( j \) by means of the conveyance \( k \). The problem is to determine the unknown quantities \( x_{ijk} \) of the product to be transported from the each of the sources \( i \) to each of the destinations \( j \) by each of the conveyances \( k \) so that the total transportation cost would be minimized.

The STP can be defined as a minimization problem with linear constraints in the following form [7].
Model-1.11:

Minimize \( Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} f_{ijk}(x_{ijk}), \)

subject to the constraints:
\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} = a_i, i = 1, 2, \ldots, m.
\]
\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} = b_j, j = 1, 2, \ldots, n.
\]
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = e_k, k = 1, 2, \ldots, l.
\]
\[
x_{ijk} \geq 0, a_i \geq 0, b_j \geq 0, e_k \geq 0, \forall i, j, k.
\]

(1.11)

Where, \( f_{ijk}(x_{ijk}) \) is a linear function. Also \( \sum_{j=1}^{n} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k \) for balanced condition.

Here (1.11) generalizes the classical TP, where there is only one conveyance available.

1.3.7 Multi-Objective Solid Transportation Problem

In many situations of the real world, to express the problem in more realistic way, it is necessary to consider more than one criteria or objectives with the related set of constraints. As far as the transportation problems are concerned, the objectives may be total cost of transportation, average time required for delivery of commodity, transportation reliability, accessibility to the users, product deterioration etc.

The posing of Multi-Objective Solid Transportation Problem (MOSTP) in addition to the assumption made in (1.11) the STP is as follows:

Model-1.12:

Minimize \( Z^p = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} f_{ijk}^p(x_{ijk}), p = 1, 2, \ldots, P. \)

subject to the constraints:
\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} = a_i, i = 1, 2, \ldots, m.
\]
\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} = b_j, j = 1, 2, \ldots, n.
\]
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = e_k, k = 1, 2, \ldots, l.
\]
\[
x_{ijk} \geq 0, a_i \geq 0, b_j \geq 0, e_k \geq 0, \forall i, j, k.
\]

(1.12)
1.3.8 Fixed Charge Solid Transportation Problem (FCSTP)

In the FCSTP, two types of costs are taking into consideration, Direct Cost and Fixed Charge. Where, direct cost is the transportation cost per unit and fixed charge is on the type of conveyance used for transportation.

For the mathematical modelling of FCSTP, some notations and assumptions are employed as follows;

**List of notations and assumptions:**

\[ i \in \{1, 2, \ldots, m\} \quad \text{Source Index} \]
\[ j \in \{1, 2, \ldots, n\} \quad \text{Destination Index} \]
\[ k \in \{1, 2, \ldots, l\} \quad \text{Conveyances Index} \]

- \(a_i\) Number of products at source \(i\) that is to be transported to \(n\) destinations
- \(b_j\) Minimal demand of products at destination \(j\)
- \(c_k\) Transportation capacity of conveyance \(k\)
- \(d_{ijk}\) Direct cost of a unit to be transported from source \(i\) to destination \(j\) by conveyance \(k\)
- \(e_{ijk}\) Fixed charge of transportation activity from source \(i\) to destination \(j\) by conveyance \(k\)
- \(x_{ijk}\) Quantity transported from source \(i\) to destination \(j\) by conveyance \(k\)

\[ y_{ijk} = \begin{cases} 1, & x_{ijk} > 0 \\ 0, & \text{otherwise} \end{cases} \]

Where, \(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, l\); respectively.

