Chapter - 4

Ratio cum Product Estimators with Known Quartiles and their Functions

4.1 Introduction

This chapter deals with further improvements on ratio cum product estimator for the population mean with known information of the auxiliary variable like quartiles and their functions. The expression for bias and MSE of proposed estimators are derived. An algebraic expression is developed for the efficiency of proposed estimators over other existing estimators. A numerical study is carried out to assess the efficiency of proposed estimators over the existing estimators with the help of some known natural populations. To assess the superiority of the proposed estimator over other estimators through the PRE

4.2 Existing Estimators

Al- Omari et al. (2009) have suggested the following MR estimators using the known information of the auxiliary variable such as first quartile ($Q_1$) and third quartile ($Q_3$)

$$\hat{Y}_{MR3} = \bar{y} \left[ \frac{x + Q_1}{x + Q_3} \right]$$

$$\hat{Y}_{MR4} = \bar{y} \left[ \frac{x + Q_3}{x + Q_3} \right]$$

Subramani & Kumarapandiyan (2012 a) have introduced the MR estimators with the known inter quartile range ,semi inter quartile range and semi inter quartile average of the auxiliary variable, these are:

$$\hat{Y}_{MR5} = \bar{y} \left[ \frac{x + Q_1}{\bar{x} + Q_1} \right]$$

$$\hat{Y}_{MR6} = \bar{y} \left[ \frac{x + Q_3}{\bar{x} + Q_3} \right]$$

$$\hat{Y}_{MR7} = \bar{y} \left[ \frac{x + Q_3}{\bar{x} + Q_3} \right]$$
MP estimators for the first and third quartiles of the auxiliary variables are given by

\[ \hat{Y}_{MP3} = \bar{y} \left[ \frac{x + Q_1}{x + Q_1} \right] \]  
\[ \hat{Y}_{MP4} = \bar{y} \left[ \frac{x + Q_3}{x + Q_3} \right] \]  

Some MP estimators with known functions of quartiles of the auxiliary variables are given by

\[ \hat{Y}_{MP5} = \bar{y} \left[ \frac{x + Q_2}{x + Q_2} \right] \]  
\[ \hat{Y}_{MP6} = \bar{y} \left[ \frac{x + Q_4}{x + Q_4} \right] \]  
\[ \hat{Y}_{MP7} = \bar{y} \left[ \frac{x + Q_5}{x + Q_5} \right] \]  

The biases and MSE of \( \hat{Y}_{MRI} \) \( i = 3, 4, 5, 6, 7 \) are given by

\[ B(\hat{Y}_{MRI}) = \delta \bar{y} \left[ \theta_i^2 c_x^2 - \theta_i p c_x c_y \right] \]  
\[ MSE(\hat{Y}_{MRI}) = \delta \bar{y}^2 \left[ c_y^2 + \theta_i^2 c_x^2 - 2\theta_i p c_x c_y \right] \]

The biases and MSE of \( \hat{Y}_{MPI} \) \( i = 3, 4, 5, 6, 7 \) are given by

\[ B(\hat{Y}_{MPI}) = \delta \bar{y} \left[ \theta_i p c_x c_y \right] \]  
\[ MSE(\hat{Y}_{MPI}) = \delta \bar{y}^2 \left[ c_y^2 + \theta_i^2 c_x^2 + 2\theta_i p c_x c_y \right] \]

Where \( \theta_i = \frac{x}{X + \omega_i}, i = 3, 4, 5, 6, 7 \) \( \omega_3 = Q_1, \omega_4 = Q_2, \omega_5 = Q_3, \omega_6 = Q_4, \omega_7 = Q_5 \)

