CHAPTER – 2

“RATIO CUM PRODUCT ESTIMATOR WITH KNOWN COEFFICIENT OF VARIATION”

2.1 Introduction
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The contents of this chapter are published in the following research paper

Chapter - 2

Ratio cum Product Estimator with Known Coefficient of Variation

2.1 Introduction

This chapter deals with the construction of an alternative estimator for the population mean of the study variable, with the help of information about the auxiliary variable. In the presence of known coefficient of variation, a new ratio cum product estimator is introduced for the finite population mean and derive an expression for the bias and MSE. The efficiency conditions under with the proposed estimator and existing estimators are obtained. Evaluation of the effectiveness of the proposed estimator with other estimators are done by algebraically and numerically. To assess the performance of the proposed estimator through the percentage relative efficiency (PRE).

2.2 Existing Estimators

In presence of known $C_x$ of the auxiliary variable, Sisodia and Dwivedi (1981) have introduced a modified ratio estimator for finite population mean and is given by

$$\hat{Y}_{MR1} = \bar{Y} \left( \frac{\bar{X} + C_x}{\bar{X} + C_x} \right)$$ (2.1)

The bias and MSE of $\hat{Y}_{MR1}$ is given by

$$B(\hat{Y}_{MR1}) = \delta \bar{Y} \left[ \theta_1^2 C_x^2 - \theta_1 C_{xy} \right]$$ (2.2)

$$MSE(\hat{Y}_{MR1}) = \delta \bar{Y}^2 \left[ C_y^2 + \theta_1^2 C_x^2 - 2 \theta_1 C_{xy} \right]$$ (2.3)

Pandey and Dubey (1988) have proposed another estimator for the population mean, is called MP estimator based on the known $C_x$ of the auxiliary variable. The MP estimator is given by

$$\hat{Y}_{MP1} = \bar{Y} \left( \frac{\bar{X} + C_x}{\bar{X} + C_x} \right)$$ (2.4)

The bias and MSE of $\hat{Y}_{MP1}$ is given by
\[ B\left(\hat{Y}_{MP1}\right) = \delta \bar{Y} [\theta_1 C_{xy}] \]  
\[ \text{MSE}\left(\hat{Y}_{MP1}\right) = \delta \bar{Y}^2 \left[ C_y^2 + \theta_1^2 C_x^2 + 2\theta_1 C_{xy} \right] \]  
Where \( \theta_1 = \frac{\bar{x}}{\bar{x}+\bar{w}_1}, \bar{w}_1 = C_x, \delta = \frac{1-f}{n}, f = \frac{n}{N} \)

### 2.3 Proposed Estimator

Subramani and Master Ajith S (2016 a) have proposed a new estimator based on the known coefficient of variation of the auxiliary variable called modified ratio cum product estimator. The suggested estimator is given by

\[ \hat{Y}_{p1} = a_1 \tau_1 \bar{y} \left( \frac{\bar{x}+C_x}{\bar{x}+C_x} \right) + (1-a_1) \nu_1 \bar{y} \left( \frac{\bar{x}+C_x}{\bar{x}+C_x} \right) \]  

Where \( \tau_1 = \frac{\bar{s}_y}{\bar{s}_y+A_1 \bar{c}_y} \) and \( \nu_1 = \frac{\bar{s}_y}{\bar{s}_y+B_1 \bar{c}_y} \), if \( A_1 = B = B(\hat{Y}_{MR1}) \) and \( B_1 = B(\hat{Y}_{MP1}) \). It is reasonable to assume that these values are known from the previous studies.

#### 2.3.1 Bias and MSE of Proposed Estimator

To derive the bias and MSE of the proposed estimator,

Consider \( e_0 = \frac{\bar{y}-\bar{y}}{\bar{y}}, e_1 = \frac{\bar{x}-\bar{x}}{\bar{x}}, \)  

\[ E(e_0) = 0, E(e_1) = 0, E(e_0^2) = \delta \bar{Y}^2 C_y^2, E(e_1^2) = \delta \bar{X}^2 C_x^2, E(e_0 e_1) = \delta C_{xy} \]  

\[ \hat{Y}_{p1} = a_1 \tau_1 \bar{y} \left( \frac{\bar{x}+C_x}{\bar{x}+C_x} \right) + (1-a_1) \nu_1 \bar{y} \left( \frac{\bar{x}+C_x}{\bar{x}+C_x} \right) \]

