CHAPTER 6
DETECTION OF BURR TYPE XII RELIABLE SOFTWARE USING SPRT

The data or software is getting increased in the internet day by day, and a vibrant research is going on towards the software reliability. In this regard, there is a need for the people to have the tools/mechanism to detect whether the software is reliable or not. Several methods came into existence to assess the software reliability. In Classical Hypothesis testing, it takes more time to come to a decision because the conclusions can be drawn only after collecting the large amounts of data. But by adopting Sequential Analysis of Statistical science it can be decided very quickly whether the software which is developed is reliable or unreliable. To implement this, Sequential Probability Ratio Test (SPRT) is applied for Burr Type XII model.

It is designed for continuous monitoring. The likelihood based SPRT proposed by Wald is very general and it can be used for many different probability distributions. In the present chapter we propose the performance of SPRT on 5 different data sets using Burr Type XII and analyzed the results. The parameters are estimated using Maximum Likelihood Estimation method. The content of this chapter is published in the following journal and the details are furnished below.

The results of this chapter are accepted for publication.
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6.1 Introduction

Sequential Probability Ratio Test (SPRT), which is usually applied in situations, requires a decision between two simple hypotheses or a single decision point. Wald’s (1947) SPRT procedure has been used to classify the software under test into one of two categories (e.g., reliable/unreliable, pass/fail, certified/noncertified) (Reckase, 1983). Wald's procedure is particularly relevant if the data is collected sequentially. Classical Hypothesis Testing is different from Sequential Analysis. In
Classical Hypothesis testing, the number of cases tested or collected is fixed at the beginning of the experiment. In this method, the analysis is made and conclusions are drawn after collecting the complete data. However, in Sequential Analysis every case is analysed directly. The data collected up to that moment is then compared with threshold values, incorporating the new information taken from the freshly collected case. This approach makes one to draw conclusions during the data collection, and ultimate conclusion can be reached at a much earlier stage. Data collection can be terminated after few cases and decisions can be taken quickly. This leads to saving in terms of cost and human life.

In the analysis of software failure data, either TBFs or failure count in a given time interval is dealt with. If it is further assumed that the average number of recorded failures in a given time interval is directly proportional to the length of the interval and the random number of failure occurrences in the interval is explained by a Poisson process. Then it is known that the probability equation of the stochastic process representing the failure occurrences is given by a Homogeneous Poisson Process with the expression

\[ P[N(t) = n] = \frac{e^{-\lambda t} (\lambda t)^n}{n!} \]  

(6.1.1)

Stieber (1997) observes that, the application of SRGMs may be difficult and reliability predictions can be misleading, if classical testing strategies are used. However, he observes that statistical methods can be successfully applied to the failure data. He demonstrated his observation by applying the well-known sequential probability ratio test of Wald (1947) for a software failure data to detect unreliable software components and compare the reliability of different software versions. In this chapter we consider the popular SRGM - a three parameter Burr Type XII model and adopt the principle of Stieber(1997) in detecting unreliable software in order to accept or reject the developed software. The theory proposed by Stieber(1997) is presented in Section 6.2 for a ready reference. Extension of this theory to the considered SRGM is presented in Section 6.3. Maximum Likelihood method is used for parameter estimation and is presented in Section 6.4. Application of the decision rule to detect unreliable software with reference to the SRGM - Burr type XII is given in Section 6.5.
6.2 Wald’s Sequential Test for a Poisson Process

A. Wald, developed the SPRT at Columbia University in 1943. A big advantage of sequential tests is that they require fewer observations (time) on the average than fixed sample size tests. SPRT is widely used for statistical quality control in manufacturing processes. The SPRT for Homogeneous Poisson Processes is described below.

Let \( \{N(t), t \geq 0\} \) be a homogeneous Poisson process with rate ‘\( \lambda \)’. In this case, \( N(t) \) = number of failures up to time ‘\( t \)’ and ‘\( \lambda \)’ is the failure rate (failures per unit time). If the system is put on test (for example a software system, where testing is done according to a usage profile and faults are corrected) and that if we want to estimate its failure rate ‘\( \lambda \)’. We cannot expect to estimate ‘\( \lambda \)’ precisely. But we want to reject the system with a high probability if the data suggest that the failure rate is larger than \( \lambda_1 \) and accept it with a high probability, if it is smaller than \( \lambda_0 \). As always with statistical tests, there is some risk to get the wrong answers. So we have to specify two (small) numbers ‘\( \alpha \)’ and ‘\( \beta \)’, where ‘\( \alpha \)’ is the probability of falsely rejecting the system. That is rejecting the system even if \( \lambda \leq \lambda_0 \). This is the “producer’s” risk. ‘\( \beta \)’ is the probability of falsely accepting the system. That is accepting the system even if \( \lambda \geq \lambda_1 \). This is the “consumer’s” risk. Wald’s classical SPRT is very sensitive to the choice of relative risk required in the specification of the alternative hypothesis. With the classical SPRT, tests are performed continuously at every time point \( t > 0 \) as additional data are collected. With specified choices of \( \lambda_0 \) and \( \lambda_1 \) such that \( 0 < \lambda_0 < \lambda_1 \), the probability of finding \( N(t) \) failures in the time span \( (0, t) \) with \( \lambda_1, \lambda_0 \) as the failure rates are respectively given by