The corresponding fixed charge will occur if the transportation activity is assigned from source \(i\) to destination \(j\) by conveyance \(k\). The objective function for total cost will be as follows;

\[
f(x, y) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} (d_{ijk} x_{ijk} + e_{ijk} y_{ijk}).
\]

(1.13)

Here, \(x\) and \(y\) are the vectors consisting of \(x_{ijk}\) and \(y_{ijk}\), where \(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, l\); Therefore, the fixed charge STP model will be as follows [156]:
Model-1.13:

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} (d_{ijk} x_{ijk} + e_{ijk} y_{ijk}). \\
\text{subject to the constraints} & \quad \sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq a_{i}, (i = 1, 2, \ldots, m). \\
& \quad \sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq b_{j}, (j = 1, 2, \ldots, n). \\
& \quad \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq c_{k}, (k = 1, 2, \ldots, l). \\
& \quad x_{ijk} \geq 0, y_{ijk} = \begin{cases} 1 & \text{if } x_{ijk} > 0, \\ 0, & \text{otherwise}. \end{cases}
\end{align*}
\]

where \(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, k = 1, 2, \ldots, l\)

(1.14)

The first constraint indicates that the total amount of products transported from source \(i\) is no more than \(a_{i}\). The second constraint in above model requires that the total amount of products transported from \(m\) sources has to satisfy the demand of destination \(b_{j}\). The third constraint is of capacity constraint. This model assumes all the values of \(a_{i}, b_{j}, c_{k}, d_{ijk}, e_{ijk}\) is constants. But, in real phenomena, the plan for transportation is made in advance, where it is impossible for us to fix the value of above all constants precisely. Suppose, there is enough historical data for these parameters are available, we can regard them as random variables. However, in the situation of lacking of historical data, or if they are invalid because of unexpected events have occurred, some domain experts to evaluate the belief degree that each event will occur. This expert data is just the subject of the uncertainty theory.
1.3.9 Fuzzy Fixed Charge Solid Transportation Problem

To model Fuzzy Fixed Charge Solid Transportation Problem (FFCSTP) we have to consider the following notations:

**List of notations and assumptions.**

\[ i \in \{1, 2, \ldots, m\} \quad \text{Source Index} \]
\[ j \in \{1, 2, \ldots, n\} \quad \text{Destination Index} \]
\[ k \in \{1, 2, \ldots, l\} \quad \text{Conveyances Index} \]

- \( a_i \): Number of products at source \( i \) that is to be transported to \( n \) destinations
- \( b_j \): Minimal demand of products at destination \( j \)
- \( c_k \): Transportation capacity of conveyance \( k \)
- \( \tilde{d}_{ijk} \): Direct cost of a unit to be transported from source \( i \) to destination \( j \) by conveyance \( k \)
- \( \tilde{e}_{ijk} \): Fuzzy Fixed charge of transportation activity from source \( i \) to destination \( j \) by conveyance \( k \)
- \( x_{ijk} \): Quantity transported from source \( i \) to destination \( j \) by conveyance \( k \)

\[
\mathcal{Y}_{ijk} = \begin{cases} 1 & x_{ijk} > 0 \\ 0, \text{otherwise} \end{cases}
\]

Where \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, l \); respectively.

The corresponding fixed charge will occur if the transportation activity is assigned from source \( i \) to destination \( j \) by conveyance \( k \). The objective function for total cost will be as follows;

\[
f(x, y) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} (d_{ijk} x_{ijk} + e_{ijk} Y_{ijk}).
\]

(1.15)

Here, \( x \) and \( y \) are the vectors consisting of \( x_{ijk} \) and \( y_{ijk} \), where \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, l \). Therefore, the fuzzy multi-objective fixed charge STP model will be as follows [156]:
Model-1.14:

\[
\text{minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} (\tilde{d}_{ijk} x_{ijk} + \tilde{e}_{ijk} y_{ijk}).
\]

subject to constraints

\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq a_i, (i = 1, 2, \ldots, m).
\]

\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq b_j, (j = 1, 2, \ldots, n).
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq c_k, (k = 1, 2, \ldots, l).
\]

\[
x_{ijk} \geq 0, y_{ijk} = \begin{cases} 1, & \text{if } i,j,k > 0, \\ 0, & \text{otherwise}. \end{cases}
\]

where \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, k = 1, 2, \ldots, l. \) \hspace{1cm} (1.16)

FFCSTP model is developed under the condition that, all the parameters of the model having fixed quantities. But, in reality, there is always complexity occurred in the modeling such that the fuzzy phenomena is taken into consideration for mathematical modelling and for that we add fuzzy variables to the model.

