CHAPTER 4
SVD and DWT based
Reversible Data Hiding
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All the previously mentioned techniques work in spatial domain and during our study; we found that in frequency domain, more space can be created for the embedding purpose. 2-D Discrete Wavelet Transform (DWT) is a widely used transform in image processing. Also, very less work has been reported in reversible data hiding literature using Singular Value Decomposition (SVD), which is a very useful transformation in image processing. Therefore, in this chapter, we have focused on developing reversible data hiding technique in transform domain by utilizing the features of both SVD and DWT.

In this chapter, a novel hybrid reversible watermarking scheme based on DWT and SVD is presented. In this scheme, we provided a double layer of security by utilizing the multiresolution property of wavelet and some prominent features of SVD. In this work, watermark is embedded into the singular values of all high-frequency subbands obtained by wavelet decomposition of the original image and at the time of extraction, watermark bits are used along with singular vectors to obtain the original image. Our scheme provides high security even after the extraction of the watermark. Without
knowing the extraction algorithm; original image cannot be recovered in its entirety.

4.1 Introduction

Although, lot of work has been carried out in the field of irreversible data hiding using SVD [8, 14, 17, 18, 26, 35, 75, 76, 81, 88, 89, 96, 100, 103, 111], but very less work has been reported in the field of lossless data hiding [146]. There are various properties of SVD, such as multiresolution, orthogonal subspaces, singular value distribution etc., that have been exploited in many simple digital watermarking schemes but rarely in reversible watermarking. In simple digital watermarking, SVD plays an eminent role due to its various properties. The SVD contains the maximum energy into few coefficients and has the capability to adjust the variations in an image [116]. Most of the developed SVD based watermarking techniques are based on the strength of singular values that specify the luminance of an image. Other than this, intrinsic algebraic image properties are represented by singular values. Therefore, small variation in singular values does not have remarkable influence on the watermarked image quality [116]. So, the SVD based digital watermarking techniques have either used the largest singular values or the lowest singular values to embed the watermark either by using quantization or by addition. In the field of reversible watermarking, these properties are still unexploited. Yoo et al. [146] proposed a reversible watermarking algorithm in which data for authentication was produced from XORing of the watermark image and singular values of one block image.
SVD provides some interesting algebraic and structural properties of an image. Use of SVD along with DWT can play a prominent role in reversible watermarking techniques, as the combination of salient features of these two provides more security in invertible data hiding. In the proposed work, we utilize the unexploited concept of SVD along with DWT to propose a novel reversible watermarking technique. Here, we have tried to utilize the properties of SVD in reversible data hiding.

The organization of the rest of the chapter is as follows: In Section 4.2, image decompositions such as DWT and SVD are discussed. In Section 4.3, proposed technique is explained in brief. Section 4.4 demonstrates the experimental results and analysis. Finally, the conclusions are drawn in Section 4.5.

4.2 Image Decompositions

In this section, the basic concepts of image decompositions such as DWT and SVD are described in detail.

4.2.1 Discrete Wavelet Transform (DWT)

2-D Discrete Wavelet Transform is a widely used transform in image processing. DWT is based on the concept of wavelets (small waves or mother wavelets) [121]. It is localized both in frequency and time domains. This reveals spatial and frequency aspects simultaneously [101, 110]. It is used for analysing an image at different resolutions into different frequency components. Due to multiresolution property, features that may go unnoticed at one resolution, may be easily detected at another.
Multiresolution analysis comprises image pyramid and subband coding theory.

For obtaining 2-D wavelet decomposition, 1-D DWT can be applied on image first in horizontal and then in vertical direction using different filters. 2-D DWT decomposes the image into two parts: approximation and detailed. Approximation part contains one low frequency subband LL and detailed part contains three high frequency subbands LH, HL and HH. Approximation part can be further decomposed into four subbands as shown in Fig. 4.1. First level decomposition of lena image is shown in Fig. 4.2. Decomposed subbands can be used to reconstruct the original image using Inverse DWT.

High frequency subbands of wavelet decomposition have Laplace distribution and this property can be utilized for the data embedding [80].