### 4.3 Proposed Estimators

Subramani and Master Ajith (2017a) introduced a class of improved ratio cum product estimators for finite population mean with known information of the quartiles and their functions of the auxiliary variable are given by

\[ \hat{Y}_{P3} = a_3 \tau_3 \bar{y} \left[ \frac{x + Q_1}{x + Q_1} \right] + (1 - a_3) \nu_3 \bar{y} \left[ \frac{x + Q_1}{x + Q_1} \right] \]  
\[ \hat{Y}_{P4} = a_4 \tau_4 \bar{y} \left[ \frac{x + Q_2}{x + Q_2} \right] + (1 - a_4) \nu_4 \bar{y} \left[ \frac{x + Q_3}{x + Q_3} \right] \]  
\[ \hat{Y}_{P5} = a_5 \tau_5 \bar{y} \left[ \frac{x + Q_3}{x + Q_3} \right] + (1 - a_5) \nu_5 \bar{y} \left[ \frac{x + Q_5}{x + Q_5} \right] \]  
\[ \hat{Y}_{P6} = a_6 \tau_6 \bar{y} \left[ \frac{x + Q_4}{x + Q_4} \right] + (1 - a_6) \nu_6 \bar{y} \left[ \frac{x + Q_5}{x + Q_5} \right] \]
\[ \hat{Y}_{p7} = a_7 \tau_7 \bar{Y} \left( \frac{\bar{x} + Q_a}{\bar{x} + Q_a} \right) + (1 - a_7) \nu_7 \bar{Y} \left( \frac{\bar{x} + Q_a}{\bar{x} + Q_a} \right) \] (4.19)

Where \( \tau_i = \frac{s_y}{s_y + A_i c_y} \), \( \nu_i = \frac{s_y}{s_y + B_i c_y} \), \( i = 3, 4, 5, 6, 7 \) \( A_i = B \left( \hat{Y}_{MRI} \right) \), and \( B_i = B \left( \hat{Y}_{MPI} \right) \). It is reasonable to assume that these values are known from the previous studies.

### 4.3.1 Bias and MSE of Proposed Estimators

To derive the bias and MSE of the proposed estimators,

Consider, \( \hat{Y}_{pi} = \bar{Y} a_i \tau_i \left( \frac{\bar{x} + \mu_i}{\bar{x} + \mu_i} \right) + (1 - a_i) \nu_i \left( \frac{\bar{x} + \mu_i}{\bar{x} + \mu_i} \right) \) by using (2.8) and (2.9)

\[ \hat{Y}_{pi} = a_i \tau_i (1 + e_0)(1 + \theta_i e_1)^{-1} + (1 - a_i) \nu_i (1 + e_0)(1 + \theta_i e_1) \]

\[ = \bar{Y} \{ a_i \tau_i(1 + e_0)(1 - \theta_i e_1 + \theta_i^2 e_1^2) + (1 - a_i) \nu_i(1 + e_0)(1 + \theta_i e_1) \} \]

\[ = \bar{Y} \{ a_i \tau_i(1 - \theta_i e_1 + \theta_i^2 e_1^2 + e_0 - \theta_i e_1 e_0) + (1 - a_i) \nu_i(1 + \theta_i e_1 + e_0 + \theta_i e_0 e_1) \} \]

\[ B \left( \hat{Y}_{pi} \right) = E(\hat{Y}_{pi} - \bar{Y}) \]

\[ = E\{ \bar{Y} \{ a_i \tau_i(1 - \theta_i e_1 + \theta_i^2 e_1^2 + e_0 - \theta_i e_1 e_0) + (1 - a_i) \nu_i(1 + \theta_i e_1 + e_0 + \theta_i e_0 e_1) - 1 \} \} \]

\[ = \bar{Y} \{ a_i \tau_i(1 + \delta \theta_i^2 C_x - \delta \theta_i \rho C_x c_y)(1 + \theta_i e_1) \nu_i(1 + \delta \theta_i \rho C_x c_y) - 1 \} \]

\[ B \left( \hat{Y}_{pi} \right) = \bar{Y} \{ a_i \tau_i + (1 - a_i) \nu_i - 1 \} + \delta \bar{Y} \{ a_i \tau_i \theta_i^2 C_x - \theta_i \rho C_x c_y(a_i \tau_i - (1 - a_i) \nu_i) \} \]