\[ = a_1 \tau_1 \bar{y}(1 + e_0)(1 + \theta_1 e_1)^{-1} + (1-a_1) \nu_1 \bar{y}(1 + e_0)(1 + \theta_1 e_1) \]

\[ = \bar{y} \{ a_1 \tau_1 (1 + e_0)(1 - \theta_1 e_1 + \theta_1^2 e_1^2) + (1-a_1) \nu_1 (1 + e_0)(1 + \theta_1 e_1) \} \]

\[ = \bar{y} \{ a_1 \tau_1 (1 - \theta_1 e_1 + \theta_1^2 e_1^2 + e_0 - \theta_1 e_1 e_0) + (1-a_1) \nu_1 (1 + \theta_1 e_1 + e_0 + \theta_1 e_0 e_1) \} \]

\[ = \bar{y} \{ a_1 \tau_1 (1 - \theta_1 e_1 + \theta_1^2 e_1^2 + e_0 - \theta_1 e_1 e_0) + (1-a_1) \nu_1 (1 + \theta_1 e_1 + e_0 + \theta_1 e_0 e_1) \} \]

\[ B(\hat{Y}_{p1}) = E(\hat{Y}_{p1} - \bar{Y}) \]
\[ E \{ \bar{Y} (a_1 \tau_1 (1 - \theta_1 e_1 + \theta_1^2 e_1^2 + e_0 - \theta_1 e_1 e_0) + (1 - a_1) v_1 (1 + \theta_1 e_1 + e_0 + \theta_1 e_0 e_1) - 1) \} \]
\[ = \bar{Y} (a_1 \tau_1 (1 + \delta \theta_1 C_x^2 - \delta \theta_1 C_{xy}) + (1 - a_1) v_1 (1 + \delta \theta_1 C_{xy}) - 1) \]
\[ B (\hat{Y}_{P_1}) = \bar{Y} (a_1 \tau_1 + (1 - a_1) v_1 - 1) + \delta \bar{Y} [a_1 \tau_1 \theta_1 C_x^2 - \theta_1 C_{xy} (a_1 \tau_1 - (1 - a_1) v_1)] \]

\[ \text{MSE} \left( \hat{Y}_{P_1} \right) = E \left\{ \bar{Y} - \bar{Y} \right\}^2 \]
\[ = E \{ \bar{Y}^2 (a_1 \tau_1^2 (1 - \theta_1 e_1 + \theta_1^2 e_1^2 + e_0 - \theta_1 e_1 e_0)^2 + (1 - a_1) v_1 (1 + \theta_1 e_1 + e_0 + \theta_1 e_0 e_1)^2 + 1 + 2a_1 \tau_1 (1 - a_1) v_1 (1 - \theta_1 e_1 + \theta_1^2 e_1^2 + e_0 - \theta_1 e_1 e_0) (1 + \theta_1 e_1 + e_0 + \theta_1 e_0 e_1) - 2a_1 \tau_1 (1 - \theta_1 e_1 + \theta_1^2 e_1^2 + e_0 - \theta_1 e_1 e_0) - 2(1 - a_1) v_1 (1 + \theta_1 e_1 + e_0 + \theta_1 e_0 e_1) \} \}
\[ = \bar{Y}^2 [a_1^2 \tau_1^2 (1 + 3 \theta_1 C_x^2 + \delta C_{xy}^2 - 4 \delta \theta_1 C_{xy}) + (1 - a_1)^2 v_1^2 (1 + \theta_1 C_x^2 - \delta C_{xy}) + 2a_1 \tau_1 (1 - a_1) v_1 (1 + \delta C_{xy}^2) - 2a_1 \tau_1 (1 + \delta \theta_1 C_x^2 - \delta \theta_1 C_{xy}) - 2(1 - a_1) v_1 (1 + \delta \theta_1 C_{xy})] \}
\[ \text{MSE} \left( \hat{Y}_{P_1} \right) = \bar{Y}^2 [(a_1 \tau_1 + (1 - a_1) v_1 - 1)^2 + \delta C_{xy}^2 (a_1 \tau_1 + (1 - a_1) v_1)^2 + \theta_1 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 v_1^2 - 2a_1 \tau_1) + 2a_1 \tau_1 (1 - a_1) v_1), - 2(1 - a_1)^2 v_1^2)]} \]
\[ \text{MSE} \left( \hat{Y}_{P_1} \right) = \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) v_1 - 1)^2 + \delta \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) v_1)^2 + \theta_1 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 v_1^2 - 2a_1 \tau_1) + 2a_1 \tau_1 (1 - a_1) v_1, - 2(1 - a_1)^2 v_1^2)]} \]