DWT of an image $I(i, j)$ of size $M \times N$ is defined as [56]

$$W_{\phi}(y_0, m, n) = \frac{1}{\sqrt{MN}} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} I(i, j) \phi_{y_0, m, n}(i, j) \quad (4.2.1)$$

$$W_{\psi}^{x}(y, m, n) = \frac{1}{\sqrt{MN}} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} I(i, j) \psi_{x, y, m, n}^{x}(i, j) \quad (4.2.2)$$

where, $W_{\phi}(y_0, m, n)$ defines approximation part of image $I(i, j)$ and $W_{\psi}^{x}(y, m, n)$ defines horizontal, vertical and diagonal parts.

For the given equations (4.2.1) and (4.2.2), inverse DWT can be defined as:
$$I(i,j) = \frac{1}{\sqrt{MN}} \sum_m \sum_n W_\phi(y_0, m, n) \phi_{y_0, m, n}(i, j) +$$

$$\frac{1}{\sqrt{MN}} \sum_{x=H,V,D} \sum_{y=y_0}^\infty \sum_m \sum_n W_\psi^x(y, m, n) \psi^x_{y, m, n}(i, j)$$  \hspace{1cm} (4.2.3)

The optical realization of 2-D DWT was proposed by Mendlovic et al. [102] in 1993. Optically, 2-D DWT is implemented by using conventional coherent correlator along with multireference matched filter.
4.2.2 Singular Value Decomposition (SVD)

SVD is a well known technique of linear algebra to decompose a matrix into a set of linearly independent components such that each of them has its own energy contribution [40]. Again, it is a numerical method used to diagonalize the matrices in numerical analysis [11, 71], however it can be seen from some other points of views also. On one hand, it can be seen as a method for changing correlated variables into a set of uncorrelated ones that better uncover the various relationships among the original data elements [68]. On the other hand, SVD is a method for finding and ordering the dimensions along which data points show the most variation [13].

SVD decomposes a matrix $A$ into matrices $U$, $S$ and $V$ such that $A = USV^T$ ($T$ represents transpose), where $U$ and $V$ contain singular vectors and $S$ is a diagonal matrix containing singular values. The SVD of a digital image $I$ of size $M \times N$ with $M \geq N$ can be defined as:

$$ I = USV^T $$

(4.2.4)

where, $U$ is a matrix of size $M \times M$ of left singular vectors, $S$ is diagonal matrix of size $M \times N$ containing singular values in decreasing order and $V$ is matrix of size $N \times N$ containing right singular vectors.

Every singular value deals with the luminance of an image while the singular vectors specify the geometrical structure of the image [55]. Therefore, any change in $U$ and $V$ can change the geometrical structure of image but any small change in singular values would not affect the image in terms of its perceptibility.
4.3 Proposed Technique

SVD is a very useful transformation that has been widely used in image processing and still has very much potential to be used in various areas. In the field of irreversible watermarking, various techniques [17, 20, 26, 96] have been proposed using SVD. In [26], authors proposed a digital watermarking technique based on SVD that exploits both singular values and singular vectors for embedding the watermark. In 2013, Benhocine et al. [17] gave technique for embedding the watermark based on SVD. In [96], authors proposed optimal watermarking technique using particle swarm optimization and SVD. In this scheme, singular values of chosen detail subband were modified by multiple-scaling factors and singular values of watermark were embedded. Therefore, it is evident that SVD has been used widely for developing irreversible watermarking techniques. It opens a spectrum of applications of SVD in reversible data hiding. So, we have proposed a reversible watermarking technique by using unexploited concept of SVD that has been discussed in this section.

The proposed reversible watermarking scheme utilizes the properties of not only DWT but also of SVD. Here, singular values of an image are utilized to hide the data in wavelet coefficient such that at the time of extraction, along with watermark, exact original image can also be recovered. Our scheme provides double layer of security: First at the SVD level and again at the level of watermark embedding in the image. The values of watermark are added into the singular values of wavelet coefficients and again, these watermark bits are embedded in the image. Thus, for extraction,
one has to extract the watermark bits and by using these bits along with singular vectors, original image can be recovered. If anyone finds out the watermark bits from watermarked image, even then original image can not be recovered without knowing the exact extracting procedure.