\[ = \bar{Y} \{ a_i \tau_i + (1 - a_i) \nu_i - 1 \} + a_i \tau_i B \left( \hat{Y}_{MRI} \right) + (1 - a_i) \nu_i B \left( \hat{Y}_{MPI} \right) \] (4.20)

\[ MSE \left( \hat{Y}_{pi} \right) = E \left( \hat{Y}_{pi} - \bar{Y} \right)^2 \]

\[ = E \{ \bar{Y} \{ a_i \tau_i(1 - \theta_i e_1 + \theta_i^2 e_1^2 + e_0 - \theta_i e_1 e_0) + (1 - a_i) \nu_i(1 + \theta_i e_1 + e_0 + \theta_i e_0 e_1) - 1 \} \} \]

\[ = E \{ \bar{Y}^2 \{ a_i \tau_i^2(1 - \theta_i e_1 + \theta_i^2 e_1^2 + e_0 - \theta_i e_1 e_0)^2 + (1 - a_i)^2 \nu_i^2(1 + \theta_i e_1 + e_0 + \theta_i e_0 e_1)^2 + 1 + 2a_i \tau_i(1 - a_i) \nu_i(1 - \theta_i e_1 + \theta_i^2 e_1^2 + e_0 - \theta_i e_1 e_0)(1 + \theta_i e_1 + e_0 + \theta_i e_0 e_1) \} \} \]
\[ \theta_0 e_0 e_1 - 2a_i \tau_i \left( 1 - \theta_i e_1 + \theta_i^2 e_1^2 + e_0 - \theta_i e_0 e_1 \right) - 2(1 - a_i) \nu_i (1 + \theta_i e_1 + e_0 + \theta_i e_0 e_1) \}

\[ = \bar{Y}^2 \left\{ a_i^2 \tau_i^2 \left( 1 + \delta \right) + \delta + \delta C_y^2 - 4 \delta \theta_i \rho C_x C_y \right) + (1 - a_i)^2 \nu_i^2 (1 + \delta \theta_i \rho C_x C_y) \right) + (1 - a_i)^2 \nu_i^2 (1 + \delta \theta_i \rho C_x C_y) \right) + (1 - a_i)^2 \nu_i^2 (1 + \delta \theta_i \rho C_x C_y) \right) \}

\[ \text{MSE} \left( \hat{\theta}_{p_i} \right) = \bar{Y}^2 \left\{ (a_i \tau_i + (1 - a_i) \nu_i - 1)^2 + \delta \left\{ C_y^2 (a_i \tau_i + (1 - a_i) \nu_i) \right) - 2(1 - a_i) \nu_i \right\} \}

\[ \text{MSE} \left( \hat{\theta}_{p_i} \right) = \bar{Y}^2 (a_i \tau_i + (1 - a_i) \nu_i - 1)^2 + \delta \bar{Y}^2 \left\{ (a_i \tau_i + (1 - a_i) \nu_i) \right) - 2(1 - a_i) \nu_i \right\} \}

\[ \text{MSE} \left( \hat{\theta}_{p_i} \right) = \bar{Y}^2 \left( (K_i - 1)^2 + \delta \bar{Y}^2 \left\{ (K_i - 1)^2 + \delta \theta_i \rho C_x C_y \right) + \theta_i^2 C_x^2 \left( K_i + (K_i + L_i)(L_i - 1) \right) - 2 \theta_i \rho C_x C_y \right) \}

\text{Where } \delta = \frac{1-\theta}{n}, \quad \theta_i = \frac{\bar{Y}}{\bar{X} + \bar{W}_i}, \quad i = 3,4,5,6,7 \quad \bar{W}_3 = Q_1, \quad \bar{W}_4 = Q_3, \quad \bar{W}_5 = Q_r, \quad \bar{W}_6 = Q_d, \quad \bar{W}_7 = Q_a

K_i = (a_i \tau_i + (1 - a_i) \nu_i) \quad \text{and} \quad L_i = (a_i \tau_i - (1 - a_i) \nu_i)