1.4 Some Preliminaries

To find the solution of transportation problems, some preliminaries are used which are defined as follows:

1.4.1 Interval Arithmetic

Many application-oriented mathematical models in OR as linear programming assume that the data used to build the model is exactly known, the constraints define a crisp set of feasible decisions, the criteria are well defined and easy to formalize. However, in the real world, most of these assumptions are only approximately true and most of the data is known only within specific limits of the true value. In LP models, data is uncertain associated with the coefficients of the model during the formulation stages. In addition, in many real life models, it is more realistic to represent some of the coefficients as interval numbers instead of fixed real numbers.

Interval arithmetic over the real numbers deals with closed and connected sets of the real numbers R. Generally, interval number denoted by lower limit and upper limit as
\[ \tilde{x} = [x^L, x^U] = \{x / x^L \leq x \leq x^U, x \in R\}. \]

Here \( x^L \) and \( x^U \) denoted as lower limit and upper limit respectively. \( R \) is the set of real numbers.

**Arithmetic Operations:**

Arithmetic operations on interval numbers are well known. Given any two positive real numbers \( \tilde{x} = [x^L, x^U] \) and \( \tilde{y} = [y^L, y^U] \); \( x^L, y^L > 0 \), and any real number \( \alpha \). The main operations of these two interval number can be represented as follows [61].

- **Addition:** \( \tilde{x} + \tilde{y} = [x^L + y^L, x^U + y^U] \)
- **Multiplication:** \( \tilde{x} \times \tilde{y} = [\alpha, \beta] \)
  \[
  \alpha = \max \{x^L y^L, x^L y^U, x^U y^L, x^U y^U\} \\
  \beta = \min \{x^L y^L, x^L y^U, x^U y^L, x^U y^U\} 
  \]
- **Division:** \( \tilde{x} / \tilde{y} = \left[\frac{x^L}{y^L}, \frac{x^U}{y^U}\right] = \left[\frac{x^L}{y^L}, \frac{1}{y^U}\right] ; 0 \notin [y^L, y^U] \)
- **Scalar multiplication:**
  \[
  \alpha \tilde{x} = [\alpha x^L, \alpha x^U] ; \alpha \geq 0 \\
  = [\alpha x^L, \alpha x^U] ; \alpha \leq 0
  \]

**Risk Attitude Parameter:**

For positive interval number \( \tilde{x} = [x^L, x^U] \), the interval mapping function is defined as follows

\[
\phi_\varepsilon(x) = m(\tilde{x}) + (\varepsilon) d(\tilde{x}); \quad (1.17)
\]

Where \( \varepsilon \) is Decision maker’s (DM) risk attitude parameter for uncertain data of real world problem and it is \( |\varepsilon| \leq 0.5, m(\tilde{x}) = \frac{x^U + x^L}{2} \) is the center of \( \tilde{x}, \ d(\tilde{x}) = x^U - x^L \) is the difference of an interval \( \tilde{x} \). For analysis process, DM gives risk attitude parameter. The risk attitude parameter can be divided into optimistic, most likely and pessimistic and the corresponding values for \( \varepsilon \) are \( -0.5 \leq \varepsilon < 0, \varepsilon = 0, 0 < \varepsilon \leq 0.5 \) respectively. Once risk
attitude parameter can be determined then interval value converts in crisp values by equation (1.17).