Where \( \theta_1 = \frac{\bar{X}}{\bar{X} + C_x}, \delta = \frac{1 - f}{n} \)
If \( A_1 = B(\hat{Y}_{MR1}) \), \( B_1 = B(\hat{Y}_{MP1}) \) and \( a_1 \) is optimum, then the proposed estimator is less biased (almost unbiased) ratio cum product estimator. The optimal value of \( a_1 \) is obtained as,

\[
\frac{\partial MSE}{\partial a_1} = 0,
\]

\[
2(a_1\tau_1 + (1 - a_1)\nu_1 - 1)(\tau_1 - \nu_1) + \delta \left( C_y^2 2(a_1\tau_1 + (1 - a_1)\nu_1)(\tau_1 - \nu_1) + \theta_1^2 C_x^2 (6a_1\tau_1^2 - 2(1 - a_1)\nu_1^2 - 2\tau_1) + 2\theta_1 C_{xy}(\tau_1 + \nu_1 - 2(2a_1\tau_1^2 + 2(1 - a_1)\nu_1^2)) \right) = 0
\]

\[
(a_1\tau_1 - a_1\tau_2)(\tau_1 - \nu_1) + \delta \left( C_y^2 (a_1\tau_1 - a_1\nu_1)(\tau_1 - \nu_1) + \theta_1^2 C_x^2 (3a_1\tau_1^2 + a_1\nu_1^2) + \theta_1 C_{xy}(-4a_1\tau_1^2 + 4a_1\nu_1^2) \right) = (\nu_1 - 1)(\nu_1 - \tau_1) + \delta \left( C_y^2 \nu_1(\nu_1 - \tau_1) + \theta_1^2 C_x^2 (\tau_1 + \nu_1^2) - \theta_1 C_{xy}(\tau_1 + \nu_1 - 4\nu_1^2) \right)
\]

\[
a_1 = \frac{(\nu_1 - 1)(\nu_1 - \tau_1) + \delta \left( C_y^2 \nu_1(\nu_1 - \tau_1) + \theta_1^2 C_x^2 (\tau_1 + \nu_1^2) - \theta_1 C_{xy}(\tau_1 + \nu_1 - 4\nu_1^2) \right)}{(\tau_1 - \nu_1)^2 + \delta ((\tau_1 - \nu_1)^2 C_y^2 + \theta_1^2 C_x^2 (3\tau_1^2 + \nu_1^2) + 4\theta_1 C_{xy}(\nu_1^2 - \tau_1^2))}
\]

### 2.4 Efficiency Comparison

The efficiencies of proposed ratio cum product estimator is compared with that of the existing estimators such as \( \bar{Y}_r, \bar{Y}_R, \bar{Y}_p, \bar{Y}_{MR1} \) and \( \bar{Y}_{MP1} \).

#### 2.4.1 Comparison with SRSWOR sample mean

By comparing the proposed estimator with that of simple random sampling (WOR) sample mean, we arrive at the efficiencies of proposed estimator \( \bar{Y}_{p1} \) is more than sample mean (SRSWOR) \( \bar{Y}_r \)

\[
V(\bar{Y}_r) \geq MSE(\bar{Y}_{p1})
\]

That is,
\[ \delta \bar{Y}^2 C_x^2 \geq \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) v_1 - 1)^2 + \delta \bar{Y}^2 \{ C_y^2 (a_1 \tau_1 + (1 - a_1) v_1)^2 + \\
\theta_1^2 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 v_1^2 - 2a_1 \tau_1) + 2\theta_1 C_{xy} ((a_1 \tau_1 - (1 - a_1) v_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 v_1^2)) \} \]
\[ \delta \bar{Y}^2 C_y^2 \geq \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) v_1 - 1)^2 + \delta \bar{Y}^2 \{ C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 v_1^2 - 2a_1 \tau_1) + 2\theta_1 C_{xy} ((a_1 \tau_1 - (1 - a_1) v_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 v_1^2)) \} \]
\[ C_y^2 \geq \left( \frac{(a_1 \tau_1 + (1 - a_1) v_1 - 1)^2 + \delta \{ \theta_1^2 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 v_1^2 - 2a_1 \tau_1) + 2\theta_1 C_{xy} ((a_1 \tau_1 - (1 - a_1) v_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 v_1^2)) \}}{\delta (1 - (a_1 \tau_1 + (1 - a_1) v_1)^2)} \right) \]