If \( a_i = B \left( \hat{\theta}_{p_i} \right), B = B \left( \hat{\theta}_{p_i} \right) \) and \( a_i \) is optimum, then the proposed estimators are less bias (almost unbiased) ratio cum product estimators. The optimal value of \( a_i \) is obtained as

\[ \frac{\partial \text{MSE}}{\partial a_i} = 0, \]
$2\bar{Y}^2(a_i\tau_i + (1-a_i)v_i - 1)(\tau_i - v_i) + 2a_i\tau_i^2\{\text{MSE}\left(\hat{\bar{Y}}_{MRL}\right) + 2\bar{Y}B\left(\hat{\bar{Y}}_{MRL}\right)\} - 2(1-a_i)v_i^2\{\text{MSE}\left(\hat{\bar{Y}}_{MRI}\right) + 2\bar{Y}B\left(\hat{\bar{Y}}_{MRI}\right)\} - 2\hat{\bar{Y}}^2(\tau_iB(\hat{\bar{Y}}_{MRI}) + v_iB(\hat{\bar{Y}}_{MRI})) + 2(\tau_i(1 - 2a_i)v_i\text{V}(\bar{Y}_r) = 0$

$$\rho_i = \frac{(\bar{Y}^2(v_i - 1)(v_i - \tau_i) + v_i^2\{\text{MSE}\left(\hat{\bar{Y}}_{MPL}\right) + 2\bar{Y}B(\hat{\bar{Y}}_{MPL})\} + \bar{Y}(\tau_iB(\hat{\bar{Y}}_{MRI}) - v_iB(\hat{\bar{Y}}_{MRI}))}{(\bar{Y}^2(v_i - 1)^2 + \tau_i^2\{\text{MSE}\left(\hat{\bar{Y}}_{MRI}\right) + 2\bar{Y}B(\hat{\bar{Y}}_{MRI})\} + v_i^2\{\text{MSE}\left(\hat{\bar{Y}}_{MPL}\right) + 2\bar{Y}B(\hat{\bar{Y}}_{MPL})\}}$$

### 4.4 Efficiency Comparison

In this section, the efficiencies of proposed estimators with that of the existing estimators such as simple random sampling (WOR) sample mean, ratio estimator, product estimator, MR estimator and MP estimator are compared.

#### 4.4.1 Comparison with SRSWOR sample mean

By comparing the proposed estimators with that of SRSWOR sample mean, we arrive at the proposed estimator $\hat{\bar{Y}}_{pi}$ is more efficient than simple random sampling (WOR) sample mean $\bar{Y}_r$

If $V(\bar{Y}_r) \geq \text{MSE}(\hat{\bar{Y}}_{pi})$

That is,

$$\delta \bar{Y}^2C_y^2 \geq \bar{Y}^2(K_i - 1)^2 + \bar{Y}^2\{K_i^2C_y^2 + \theta_i^2C_x^2\left(K_i^2 + (K_i + L_i)(L_i - 1)\right) - 2\theta_i\rho_iC_xC_yL_i(2K_i - 1)\}$$

$$\delta \bar{Y}^22\theta_i\rho_iC_xC_yL_i(2K_i - 1) \geq \bar{Y}^2(K_i - 1)^2 + \bar{Y}^2\{(K_i^2 - 1)C_y^2 + \theta_i^2C_x^2\left(K_i^2 + (K_i + Q_i)(Q_i - 1)\right)\}$$

$$2\delta \rho_iC_xC_y\theta_iL_i(2K_i - 1) \geq (K_i - 1)^2 + \delta\left(K_i^2 - 1\right)C_y^2 + \theta_i^2C_x^2\left(K_i^2 + (K_i + L_i)(L_i - 1)\right)$$

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\[ \rho \geq \frac{(K_i - 1)^2 + \delta [C_y^2(K_i^2 - 1) + \theta_i^2C_x^2(K_i^2 + (K_i + L_i)(L_i - 1))]}{2\delta \theta_iC_xC_yK_i(2L_i - 1)} \]