The mathematical theory behind our proposed scheme is described as follows:

Let $A$ be a matrix of size $M \times N$, then SVD of matrix $A$ is

$$A = USV^T$$

Now, $US_1V^T = A_1$ (say), where $S_1 = S_{ii} + W_i$; $W \in \{0, 1\}$

Again,

$$V' = V_{ij}^T \ast W_i$$

(Scalar Multiplication of $W_i$ with the $i^{th}$ row of $V^T$)

$$E = UV'$$

where, $E$ is the matrix of extra terms which came due to addition of string $W$ into singular values $S_{ii}$

Now, $A_1 - E = A$ (Original Matrix)

Following is the explanation of above theory for a $2 \times 2$ matrix $A$. Let us consider the singular value decomposition of a matrix $A$ be $A = USV^T$

where,

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, S = \begin{pmatrix} e & 0 \\ 0 & f \end{pmatrix}, V = \begin{pmatrix} g & h \\ i & j \end{pmatrix}$$

(4.3.5)
Therefore,

\[ A = U S V^T = \begin{pmatrix} aeg + bh & aei + bfj \\ ceg + dfh & cei + dfj \end{pmatrix} \]  \quad (4.3.6)

Now, let us consider a watermark \( W = (w_1, w_2) \) to be added to the diagonal elements of the diagonal matrix \( S \). After the addition of watermark in singular values, the matrix \( S \) becomes

\[ S_1 = \begin{pmatrix} e + w_1 & 0 \\ 0 & f + w_2 \end{pmatrix} \]  \quad (4.3.7)

Now, let \( A_1 = U S_1 V^T \)

\[
A_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e + w_1 & 0 \\ 0 & f + w_2 \end{pmatrix} \begin{pmatrix} g & i \\ h & j \end{pmatrix}
\]

\[
A_1 = \begin{pmatrix} aeg + aw_1g + bh + bw_2h & aei + aw_1i + bfj + bw_2j \\ ceg + cw_1g + dfh + dw_2h & cei + cw_1i + dfj + dw_2j \end{pmatrix} \]  \quad (4.3.8)

It can be analysed from above that there are extra terms (underlined) in every element of \( A_1 \) due to the watermark bits added to the singular values, which includes the values from \( U \) and \( V \) also. Therefore, these extra terms have been adjusted in such a way (discussed below) that original value of \( A \) can be obtained.

Now, by using watermark \( W \), matrix \( A_2 \) is computed such that

\[ A_2 = V_{ij}^T \ast W_i \]  \quad (4.3.9)

(Scalar Multiplication of \( W_i \) with the \( i^{th} \) row of \( V^T \))

Now, \( A_3 = U A_2 \)
Now, \( B = A_1 - A_3 \)

\[
B = \begin{pmatrix}
aw_1g + bw_2h & aw_1i + bw_2j \\
cw_1g + dw_2h & cw_1i + dw_2j
\end{pmatrix}
\]

It is evident from above example that the matrix \( A \) (original image) can be recovered from \( A_1 \) (watermarked image). It shows that by an adjustment of singular vectors and watermark, the original image can be recovered even after addition of watermark in its singular values.

The flow charts of embedding and extracting algorithms are shown in Fig. 4.3. The proposed embedding and extracting algorithms are described as follows:

### 4.3.1 Embedding Algorithm

1. Apply DWT on an original image \( I \) and decompose \( I \) into four subbands LL, LH, HL and HH.

2. Apply SVD on each subband except LL.

3. Embed Watermark bits in singular values of each subband such that

\[
S_1(i, j) = \begin{cases} 
S(i, j) + W(i); & \text{if } i = j \\
S(i, j) & \text{otherwise}
\end{cases}
\]
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4. Reconstruct each high frequency subband as $US_1V^T$.

5. Apply inverse DWT to obtain the intermediate watermarked image.

6. Now embed watermark bits in image as follows:

   (a) Choose any column(s) or row(s) of the image, notably last column.

   (b) Here values are in double data type; extract integer part, left shift it. Make Least Significant Bit (LSB) as 1 if bit to be embedded is 1 otherwise keep LSB as it is. Combine integer part with fraction part and obtain final watermarked image $I'$.