2.4.2 Comparison with Ratio estimator

By comparing the proposed estimator with that of ratio estimator, we arrive at the efficiencies of the proposed estimator \( \hat{Y}_{p1} \) is more than ratio estimator \( \hat{Y}_R \)

if \[ MSE( \hat{Y}_R ) \geq MSE( \hat{Y}_{p1} ) \]

That is,
\[ \delta \bar{Y}^2 \{ C_y^2 + C_x^2 - 2\rho C_{xy} \} \geq \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) v_1 - 1)^2 + \delta \bar{Y}^2 \{ C_y^2 (a_1 \tau_1 + (1 - a_1) v_1)^2 + \\
\theta_1^2 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 v_1^2 - 2a_1 \tau_1) + 2\theta_1 C_{xy} ((a_1 \tau_1 - (1 - a_1) v_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 v_1^2)) \} \]
\[ \delta \bar{Y}^2 \{ C_y^2 (\theta_1^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 v_1^2 - 2a_1 \tau_1) - 1) + 2C_{xy} (\theta_1 (a_1 \tau_1 - (1 - a_1) v_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 v_1^2)) + 1) \} \]
\[ \delta C_y^2 (1 - (a_1 \tau_1 + (1 - a_1) v_1)^2) \geq \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) v_1 - 1)^2 + \delta \bar{Y}^2 \{ C_y^2 (\theta_1^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 v_1^2 - 2a_1 \tau_1) - 1) + 2C_{xy} (\theta_1 (a_1 \tau_1 - (1 - a_1) v_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 v_1^2)) + 1) \} \]
\[ C_y^2 \geq \left( \frac{\bar{Y}^2(a_1\tau_1+(1-a_1)v_1-1)^2 + \delta \bar{Y}^2 \{ c_x^2 \left( \theta_1^2(3a_1^2\tau_1^2+(1-a_1)^2v_1^2-2a_1\tau_1)-1 \right) \} + 2c_{xy}(\theta_1((a_1\tau_1-(1-a_1)v_1)-2(a_1^2\tau_1^2+(1-a_1)^2v_1^2))\right)}{\delta(1-(a_1\tau_1+(1-a_1)v_1)^2)} \]

2.4.3 Comparison with Product estimator

By comparing the proposed estimator with that of product estimator, we arrive at the efficiency of proposed estimator \( \hat{Y}_{p1} \) is more than the product estimator \( \hat{Y}_p \)

if \( MSE(\hat{Y}_p) \geq MSE(\hat{Y}_{p1}) \)

That is,

\[ \delta \bar{Y}^2 \{ c_x^2 + 2\rho c_{xy} \} \geq \bar{Y}^2(a_1\tau_1+(1-a_1)v_1-1)^2 + \delta \bar{Y}^2 \{ c_y^2(a_1\tau_1+(1-a_1)v_1)^2 + \theta_1^2 c_x^2(3a_1^2\tau_1^2+(1-a_1)^2v_1^2-2a_1\tau_1) + 2\theta_1 c_{xy}(\theta_1((a_1\tau_1-(1-a_1)v_1)-2(a_1^2\tau_1^2+(1-a_1)^2v_1^2)) \}
\]

\[ \delta c_y^2 c_x^2(1-(a_1\tau_1+(1-a_1)v_1)^2) \geq \bar{Y}^2(a_1\tau_1+(1-a_1)v_1-1)^2 + \delta \bar{Y}^2 \{ c_x^2(\theta_1^2(3a_1^2\tau_1^2+(1-a_1)^2v_1^2-2a_1\tau_1)-1) + 2c_{xy}(\theta_1((a_1\tau_1-(1-a_1)v_1)-2(a_1^2\tau_1^2+(1-a_1)^2v_1^2)) \}
\]