1.4.2 Fuzzy Optimization

In past few years, optimization models have been mostly developed in a deterministic and crisp environment. In such models, the objective(s) and the constraints are formulated in a difficult, crisp manner leaving no scope for uncertainty, and the solutions are either feasible or infeasible, either above a certain aspiration level or below. Such models frequently lead to approximating real-world problems, the solutions of which may not always be acceptable by the user. This is particularly true if the problem under consideration includes human evaluation, imprecision, vaguely defined relationships and uncertainties inherent in the parameters. There are two types of uncertainties pointed out by researchers as Ambiguity and Vagueness [62]. “Ambiguity is concerned with those situations in which the choice among two or more alternatives is left indefinite”, while “vagueness is concerned with those domains of interest that cannot be defined by sharp boundaries”. The vagueness in DM’s understanding of the objective(s) and constraints of an optimization problem, as well as the ambiguity naturally in the parameters involved, can be well modelled by the fuzzy set theory.

As a way to handle vagueness or the uncertainty occurred in the complex structure, the Fuzzy Set Theory was build up by Zadeh [8] in the year of 1965. After then, it has been using in many fields like OR, Management Science, Artificial Intelligence, Control Theory etc. Basically, it is a simplified classical set theory. In classical set theory, each element X is either belongs to set A or B, where in fuzzy approach it belongs to a set A with certain degree of membership.

Fuzzy Set:
A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the interval [0,1].

A fuzzy set A in a universe of X is defined as the following set of pairs:
\[ A(x) = \{ (x, \mu_A(x)) \mid x \in X \} \]
Here $\mu_A(x) : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set $A$ and $\mu_A(x)$ is called a membership value of $x \in X$ in the fuzzy set $A$.

**Support of a Fuzzy Set:**
Let $A$ be a fuzzy set in $X$ then a support of $A$, denoted by $S(A)$, is the crisp set given by

$$S(A) = \{ x \in X : \mu_A(x) > 0 \}.$$ 

**Normal Fuzzy Set:**
Let $A$ be a fuzzy set in the universe of discourse $X$ is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A(x) = 1$.

**Fuzzy Membership Function:**
Any set can be represented by mathematical function. In a classical set theory, the characteristic function, denoted by $\mu_A(x)$ is used to represent a crisp set while with reference to the fuzzy set theory the characteristic function is known as membership function denoted by $\mu_A(x)$ and is non-negative real number whose value lies in the interval $[0,1]$. A fuzzy set may be represented by using this membership function. This function gives the grade (degree of membership) within the set of any element of the universe of discourse. The membership functions are used to process numeric input data. A fuzzy set $A(x)$ is represented by a pair of two-things, element $x$ and its membership function $\mu_A(x)$ and denoted as $A(x) = \{ (x, \mu_A(x)) , x \in X ; \mu_A(x) \in [0,1] \}$. A membership function is a curve that defines how each point in the input space is mapped to a membership value between 0 and 1. Let $Z^U_k$ and $Z^L_k$ are upper and lower bound of objective $Z_k$ respectively then linear membership function and exponential membership function defined as follows:
Linear membership function:
Fuzzy linear membership function is defined as follows:

\[
\mu_k(Z^k_y) = \begin{cases} 
1; & \text{if } Z_k \leq Z^L_k \\
\frac{Z^U_k - Z^L_k}{Z^U_k - Z^L_k}; & \text{if } Z^L_k < Z_k < Z^U_k \\
0; & \text{if } Z_k \geq Z^U_k
\end{cases}
\]

(1.18)

Exponential membership function:

\[
\mu^E_k(x) = \begin{cases} 
1; & \text{if } Z_k \leq Z^L_k \\
\frac{e^{-S\psi_k(x)} - e^{-S}}{1 - e^{-S}}, & \text{if } Z^L_k < Z_k < Z^U_k \\
0; & \text{if } Z_k \geq Z^U_k
\end{cases}
\]

(1.19)

Where, \(\psi_k(x) = \frac{Z_k - Z^U_k}{Z^L_k - Z^U_k}\) and S is non-zero shape parameter given by DM that

\[0 \leq \mu^E_k(x) \leq 1.\] For \(S > 0 (S < 0),\) the membership function is strictly concave (convex) in \([Z^L_k, Z^U_k].\)