4.3.2 Extracting Algorithm

Extracting Watermark

1. Consider the watermarked image $I'$.

2. Consider the last column(s) or row(s) as used in embedding procedure and extract the integer part from it.

3. Number will be in $2^p$ or $2^p+1$ form where $p$ is the pixel value. Convert it into binary.

4. If LSB is 1, extract watermark security key bit as 1, subtract the LSB from the number, divide the resultant by 2 and construct values by putting together obtained quotient and fraction part otherwise set watermark bit as 0 and to
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Figure 4.3: An Overview of the proposed scheme (a) Embedding Procedure; (b) Extraction Procedure.

obtain the values, divide the number by 2 and put together with its fractional part.

(a) For example, if the value from last column is obtained as 23.9504 then binary representation of integer part 23 would be 00010111. Here LSB is 1, so 1 is stored as watermark bit, and 1 is subtracted from the number and resultant is 22. After dividing 22 by 2 and putting the quotient with the fractional part of the number, 11.9504 is obtained which is the pixel
value after extracting the watermark bit.

5. Repeat step 2 by the number of times of the length of the watermark to extract
the watermark $W$.

Following are the steps to recover original image using extracted watermark:

**Recovering Original Image**

1. Apply DWT on the image left after the extraction of watermark $W$.

2. Apply SVD on each subband except LL to obtain the values $U'$, $S'$ and $V'$.

3. Use extracted watermark for recovering original image. Make a matrix $B_1$ by
scalar multiplication of $W_i$ with the $i^{th}$ rows of $V'^T$, such that

$$B_1 = V'^T W_i \quad (4.3.10)$$

Now,

$$B_2 = U' B_1 \quad (4.3.11)$$

Apply this procedure on each high frequency subband.

Subtract $B_2$ from original values of that subband i.e.

$$B = (U' S' V'^T) - B_2; \quad (4.3.12)$$

4. Apply inverse DWT on subbands obtained from step 3.

5. The image obtained from the step 4 is the recovered original image.
4.4 Experimental Results and Discussion

The proposed algorithm has been applied on various test images using MATLAB. The images used for testing the proposed scheme are shown in Fig. 4.4. The proposed scheme can be applied on many levels of decompositions of DWT and therefore security will be increased but it may increase the computational complexity. At first level of wavelet decomposition of image of size $N \times N$, four subbands of size $N/2 \times N/2$ would be obtained. The maximum embedding that can be done in one subband using the proposed scheme is the total number of singular values of that subband i.e. $N/2$. Thus, total embedding capacity at first level of decomposition would be $3N/2$ when embedding is done in detailed parts i.e. in three high frequency subbands. If the image is of size $512 \times 512$, for one level decomposition, the number of embedded bits $= (3 \times 512)/2 = 768$. The embedding capacity at the $k$ level decomposition $= 3N \sum 1/2^k \rightarrow 3N$ as $k \rightarrow \infty$. Therefore, the maximum embedding capacity would converge towards $3N$ for any level of decomposition using DWT of an image of size $N \times N$.

Two layer of embedding provides higher security in terms such that after extracting watermark at first level, if extraction algorithm is not known, the original image cannot be fully recovered.

Computed PSNR values for different test images at different capacities have been shown in the Table 4.1. These PSNR values are obtained when the maximum embedding capacity is utilized (all watermark bits as 1 in all the diagonal elements).
Table 4.1: PSNR, UIQI and SSIM values for various test images at different capacity using Proposed scheme