2.4.4 Comparison with MR estimator

By comparing the proposed estimator with that of modified ratio estimator, we arrive at the efficiency of proposed estimator \( \hat{Y}_{p1} \) is more than MR estimator \( \hat{Y}_{MR1} \)

if \( MSE(\hat{Y}_{MR1}) \geq MSE(\hat{Y}_{p1}) \)
That is,
\[
\delta \bar{Y}^2 \{ C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 \rho C_{xy} \} \geq \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) \nu_1 - 1)^2 + \delta \bar{Y}^2 \{ C_y^2 (a_1 \tau_1 + (1 - a_1) \nu_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 \nu_1^2)) \}
\]
\[
\delta \bar{Y}^2 C_y^2 - \delta \bar{Y}^2 C_x^2 (a_1 \tau_1 + (1 - a_1) \nu_1)^2 \geq \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) \nu_1 - 1)^2 + \delta \bar{Y}^2 \{ \theta_1^2 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 \nu_1^2 - 2a_1 \tau_1 - 1) + 2\theta_1 C_{xy} ((a_1 \tau_1 - (1 - a_1) \nu_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 \nu_1^2)) - 1) \}
\]
\[
\delta C_y^2 (1 - (a_1 \tau_1 + (1 - a_1) \nu_1)^2) \geq (a_1 \tau_1 + (1 - a_1) \nu_1 - 1)^2 + \delta \{ \theta_1^2 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 \nu_1^2 - 2a_1 \tau_1 - 1) + 2\theta_1 C_{xy} ((a_1 \tau_1 - (1 - a_1) \nu_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 \nu_1^2) - 1) \}
\]
\[
C_y^2 \geq \frac{\left( (a_1 \tau_1 + (1 - a_1) \nu_1 - 1)^2 + \delta \{ \theta_1^2 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 \nu_1^2 - 2a_1 \tau_1 - 1) + 2\theta_1 C_{xy} ((a_1 \tau_1 - (1 - a_1) \nu_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 \nu_1^2) - 1) \right)}{\delta (1 - (a_1 \tau_1 + (1 - a_1) \nu_1)^2)}
\]

### 2.4.5 Comparison with MP estimator

By comparing the proposed estimator with that of MP estimator, we arrive at the efficiency of proposed estimator $\bar{Y}_{p1}$ is more than the MP estimator $\bar{Y}_{MP1}$ if
\[
MSE(\bar{Y}_{MP1}) \geq MSE(\bar{Y}_{p1})
\]
That is,
\[
\delta \bar{Y}^2 \{ C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 \rho C_{xy} \} \geq \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) \nu_1 - 1)^2 + \delta \bar{Y}^2 \{ C_y^2 (a_1 \tau_1 + (1 - a_1) \nu_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 \nu_1^2)) \}
\]
\[
\delta \bar{Y}^2 C_y^2 - \delta \bar{Y}^2 C_x^2 (a_1 \tau_1 + (1 - a_1) \nu_1)^2 \geq \bar{Y}^2 (a_1 \tau_1 + (1 - a_1) \nu_1 - 1)^2 + \delta \bar{Y}^2 \{ \theta_1^2 C_x^2 (3a_1^2 \tau_1^2 + (1 - a_1)^2 \nu_1^2 - 2a_1 \tau_1 - 1) + 2\theta_1 C_{xy} ((a_1 \tau_1 - (1 - a_1) \nu_1) - 2(a_1^2 \tau_1^2 - (1 - a_1)^2 \nu_1^2) - 1) \}
\]
\[ \delta C_Y^2 (1 - (a_1\tau_1 + (1 - a_1)v_1)^2) \geq (a_1\tau_1 + (1 - a_1)v_1 - 1)^2 + \delta \{ \theta_1 C_x^2 (3a_1^2\tau_1^2 + (1 - a_1)^2v_1^2 - 2a_1\tau_1 - 1) + 2\theta_1 C_{xy}((a_1\tau_1 - (1 - a_1)v_1) - 2(a_1^2\tau_1^2 - (1 - a_1)^2v_1^2) - 1) \}
\]

\[ C_y^2 \geq \frac{(a_1\tau_1 + (1 - a_1)v_1 - 1)^2 + \delta \{ \theta_1 C_x^2 (3a_1^2\tau_1^2 + (1 - a_1)^2v_1^2 - 2a_1\tau_1 - 1) + 2\theta_1 C_{xy}((a_1\tau_1 - (1 - a_1)v_1) - 2(a_1^2\tau_1^2 - (1 - a_1)^2v_1^2) + 1) \}}{\delta(1 - (a_1\tau_1 + (1 - a_1)v_1)^2)} \]

### 2.5 Numerical Study

In this section, four natural populations are considered to assess the efficiency of proposed estimator with that of Simple random sampling (SRSWOR) sample mean, ratio estimator, product estimator, MR and MP estimator with known coefficient of variation. The four natural populations are, population A is taken from Khoshnevisan et al. (2007), population B is taken from Cochran (1977), population C and D are taken from Singh and Chaudhary (1986).