\[
P_1 = \frac{e^{-\lambda_1 t}[\lambda_1 t]^{N(t)}}{N(t)!}
\]

\[
P_0 = \frac{e^{-\lambda_0 t}[\lambda_0 t]^{N(t)}}{N(t)!}
\]

The ratio \( \frac{P_1}{P_0} \) at any time ‘\( t \)’ is considered as a measure of deciding the truth towards \( \lambda_0 \) or \( \lambda_1 \), given a sequence of time instants say \( t_1 < t_2 < \cdots < t_k \) and the
corresponding realizations $N(t_1), N(t_2) ... N(t_k)$ of $N(t)$. Simplification of $\frac{P_1}{P_0}$ gives

$$\frac{P_1}{P_0} = \exp(\lambda_0 - \lambda_1) t + \left[ \frac{\lambda_1}{\lambda_0} \right]^{N(t)}$$

The decision rule of SPRT is to decide in favour of $\lambda_1$, in favour of $\lambda_0$ or to continue by observing the number of failures at a later time than 't' according as $\frac{P_1}{P_0}$ is greater than or equal to a constant say A, less than or equal to a constant say B or in between the constants A and B. That is, we decide the given software product as unreliable, reliable or continue (Satya Prasad 2007) the test process with one more observation in failure data, according to

$$\frac{P_1}{P_0} \geq A \quad (6.2.3)$$

$$\frac{P_1}{P_0} \leq B \quad (6.2.4)$$

$$B < \frac{P_1}{P_0} < A \quad (6.2.5)$$

The approximate values of the constants A and B are taken as

$$A \approx \frac{1-\beta}{\alpha}, \quad B \approx \frac{\beta}{1-\alpha}$$

Where ‘$\alpha$ ’ and ‘$\beta$ ’ are the risk probabilities as defined earlier. A simplified version of the above decision processes is to reject the system as unreliable if $N(t)$ falls for the first time above the line

$$N_U(t) = at + b_2 \quad (6.2.6)$$

To accept the system to be reliable if $N(t)$ falls for the first time below the line

$$N_L(t) = at - b_1 \quad (6.2.7)$$
To continue the test with one more observation on \([t, N(t)]\) as the random graph of \([t, N(t)]\) is between the two linear boundaries given by Equations (6.2.6) and (6.2.7) where

\[
a = \frac{\lambda_1 - \lambda_0}{\log\left(\frac{\lambda_1}{\lambda_0}\right)}
\]

(6.2.8)

\[
b_1 = \frac{\log\left(\frac{1-\alpha}{\beta}\right)}{\log\left(\frac{\lambda_1}{\lambda_0}\right)}
\]

(6.2.9)

\[
b_2 = \frac{\log\left(\frac{1-\beta}{\alpha}\right)}{\log\left(\frac{\lambda_1}{\lambda_0}\right)}
\]

(6.2.10)

The parameters \(\alpha, \beta, \lambda_0\) and \(\lambda_1\) can be chosen in several ways. One way suggested by Stieber (1997) is

\[
\lambda_0 = \frac{\lambda \log(q)}{q-1}
\]

\[
\lambda_1 = q \frac{\lambda \log q}{q-1} \quad \text{where} \quad q = \frac{\lambda_1}{\lambda_0}
\]

If \(\lambda_0\) and \(\lambda_1\) are chosen in this way, the slope of \(N_U(t)\) and \(N_L(t)\) equals \(\lambda\). The other two ways of choosing \(\lambda_0\) and \(\lambda_1\) are from past projects (for a comparison of the projects) and from part of the data to compare the reliability of different functional areas (components).
6.3 Sequential Probability Ratio Test for Burr Type XII SRGM

In Section 6.2, for the Poisson process we know that the expected value of \( N(t) = \lambda(t) \) called the average number of failures experienced in time \( t \) . This is also called the mean value function of the Poisson process. On the other hand if we consider a Poisson process with a general function (not necessarily linear) \( m(t) \) as its mean value function the probability equation of such a process is

\[
P[N(t) = Y] = \frac{[m(t)]^y}{y!} e^{-m(t)}, y = 0,1,2 \ldots
\]