### 4.4.2 Comparison with Ratio estimator

By comparing the proposed estimators with that of ratio estimator, we arrive at the proposed estimator \( \hat{Y}_{pi} \) is more efficient than the ratio estimator \( \hat{Y}_R \) if

\[ MSE(\hat{Y}_R) \geq MSE(\hat{Y}_{pi}) \]

That is,

\[ \delta \bar{Y}^2(C_y^2 + C_x^2 - 2\rho C_xC_y) \geq \bar{Y}^2(K_i - 1)^2 + \delta \bar{Y}^2 (K_i^2C_y^2 + \theta_i^2C_x^2(L_i^2 + (K_i + L_i)(L_i - 1)) - \theta_i\rho C_xC_yL_i(2K_i - 1)) \]

\[ \delta \bar{Y}^22\theta_i\rho C_xC_yL_i(2K_i - 1) - \delta \bar{Y}^22\rho C_xC_y \geq \bar{Y}^2(K_i - 1)^2 + \delta \bar{Y}^2 ((K_i^2 - 1)C_y^2 + C_x^2(\theta_i^2(K_i^2 + (K_i + L_i)(L_i - 1) - 1)) \]

\[ 2\delta \rho C_xC_y(2\theta_iL_i(2K_i - 1) - 1) \geq (K_i - 1)^2 + \delta((K_i^2 - 1)C_y^2 + C_x^2(\theta_i^2(K_i^2 + (K_i + L_i)(L_i - 1) - 1)) \]

\[ \rho \geq \frac{(K_i - 1)^2 + \delta [C_y^2(K_i^2 - 1) + \theta_i^2(K_i^2 + (K_i + L_i)(L_i - 1) - 1)]}{2\delta C_xC_y(\theta_iL_i(2K_i - 1) - 1)} \]

### 4.4.3 Comparison with Product estimator

By comparing the proposed estimators with that of product estimator, we arrive at the proposed estimator \( \hat{Y}_{pi} \) is more efficient than the product estimator \( \hat{Y}_p \) if

\[ MSE(\hat{Y}_p) \geq MSE(\hat{Y}_{pi}) \]

That is,

\[ \delta \bar{Y}^2(C_y^2 + C_x^2 + 2\rho C_xC_y) \geq \bar{Y}^2(K_i - 1)^2 + \delta \bar{Y}^2 (K_i^2C_y^2 + \theta_i^2C_x^2(K_i^2 + (K_i + L_i)(L_i - 1)) - 2\theta_i\rho C_xC_yL_i(2K_i - 1)) \]
\[ \delta \bar{Y}^2 2\theta_l \rho C_x C_y L_i (2K_i - 1) + \delta \bar{Y}^2 2\rho C_x C_y \geq \bar{Y}^2 (K_i - 1)^2 + \delta \bar{Y}^2 \{ (K_i^2 - 1)C_y^2 + C_x^2 (K_i^2 + (K_i + L_i)(L_i - 1)) - 1 \} \]

\[ 2\delta \rho C_x C_y (\eta_i L_i (2K_i - 1) + 1) \geq (K_i - 1)^2 + \delta \{ (K_i^2 - 1)C_y^2 + C_x^2 (\theta_i^2 (K_i^2 + (K_i + L_i)(L_i - 1)) - 1) \} \]

\[
\rho \geq \frac{(K_i - 1)^2 + \delta \{ C_y^2 (K_i^2 - 1) + C_x^2 (\theta_i^2 (K_i^2 + (K_i + L_i)(L_i - 1)) - 1) \}}{2\delta \rho C_x C_y (1 + \theta_i L_i (2K_i - 1))}
\]

### 4.4.4 Comparison with MR estimators

By comparing the proposed estimators with that of MR estimators, we arrive at the proposed estimator \( \hat{Y}_{Pi} \) is more efficient than the existing MR estimator \( \hat{Y}_{MRI} \) \( (i = 3, 4, 5, 6, 7) \)

if \[ MSE(\hat{Y}_{MRI}) \geq MSE(\hat{Y}_{Pi}) \]