The various constants and parameters are given in the following tables.

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<td>0.996</td>
<td>0.922</td>
<td>0.955</td>
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<td>( a_1 )</td>
<td>0.070</td>
<td>0.868</td>
<td>0.765</td>
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</table>
Table 2.2: Bias and MSE of Estimators

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>Bias</td>
<td>MSE</td>
<td>Bias</td>
<td>MSE</td>
<td>Bias</td>
<td>MSE</td>
<td>Bias</td>
<td>MSE</td>
</tr>
<tr>
<td>( \hat{\bar{y}}_{P1} )</td>
<td>1.23E-06</td>
<td>0.544</td>
<td>3.19E-15</td>
<td>31.927</td>
<td>6.39E-14</td>
<td>66164.15</td>
<td>-7.82E-14</td>
<td>109154.9</td>
</tr>
<tr>
<td>( \hat{\bar{y}}_r )</td>
<td>-</td>
<td>3.616</td>
<td>-</td>
<td>55.660</td>
<td>-</td>
<td>91690.37</td>
<td>-</td>
<td>163356.4</td>
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<tr>
<td>( \hat{\bar{y}}_R )</td>
<td>0.416</td>
<td>15.459</td>
<td>0.113</td>
<td>35.044</td>
<td>35.374</td>
<td>87325.38</td>
<td>63.024</td>
<td>155579.7</td>
</tr>
<tr>
<td>( \hat{\bar{y}}_p )</td>
<td>-0.188</td>
<td>0.686</td>
<td>0.317</td>
<td>163.283</td>
<td>40.471</td>
<td>225967.1</td>
<td>72.104</td>
<td>402585.0</td>
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<tr>
<td>( \hat{\bar{y}}_{MR1} )</td>
<td>0.403</td>
<td>15.126</td>
<td>0.111</td>
<td>34.992</td>
<td>34.993</td>
<td>87117.86</td>
<td>62.344</td>
<td>155210.0</td>
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<tr>
<td>( \hat{\bar{y}}_{MP1} )</td>
<td>-0.185</td>
<td>0.657</td>
<td>0.316</td>
<td>162.936</td>
<td>40.332</td>
<td>225282.9</td>
<td>71.856</td>
<td>401366.2</td>
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</table>

Table 2.3: PRE of the Proposed Estimator

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</thead>
<tbody>
<tr>
<td></td>
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<td>MSE</td>
<td>Bias</td>
<td>MSE</td>
<td>Bias</td>
<td>MSE</td>
<td>Bias</td>
<td>MSE</td>
</tr>
<tr>
<td>( \hat{\bar{y}}_r )</td>
<td>664.62</td>
<td>174.33</td>
<td>109.76</td>
<td>131.98</td>
<td>142.53</td>
<td>368.82</td>
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</tr>
<tr>
<td>( \hat{\bar{y}}_R )</td>
<td>2841.06</td>
<td>109.76</td>
<td>131.98</td>
<td>142.53</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \hat{\bar{y}}_p )</td>
<td>126.25</td>
<td>511.42</td>
<td>341.52</td>
<td>368.82</td>
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<tr>
<td>( \hat{\bar{y}}_{MR1} )</td>
<td>2779.82</td>
<td>109.60</td>
<td>131.67</td>
<td>142.19</td>
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<td></td>
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<tr>
<td>( \hat{\bar{y}}_{MP1} )</td>
<td>120.83</td>
<td>510.34</td>
<td>340.49</td>
<td>367.70</td>
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</tbody>
</table>

The PRE is ranging from

- 140.71 to 689.13 in case of SRSWOR sample mean
- 134.01 to 2945.78 in case of ratio estimator
- 130.88 to 511.65 in case of product estimator
- 109.65 to 2888.23 in case of MR estimator
- 125.26 to 510.57 in case of MP estimator
2.6 Conclusion

In this chapter, we have introduced a new ratio cum product estimator with known coefficient of variation of the auxiliary variable and analyzed the effectiveness of the proposed estimator. Obtained the expression for bias and MSE of the proposed estimator. The conditions under which the MSE of the proposed estimator is better than that of other existing estimators such as simple random sampling (SRSWOR) sample mean, ratio, product, MR and MP estimators are derived. Numerical comparisons are done with the help of some known natural populations. From the numerical comparisons, it is clear that the proposed estimator is more effective than the existing estimators. The PREs of proposed estimator with respect to the existing estimators are more than 100.