Hyperbolic membership function:
A hyperbolic membership functions is defined by

\[
\mu^H_k(Z^k_y) = \begin{cases} 
1; & \text{if } Z_k \leq L_k \\
\frac{1}{2} \left[ e^{-\left(\frac{(Z_k - L_k)}{2}\right)} - e^{-\left(\frac{(U_k - L_k)}{2}\right)} \right]_{x_i} - \frac{1}{2} \left[ e^{-\left(\frac{(Z_k - U_k)}{2}\right)} + e^{-\left(\frac{(L_k + L_k)}{2}\right)} \right]_{x_i} + \frac{1}{2} \text{ if } L_k < Z_k < U_k \\
0; & \text{if } Z_k \geq U_k
\end{cases}
\]

(1.20)

Where, \(a_k = \frac{6}{(U_k - L_k)}.\)

\(\alpha - \text{Cut}:\)
Let \(A\) be a fuzzy set in \(X\) and \(\alpha \in (0,1].\) The \(\alpha\) -cut of fuzzy set \(A\) is the crisp set \(A_\alpha\) given by \(A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}.\) From the definition of \(\alpha\) -cut, for any fuzzy set \(X\) and pair \(\alpha_1, \alpha_2 \in (0,1], \alpha_1 \leq \alpha_2,\) we have \(A_{\alpha_1} \subseteq A_{\alpha_2}.\) Therefore, all \(\alpha\) -cuts of any fuzzy set from families of crisp sets which can be used to represent a given fuzzy set \(A\) in \(X\).
**Fuzzy Number:**

A fuzzy set $A$ in $\mathbb{R}$ is called a fuzzy number if it satisfies the following conditions

1. $A$ is normal

2. $A_\alpha$ is a closed interval for every $\alpha \in (0,1]$,

3. The support of $A$ is bounded.

Let $A$ be a fuzzy number. Then $A^L_\alpha$ and $A^U_\alpha$ are defined as $A^L_\alpha = \inf_{\mu(x) \geq \alpha} (x)$ and $A^U_\alpha = \sup_{\mu(x) \geq \alpha} (x)$

**Triangular Fuzzy Number:**

Triangular fuzzy number denoted by $A = (a, b, c)$, of crisp number with $a < b < c$ and represented as

![Triangular fuzzy number](Figure-1.1 Triangular fuzzy number)

And its membership function is defined as 

$$
\mu(x) = \begin{cases} 
\frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\
\frac{c-x}{c-b}, & \text{if } b \leq x \leq c \\
0, & \text{otherwise}
\end{cases}
$$

The $\alpha$–cut of the triangular fuzzy number $A = (a, b, c)$ is defined as

$$
A_\alpha = [A^L_\alpha, A^U_\alpha] = [(b - a)\alpha + a, (b - c)\alpha + c], \alpha \in (0,1].
$$
Trapezoidal Fuzzy Number:

Trapezoidal fuzzy number denoted as $A = (a, b, c, d)$ of crisp number with $a < b < c < d$ and represented as

![Trapezoidal fuzzy number](image)

Figure- 1.2: Trapezoidal fuzzy number

The membership function is defined as

$$
\mu(x) = \begin{cases} 
\frac{x-a}{b-a} & \text{if } a \leq x \leq b \\
1 & \text{if } b \leq x \leq c \\
\frac{d-x}{d-c} & \text{if } c \leq x \leq d \\
0 & \text{otherwise}
\end{cases}
$$

The $\alpha$ – cut of the trapezoidal fuzzy number $A = (a, b, c, d)$ is defined as

$$
A_{\alpha} = [A^L_{\alpha}, A^K_{\alpha}] = [(b-a)\alpha + a, (c-d)\alpha + d], \alpha \in (0,1].
$$

Fuzzy set theory appears to be an ideal approach to deal with decision problems that are formulated as optimization problems under uncertainty. Fuzzy optimization problems are selected for such purpose.