That is,

\[
\delta \bar{Y}^2 (C_y^2 + \theta_i^2 C_x^2 - 2\theta_i \rho C_x C_y) \geq \bar{Y}^2 (K_i - 1)^2 + \delta \bar{Y}^2 \{ K_i^2 C_y^2 + \theta_i^2 C_x^2 (K_i^2 + (K_i + L_i)(L_i - 1)) - 2\theta_i \rho C_x C_y L_i (2K_i - 1) \} \]

\[
\delta \bar{Y}^2 2\theta_i \rho C_x C_y L_i (2K_i - 1) - \delta \bar{Y}^2 2\theta_i \rho C_x C_y \geq \bar{Y}^2 (K_i - 1)^2 + \delta \bar{Y}^2 \{ (K_i^2 - 1)C_y^2 + C_x^2 \theta_i^2 (K_i^2 + (K_i + L_i)(L_i - 1)) - 1 \} \]

\[ 2\delta \theta_i \rho C_x C_y (L_i (2K_i - 1) - 1) \geq (K_i - 1)^2 + \delta \{ (K_i^2 - 1)C_y^2 + C_x^2 \theta_i^2 (K_i^2 + (K_i + L_i)(L_i - 1)) - 1 \} \]

\[
\rho \geq \frac{(K_i - 1)^2 + \delta \{ C_y^2 (K_i^2 - 1) + \theta_i^2 C_x^2 (K_i^2 + (K_i + L_i)(L_i - 1)) - 1 \}}{2\delta \theta_i C_x C_y (L_i (2K_i - 1) - 1)}
\]

### 4.4.5 Comparison with MP estimators

By comparing the proposed estimators with that of modified product estimators, we arrive at the proposed estimator \( \hat{Y}_{Pi} \) is more efficient than MP estimator \( \hat{Y}_{MPI} \)

if \[ MSE(\hat{Y}_{MPI}) \geq MSE(\hat{Y}_{Pi}) \]
That is,
\[ \delta \bar{Y}^2 (C_y^2 + \theta_i^2 C_x^2 + 2 \theta_i \rho C_x C_y) \geq \bar{Y}^2 (K_i - 1)^2 + \delta \bar{Y}^2 \{ K_i^2 C_y^2 + \theta_i^2 C_x^2 (K_i^2 + (K_i + L_i)(L_i - 1)) - 2 \theta_i \rho C_x C_y L_i (2K_i - 1) \}
\[ \delta \bar{Y}^2 2 \theta_i \rho C_x C_y L_i (2K_i - 1) + \delta \bar{Y}^2 2 \theta_i \rho C_x C_y \geq \bar{Y}^2 (K_i - 1)^2 + \delta \bar{Y}^2 \{ (K_i^2 - 1) C_y^2 + C_x^2 \theta_i^2 (K_i^2 + (K_i + L_i)(L_i - 1) - 1) \}
\[ 2 \delta \theta_i \rho C_x C_y (L_i(2K_i - 1) + 1) \geq (K_i - 1)^2 + \delta [(K_i^2 - 1) C_y^2 + C_x^2 \theta_i^2 (K_i^2 + (K_i + L_i)(L_i - 1) - 1)]
\]
\[ \rho \geq \frac{(K_i - 1)^2 + \delta [C_y^2 (K_i^2 - 1) + \theta_i^2 C_x^2 (K_i^2 + (K_i + L_i)(L_i - 1) - 1)]}{2 \delta \theta_i C_x C_y (1 + L_i(2K_i - 1))} \]

Where \( K_i = (a_i \tau_i + (1 - a_i) v_i) \) and \( L_i = (a_i \tau_i - (1 - a_i) v_i, \theta_i = \frac{\bar{X}}{\bar{X} + \bar{W}_i}, i = 3, 4, 5, 6, 7 \)

\( \bar{W}_3 = Q_1, \bar{W}_4 = Q_3, \bar{W}_5 = Q_5, \bar{W}_6 = Q_6, \bar{W}_7 = Q_7 \)

### 4.5 Numerical Study

In this section, we consider two natural populations, population A is taken from Cochran (1977) and population B is taken from Singh and Chaudhary (1986), for accessing the efficiency of the proposed estimators.