Depending on the forms of \( m(t) \) we get various Poisson processes called NHPP, for our Burr type XII model. The mean value function is given as

\[
m(t) = a \left[ 1 - \left( 1 + t^c \right)^{-b} \right], \quad t \geq 0
\]

We may write

\[
P_1 = \frac{e^{-m_1t}[m_1t]^{N(t)}}{N(t)!}
\]

\[
P_0 = \frac{e^{-m_0t}[m_0t]^{N(t)}}{N(t)!}
\]

Where \( m_1(t), m_0(t) \) are values of the mean value function at specified sets of its parameters indicating reliable software and unreliable software respectively. The mean value function \( m(t) \) contains the parameters ‘\( a, b \) and \( c \)’. Let \( P_0, P_1 \) be values of the NHPP at two specifications of \( b \) say \( b_0, b_1 \) where \( b_0 < b_1 \) and two specifications of \( c \) say \( c_0, c_1 \) where \( c_0 < c_1 \). It can be shown that for our model \( m(t) \) at \( b_1 \) is greater than that at \( b_0 \) and \( m(t) \) at \( c_1 \) is greater than that at \( c_0 \). symbolically \( m_0(t) < m_1(t) \). Then the SPRT procedure is as follows:

Accept the system to be Reliable if \( \frac{P_1}{P_0} \leq B \)

i.e.,

\[
\frac{e^{-m_1(t)}[m_1(t)]^{N(t)}}{e^{-m_0(t)}[m_0(t)]^{N(t)}} \leq B
\]
i.e., \( N(t) \leq \frac{\log\left(\frac{\beta}{1-\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \) \hspace{1cm} (6.3.1)

Decide the system to be unreliable and Reject if \( \frac{p_1}{p_0} \geq A \)

i.e., \[ \frac{e^{-m_1(t)} \left[m_1(t)\right]^{N(t)}}{e^{-m_0(t)} \left[m_0(t)\right]^{N(t)}} \geq A \]

i.e., \( N(t) \geq \frac{\log\left(\frac{1-\beta}{\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \) \hspace{1cm} (6.3.2)

Continue the test procedure as long as

\[ \frac{\log\left(\frac{\beta}{1-\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} < N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \] \hspace{1cm} (6.3.3)

Substituting the appropriate expressions of the respective mean value function \( m(t) \), we get the respective decision rules and are given in followings lines.

Acceptance Region:

\[ N(t) \leq \frac{\log \left( \frac{\beta}{(1-\alpha)} \right) + a \left[ \left(1 + t^a\right)^{-h} - \left(1 + t^b\right)^{-h} \right]}{\log a \left[ \left(1 + t^a\right)^{-h} \right] \left[ \left(1 + t^b\right)^{-h} \right]} \] \hspace{1cm} (6.3.4)
Rejection Region:

\[
N(t) \geq \log \left( \frac{1 - \beta}{\alpha} \right) + a \left[ \left( 1 + t_0^s \right)^{-b_0} - \left( 1 + t_1^s \right)^{-b_1} \right] - \log a \left[ \left( 1 + t_0^s \right)^{-c_0} - \left( 1 + t_1^s \right)^{-c_1} \right]
\]  

(6.3.5)

Continuation Region:

\[
\log \left( \frac{\beta}{1 - \alpha} \right) + a \left[ \left( 1 + t_0^s \right)^{-b_0} - \left( 1 + t_1^s \right)^{-b_1} \right] < N(t) < \log \left( \frac{1 - \beta}{\alpha} \right) + a \left[ \left( 1 + t_0^s \right)^{-b_0} - \left( 1 + t_1^s \right)^{-b_1} \right] - \log a \left[ \left( 1 + t_0^s \right)^{-c_0} - \left( 1 + t_1^s \right)^{-c_1} \right] < \log \left( \frac{\beta}{1 - \alpha} \right) + a \left[ \left( 1 + t_0^s \right)^{-b_0} - \left( 1 + t_1^s \right)^{-b_1} \right]
\]  

(6.3.6)

It may be noted that in the proposed model the decision rules are exclusively based on the strength of the sequential procedure \((\alpha, \beta)\) and the values of the respective mean value functions namely, \(m_0(t), m_1(t)\). If the mean value function is linear in 't' passing through origin, that is, \(m(t) = \lambda t\) the decision rules become decision lines as described by Stieber (1997). In that sense Equations (6.3.1), (6.3.2) and (6.3.3) can be regarded as generalizations to the decision procedure of Stieber (1997).