1.5 Literature Review

The usual transportation problem obtains the optimum distribution of products to be transported from supplier to consumer in such a way so that the total transportation cost should be minimum. The several methods are available to satisfy this objective and to get the initial and feasible solution of transportation problem like as, North-West Corner, Row Minima, Column Minima, Matrix Minima, Vogel Approximation etc. The optimum solution of transportation problem is obtained by Modified Distribution (MODI) Method. Also the alternative way to find optimum solution of transportation problem is Stepping Stone Method (SSM) developed by Charnes and Cooper [9]. To identify the all non-
dominated solutions for a linear MOTP an algorithm was introduced by Diaz [15] and Isermann [17]. Current et al [27] reviewed the design of network of MOTP and to generate all non-dominated solutions of it and an alternative procedure was presented by Diaz [16]. Ringuest et al.[18] has given two interactive algorithm for the same. Bit et al [19] obtained optimal compromise solution of MOTP by applying fuzzy programming technique with linear membership function. Also, with some non-linear membership functions a MOTP was solved by Verma et al. [20] using fuzzy programming techniques by considering some other factor which are equally affecting the decision-making process like, type of goods to be transported, the distance of journey, the time and cost sensitivity, reliability of transportation system etc. [21].

The Solid Transportation Problem (STP) is a generalization of classical TP was introduced by Shell [7] in 1955. In which, instead of two items, Source and destinations, he considered three properties in the set of constraints. Further, he has explained, in which situation STP is raised. An algorithm to get optimum solution of solid fixed charge linear transportation problem was given by Basu et al [22]. In which he fixed all the coefficients of objective function, availability at source, requirement of destinations, conveyance capacities in crisp manner. The transportation cost per unit is never be constant forever such that the above parameter cannot be fixed and the satisfied always. To deal with this uncertainty, the interval programming techniques have been developed by many researchers [10, 11, 12, 23].

The STP performs a vital role in optimization especially in the environment of uncertainty. Many algorithms have been proposed by researchers [28, 29]. A mathematical model for the fixed cost bi-criteria indefinite quadratic transportation problem was given by A. Nagarajan and K. Jeyaraman. Also for multi-objective interval solid transportation problem they developed Expected Value Goal Programming and Chance Constrained Goal Programming Model [24, 25, 26]. Using S.K. Das et al. [30], developed a methodology for MOTP using interval cost, source and destination parameters. A fuzzy expected value model was given by Baoding Liu and Yian-Kui Liu [31]. Fuzzy set theory was firstly introduced by Zadeh [8], then Bit et al [32] and Jimenez and Verdegay [33] gave the fuzzy based model for MOSTP in which the supplies, demands and capacity of conveyance was in interval and fuzzy number. Jimenez and Verdegay [34] gave an evolutionary algorithm for fuzzy STP based on the
parametric approach. To solve MOSTP the Neural Network and Genetic Algorithm approach with fuzzy numbers was given by Li et al. [35] [36]. Gen et al [45] also used the genetic algorithm to solve bi-criteria FSTP.

The fixed charge transportation problem (FCTP) [37] is an extended version of TP has been studying by many researchers, Steinberg [38], Sun et al [39] etc. in which the cost solid transportation problem (CSTP) was modeled, based on uncertainty theory. In 2007 Liu [40] found the theory of uncertainty and it was refined by him in 2011 [41]. It was noted that a TP has uncertain cost parameter known as Uncertain Cost Transportation Problem (UCTP) [42] [43]. UCTP is most significance because in real practices, there are always been changes in supply and demand of market, uncertainty in the weather conditions, sometime bad road conditions and other external uncertainty factors are involved. Theoretical aspects, uncertainty is equally important in order to construct model for STP in uncertain environment and for that some knowledge of uncertainty theory is required in prior. For that Liu [44] has given a process, a sequence of uncertain variables indexed by uncertain programming model for STP. So there is huge literature available for different kind of transportation problems and their solutions, out of them some have been mentioned here and remaining will be discussed in detail at particular chapters.