The computed values of constants and parameters of these populations are given below:

**Table 4.1: Population A (Cochran, 1977)**

<table>
<thead>
<tr>
<th>N = 10</th>
<th>n=3</th>
<th>( \bar{Y} = 101.1 )</th>
<th>( \bar{X} = 58.8 )</th>
<th>( \rho = 0.652 )</th>
<th>( S_x = 7.941 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_x = 15.445 )</td>
<td>( C_x = 0.135 )</td>
<td>( C_y = 0.153 )</td>
<td>( \beta_1 = 0.236 )</td>
<td>( \beta_2 = 2.239 )</td>
<td>( Q_1 = 53 )</td>
</tr>
<tr>
<td>( Q_3 = 61.5 )</td>
<td>( Q_4 = 4.25 )</td>
<td>( Q_5 = 57.25 )</td>
<td>( Q_6 = 8.5 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.2: Population B (Singh and Chaudhary, 1986)**

<table>
<thead>
<tr>
<th>N = 34</th>
<th>n=5</th>
<th>( \bar{Y} = 856.41 )</th>
<th>( \bar{X} = 199.44 )</th>
<th>( \rho = 0.44 )</th>
<th>( S_x = 733.14 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_x = 150.21 )</td>
<td>( C_x = 0.85 )</td>
<td>( C_y = 0.75 )</td>
<td>( \beta_1 = 7.95 )</td>
<td>( \beta_2 = 13.36 )</td>
<td>( Q_1 = 402.5 )</td>
</tr>
<tr>
<td>( Q_4 = 1049 )</td>
<td>( Q_4 = 323.25 )</td>
<td>( Q_5 = 725.75 )</td>
<td>( Q_6 = 646.5 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.3: Constants and parameters of the population

<table>
<thead>
<tr>
<th>Constants</th>
<th>Population A</th>
<th></th>
<th>Population B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$A_i$</td>
<td>-0.048</td>
<td>-0.052</td>
<td>0.079</td>
<td>-0.050</td>
</tr>
<tr>
<td>$B_i$</td>
<td>0.167</td>
<td>0.155</td>
<td>0.296</td>
<td>0.161</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>1.000</td>
<td>1.001</td>
<td>0.999</td>
<td>1.000</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>0.998</td>
<td>0.998</td>
<td>0.997</td>
<td>0.998</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>1.200</td>
<td>1.253</td>
<td>0.894</td>
<td>1.227</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>0.526</td>
<td>0.489</td>
<td>0.933</td>
<td>0.507</td>
</tr>
</tbody>
</table>

Table 4.4: Bias and MSE of Proposed Estimators

<table>
<thead>
<tr>
<th>Proposed Estimators</th>
<th>Population A</th>
<th></th>
<th>Population B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bias</td>
<td>MSE</td>
<td>Bias</td>
<td>MSE</td>
</tr>
<tr>
<td>$\hat{y}_{p3}$</td>
<td>-1.01E-14</td>
<td>32.083</td>
<td>-2.44E-15</td>
<td>2902.789</td>
</tr>
<tr>
<td>$\hat{y}_{p4}$</td>
<td>-1.31E-14</td>
<td>32.089</td>
<td>-1.65E-14</td>
<td>3056.961</td>
</tr>
<tr>
<td>$\hat{y}_{p5}$</td>
<td>4.39E-15</td>
<td>31.959</td>
<td>-6.88E-15</td>
<td>2985.792</td>
</tr>
<tr>
<td>$\hat{y}_{p6}$</td>
<td>-1.20E-14</td>
<td>32.086</td>
<td>2.02E-14</td>
<td>2862.811</td>
</tr>
<tr>
<td>$\hat{y}_{p7}$</td>
<td>3.19E-15</td>
<td>31.985</td>
<td>5.44E-15</td>
<td>3004.384</td>
</tr>
</tbody>
</table>