The applications of these results for live software failure data are presented with analysis in Section 6.4.

### 6.4 SPRT Analysis of Live Datasets

In this section, the SPRT methodology is applied on five different data sets that are borrowed from Pham (2006), Xie et al. (2002) and SONATA software services. The decisions are evaluated based on the considered mean value function. Based on the estimates of the parameter 'b' in each mean value function, we have chosen the specifications of \(b_0 = b - \delta\), \(b_1 = b + \delta\) and \(c_0 = c - \delta\), \(c_1 = c + \delta\) and apply SPRT
such that $b_0 < b < b_1$ and $c_0 < c < c_1$. Assuming the $\delta$ value is 0.5, the choices are given in the following Table 6.4.1.

Table 6.4.1. Estimates of $a, b, c$ & specifications of $b_0, b_1, c_0, c_1$

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Estimate of ‘a’</th>
<th>Estimate of ‘b’</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>Estimate of ‘c’</th>
<th>$c_0$</th>
<th>$c_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xie</td>
<td>30.040800</td>
<td>0.999825</td>
<td>0.499825</td>
<td>1.499825</td>
<td>0.999619</td>
<td>0.499619</td>
<td>1.499619</td>
</tr>
<tr>
<td>AT &amp; T</td>
<td>22.032465</td>
<td>0.999859</td>
<td>0.499859</td>
<td>1.499859</td>
<td>0.999611</td>
<td>0.499611</td>
<td>1.499611</td>
</tr>
<tr>
<td>IBM</td>
<td>15.051045</td>
<td>0.999530</td>
<td>0.499530</td>
<td>1.499530</td>
<td>0.999196</td>
<td>0.499196</td>
<td>1.499196</td>
</tr>
<tr>
<td>NTDS</td>
<td>26.105273</td>
<td>0.998899</td>
<td>0.498899</td>
<td>1.498899</td>
<td>0.998903</td>
<td>0.498903</td>
<td>1.498903</td>
</tr>
<tr>
<td>SONATA</td>
<td>30.016391</td>
<td>0.999958</td>
<td>0.499958</td>
<td>1.499958</td>
<td>0.999920</td>
<td>0.499920</td>
<td>1.499920</td>
</tr>
</tbody>
</table>

Using the specification $b_0, b_1$ and $c_0, c_1$ the mean value functions $m_0(t)$ and $m_1(t)$ are computed for each ‘t’. Later the decisions are made based on the decision rules specified by the Equations (6.3.4) and (6.3.5) for the data sets. At each ‘t’ of the data set, the strengths ($\alpha, \beta$) are considered as (0.05, 0.2). SPRT procedure is applied on five different data sets and the necessary calculations are given in the Table 6.4.2.

Table 6.4.2. SPRT Analysis for 5 data sets

<table>
<thead>
<tr>
<th>Data Set</th>
<th>$T$</th>
<th>$N(t)$</th>
<th>R.H.S. of Equation (6.3.4) Acceptance region ($\leq$)</th>
<th>R.H.S. of Equation (6.3.5) Rejection region ($\geq$)</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xie</td>
<td>30.02</td>
<td>1</td>
<td>0.003606</td>
<td>0.005130</td>
<td>Rejection</td>
</tr>
<tr>
<td>AT &amp; T</td>
<td>5.50</td>
<td>1</td>
<td>0.115010</td>
<td>0.164486</td>
<td>Rejection</td>
</tr>
<tr>
<td>IBM</td>
<td>10</td>
<td>1</td>
<td>0.023281</td>
<td>0.040821</td>
<td>Rejection</td>
</tr>
<tr>
<td>NTDS</td>
<td>9</td>
<td>1</td>
<td>0.047184</td>
<td>0.065189</td>
<td>Rejection</td>
</tr>
<tr>
<td>SONATA</td>
<td>52.50</td>
<td>1</td>
<td>0.001000</td>
<td>0.001499</td>
<td>Rejection</td>
</tr>
</tbody>
</table>
6.5 Conclusion

The SPRT methodology for the proposed software reliability growth model Burr type XII is applied for the software failure data sets IBM (2002), NTDS (2005), Xie (2002), SONATA (2010), S2 (1996) and AT &T (2005) is illustrated in Table 6.4.2. Hence, it is observed that we are able to come to a conclusion in less time regarding the reliability or unreliability of a software product. The results exemplifies that the model has given a decision of rejection for all the data sets at various time instant of the data. Therefore, we may conclude that, applying SPRT on different data sets we can come to an early conclusion of reliable / unreliable of software.