1.6 Motivation of the Work

Multi-objective transportation problems are a difficult task faced by industrial organization, manufacturing system, developing service system etc., because of uncertainty of resources. Solving such a real world problems manually, often requires a large amount of time and expensive resources. In order to handle the complexity of the problems and to provide decision related to the problems, many researches have invested their efforts for over the years by using various traditional and non- traditional methods and provided the solution. However, to find the solution of such problems, the multi-objective optimization problem converted into single objective optimization problem (NP-hard problem) with some realistic constraints. In recent years Antunes [68] put forward a multi objective model for optimization problems, based on method of linear programming about economy, energy, environment factors in the production of manufacturing, including power approach, financial expansion, social welfare,
environment friendly elements and other indicators. Onut [83] applied fuzzy network analysis mode of transportation for some quantitative and qualitative criteria in order to obtain a comprehensive evaluation of the most suitable mode, and used a case of Turkey logistics Service Company for the purpose of model verification. Liu Xinvang [84] proposed a fuzzy reasoning, based on the objective function, an integrated approach through the fuzzy rules directly to express the decision maker’s preference knowledge structure. Zhang Liang, Wang Duan Ming [85] were taking into account the time and safety factors during the war in MOTPs and the establishment of a mathematical model of transportation, using a fuzzy programming algorithm distributing in the partial big scale parabola, through an improved tabular method obtained the best compromise solution. But it lacked effective examples to demonstrate the model. So, time to time new theory was developed and by using this new theory different solution approaches have been developed. Hence, up till now several approaches have been developed to solve MOOPs but each and every method have their own limitations like, complex multi objective optimization problem is unable to solve with this approaches or some time uncertainty of the problem cannot be handle properly. Such that, in this study, we have developed a Grey situation decision making theory and fuzzy programming technique based approach to solve different kind of single and multi-objective transportation problems. Which is highly benefited when, we are dealing with uncertain multi objective optimization problem. It easily solves any kind of complex structure based transportation problems.

1.7 Objective of the Study

The primary objectives of this study are as follows:

- To solve multi-objective transportation problems using grey situation decision making theory and fuzzy programming technique based approach.
- To develop an approach and algorithm to provide efficient solution for multi-objective transportation problems.
- To find alternative solution approach of multi-objective transportation problems.
1.8 Problem Statement

On the basis of motivation and objectives discussed above, we have decoded the thesis title as follows

“A Study on Multi Objective Transportation Problems and their Solutions with Grey Situation Decision Making Theory”

1.9 Structure of the Thesis

- **Chapter 1**: Discusses basics related to transportation problems and concepts which are useful to solve such problems with detail literature.

- **Chapter 2**: Discusses the basic concepts to understand the Multi-objective Grey Situation Decision Making Theory (MGSD) model. The useful terminologies, process of MGSD MODEL and advantages are also presented in this chapter.

- **Chapter 3**: Discusses the mathematical model of multi-objective transportation problem and how to find the solution using Grey situation decision making theory and fuzzy programming technique approach.

- **Chapter 4**: Discusses the solution of multi-objective interval transportation problem by grey situation decision making theory and fuzzy programming technique approach.

- **Chapter 5**: Discusses fixed charge transportation problem and its solution by using grey situation decision making theory.

- **Chapter 6**: Discusses solid transportation problem and its solution by using grey situation decision making theory and fuzzy programming technique approach.

- **Chapter 7**: Discusses uncertain solid transportation problem with fixed charge and its solution is presented in details using grey situation decision making theory and fuzzy programming technique approach.
The proposed alternative mathematical approach for the solution of Multi-Objective Transportation Problem using fuzzy programming techniques along with Lingo as a mathematical tool is described in forth coming chapters.