<table>
<thead>
<tr>
<th>Images</th>
<th>Capacity (no. of bits)</th>
<th>PSNR (db)</th>
<th>UIQI</th>
<th>SSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>256</td>
<td>43.1746</td>
<td>0.9992</td>
<td>0.9978</td>
</tr>
<tr>
<td></td>
<td>512</td>
<td>39.2699</td>
<td>0.9984</td>
<td>0.9946</td>
</tr>
<tr>
<td></td>
<td>768</td>
<td>38.6632</td>
<td>0.9974</td>
<td>0.9937</td>
</tr>
<tr>
<td>Mandrill</td>
<td>256</td>
<td>42.9822</td>
<td>0.9996</td>
<td>0.9964</td>
</tr>
<tr>
<td></td>
<td>512</td>
<td>40.0038</td>
<td>0.9987</td>
<td>0.9929</td>
</tr>
<tr>
<td></td>
<td>768</td>
<td>38.5014</td>
<td>0.9981</td>
<td>0.9899</td>
</tr>
<tr>
<td>Barbara</td>
<td>256</td>
<td>42.0653</td>
<td>0.9992</td>
<td>0.9973</td>
</tr>
<tr>
<td></td>
<td>512</td>
<td>39.2615</td>
<td>0.9985</td>
<td>0.9949</td>
</tr>
<tr>
<td></td>
<td>768</td>
<td>38.0083</td>
<td>0.9976</td>
<td>0.9932</td>
</tr>
<tr>
<td>Airplane</td>
<td>256</td>
<td>38.5510</td>
<td>0.9990</td>
<td>0.9912</td>
</tr>
<tr>
<td></td>
<td>512</td>
<td>35.0109</td>
<td>0.9980</td>
<td>0.9805</td>
</tr>
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<td></td>
<td>768</td>
<td>34.9578</td>
<td>0.9975</td>
<td>0.9803</td>
</tr>
<tr>
<td>Boat</td>
<td>256</td>
<td>41.5999</td>
<td>0.9993</td>
<td>0.9967</td>
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<tr>
<td></td>
<td>512</td>
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<td>0.9982</td>
<td>0.9912</td>
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<td></td>
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<td>0.9906</td>
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<td>Goldhill</td>
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<td>42.7617</td>
<td>0.9993</td>
<td>0.9972</td>
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<tr>
<td></td>
<td>512</td>
<td>42.3330</td>
<td>0.9988</td>
<td>0.9969</td>
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<td></td>
<td>768</td>
<td>41.0620</td>
<td>0.9982</td>
<td>0.9958</td>
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<tr>
<td>House</td>
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<td>40.2097</td>
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<td>0.9938</td>
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<td>36.0883</td>
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<td>0.9842</td>
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<td>Zelda</td>
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<td>0.9991</td>
<td>0.9967</td>
</tr>
<tr>
<td></td>
<td>512</td>
<td>41.1350</td>
<td>0.9983</td>
<td>0.9940</td>
</tr>
<tr>
<td></td>
<td>768</td>
<td>40.4958</td>
<td>0.9974</td>
<td>0.9931</td>
</tr>
</tbody>
</table>
Figure 4.4: Various standard test images (a) Lena; (b) Mandrill; (c) Barbara; (d) Airplane; (e) Boat; (f) Goldhill; (g) House; (h) Man; (i) Zelda.

It is evident from the PSNR values at different capacity that our scheme provides good visual quality of watermarked image.

The values of UIQI and SSIM for proposed scheme are also shown in Table 4.1. For proposed scheme, the values of UIQI between original images and watermarked images are almost near to 1 which shows the effectiveness of the proposed scheme. SSIM values near to 1 clearly indicate that the structural information is almost preserved even after watermark embedding.
4.5 Conclusions

A new and effective scheme on reversible watermarking is presented in this chapter, to deal with the extraction of watermark and also the recovery and security of the original image. The scheme discussed in this chapter is based on the concept of discrete wavelet transform and singular values of high frequency subbands. The scheme is tested on various images and experimental results are obtained which are visually good and PSNR values are also high. The high PSNR values at maximum embedding capacity indicate that our proposed scheme is good enough to produce high quality watermarked images. UIQI values for proposed scheme also show that the degradation is almost negligible in watermarked images. Proposed scheme provides a strong reversible feature of singular values that is used in proposed extraction algorithm to get back the original singular values for recovering the original image exactly. Two layers of embedding and salient algebraic property of the singular values provides very strong aspect of security. Properties of SVD are still not exploited to its fullest in the area of lossless data hiding and more work can be done on this aspect in future. Location map can be used to eliminate the restriction of using specific column(s) or row(s) at the time of watermark embedding in an image.

When conventional wavelet transform is used, it may not be fully reversible in some cases. For example, if conventional wavelet transform is used to decompose an image, it may be possible that wavelet coefficients after watermark embedding are in floating point. There may be loss of information if any floating point value is
truncated and thus, the cover image would not be recovered bit by bit. Integer-to-
Integer Wavelet Transform (IWT) can provide solution for this issue and therefore,
in next chapter, a novel reversible data hiding technique using IWT is presented.