Table 4.5: Bias and MSE of Existing Estimators

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Population A</th>
<th></th>
<th>Population B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bias</td>
<td>MSE</td>
<td>Bias</td>
<td>MSE</td>
</tr>
<tr>
<td>$\hat{y}_r$</td>
<td>-</td>
<td>55.660</td>
<td>-</td>
<td>3849.248</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>0.113</td>
<td>35.044</td>
<td>15.164</td>
<td>4925.325</td>
</tr>
<tr>
<td>$\hat{p}_p$</td>
<td>0.311</td>
<td>163.283</td>
<td>9.768</td>
<td>12718.48</td>
</tr>
<tr>
<td>$\hat{p}_{MB3}$</td>
<td>-0.047</td>
<td>33.971</td>
<td>4.892</td>
<td>3499.732</td>
</tr>
<tr>
<td>$\hat{p}_{MB8}$</td>
<td>-0.052</td>
<td>34.713</td>
<td>0.646</td>
<td>3102.441</td>
</tr>
<tr>
<td>$\hat{p}_{MR5}$</td>
<td>0.078</td>
<td>33.699</td>
<td>2.529</td>
<td>3243.522</td>
</tr>
<tr>
<td>$\hat{p}_{MR6}$</td>
<td>-0.050</td>
<td>34.340</td>
<td>6.048</td>
<td>3641.236</td>
</tr>
<tr>
<td>$\hat{p}_{MR7}$</td>
<td>0.051</td>
<td>32.847</td>
<td>2.017</td>
<td>3197.035</td>
</tr>
<tr>
<td>$\hat{y}_{MP3}$</td>
<td>0.166</td>
<td>101.416</td>
<td>6.645</td>
<td>8801.253</td>
</tr>
<tr>
<td>$\hat{y}_{MP4}$</td>
<td>0.154</td>
<td>97.393</td>
<td>4.390</td>
<td>6605.174</td>
</tr>
<tr>
<td>$\hat{y}_{MP5}$</td>
<td>0.295</td>
<td>153.293</td>
<td>5.566</td>
<td>7684.333</td>
</tr>
<tr>
<td>$\hat{y}_{MP6}$</td>
<td>0.160</td>
<td>99.316</td>
<td>7.091</td>
<td>9298.914</td>
</tr>
<tr>
<td>$\hat{y}_{MP7}$</td>
<td>0.277</td>
<td>144.889</td>
<td>5.287</td>
<td>7415.407</td>
</tr>
</tbody>
</table>

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From table 4.6 we have observed the following:

- In the case of SRSWOR sample mean, the PREs are ranging from 125.918 to 174.157.
- In the case of ratio estimator, the PRE are ranging from 109.208 to 172.039.
- In the case of product estimator, the PRE are ranging from 416.05 to 510.899.

From table 4.7 we have observed the following:

- In population A in the case of MR estimators, the PRE are ranging from 102.696 to 108.176.
- In the case of population B PRE are ranging from 101.488 to 127.187.
- In population A in the case of MP estimators, the PRE are ranging from 303.504 to 479.644.
- In the case of population B PRE are ranging from 216.070 to 324.806.
4.6 Conclusion

In this chapter, we have proposed a set of modified ratio cum product estimators for the estimation of finite population mean with known parameters of the auxiliary variables such as first and third quartiles and their functions. The equations for biases and MSE are derived. The efficiency conditions under which the proposed estimators perform better than the existing estimators have been derived. By using some known natural populations, the performances of the proposed estimators are assessed. It shows the proposed estimators have less bias (almost unbiased) and less MSE than all the existing estimators and proposed estimators are more preferable than all the existing estimators.
CHAPTER– 5

“RATIO CUM PRODUCT ESTIMATOR WITH KNOWN COEFFICIENT OF KURTOSIS”

5.1 Introduction
5.2 Existing estimators
5.3 Proposed Estimator
5.4 Efficiency Comparison
5.5 Numerical Study
5.6 Conclusion

The contents of this chapter are published in the following research paper