A NEW DECISION MAKING MODEL FOR MAGDM WITH UNKNOWN ATTRIBUTES WEIGHT INFORMATION
5.1. INTRODUCTION AND LITERATURE REVIEW

Most of the real world decision making problems involve more than one DM. When attribute weights are unknown prior, each DM may have different perception regarding the weights of attributes. Consequently their assignment of alternatives performance against each attribute will differ. If the weights of attributes are obtained using performance of alternatives, assigning unique attribute weights, may lead to wrong identification of best alternative.

Several authors have studied MAGDM problems of unknown criterion weight information with IFSs, IVIFSs, IVTFSs and IVTrIFSs. Chen (2012) presented a method of MAGDM when the information of performance of alternatives is given in IVTFNs. Signed distance measure is used to order and compare the alternatives. Further, they developed an integrated programming model to estimate criterion weights, when the weights of criterion is partially known. Liu and Fin (2012) presented a MCGDM method when criteria values information is represented in generalized interval valued trapezoidal fuzzy numbers (GITFNs). They have defined weighted geometric, ordered weighted geometric and hybrid geometric aggregation operators on GITFNs and utilized them for aggregation in the process of decision making. Tan and Chen (2013) presented an approach for MCGDM based on VIKOR and Choquet integral when performance of alternatives is represented by IVIFNs. Additionally, they studied the MCGDM problems when the interactive criteria are computed by shapely value. Chen et al., (2013) defined new entropy measure using cotangent function for IVIFSs. They applied to find weights of criteria and solved MCGDM with unknown weight information.

Only a few researchers have studied Multi attribute Group Decision Making (MAGDM) problems with IVTIFNs. Jiang and Wang (2014) introduced new distance measure between two interval valued intuitionistic trapezoidal fuzzy matrices and investigated an approach to MCGDM with IVTIFNs when the expert weights are unknown based on interval valued trapezoidal intuitionistic fuzzy ordered weighted geometric (IVTIFOWG) and interval valued trapezoidal intuitionistic fuzzy hybrid geometric (IVTIFHG) operators for decision making. Wu and Liu (2013) obtained the criteria weights using
normal distribution given by Xu (2005) and found the best alternative using IVITFOWG operator. But, this method assigns the weights depending on number of attributes involved without considering the alternatives performance against each attribute.

Another issue that should be taken care of is aggregation of data. The aggregation operators IVTIFOWG, IVTIFHG, IVTIFOWG are based on algebraic operational laws of IVTIFNs to aggregate the data. Deschrijver and Kerre (2002) defined union, intersection, sum and product of IFNs using generalized triangular norms (t-norm) and triangular conorm (t-conorm). Using general t-norm and t-conorm, they analyzed the properties of generalized union and intersections of IFSs and proved the idempotency, comutativity, absorption laws, associativity, disributivity and De Morgan properties for these defined operators. Later on, generalized union and intersection of IVIFNs is constructed using general t-norm and t-conorm. Thus, “various t-norms and t-conorms families can be used to perform the corresponding intersections and unions of IVIFNs”. The Einstein t-norms and product are two classical examples of the class of strict Archimedian t-norms (Klement et al., 2000 & 2004). Hence, for an intersection and union of IVIFNs, “a better alternative to the algebraic product and sum is the Einstein product and Einstein sum”, as it gives the same estimates as the algebraic product and algebraic sum. Wang and Liu (2011, 2012) defined Einstein operations on IVIFNs and developed the interval-valued intuitionistic fuzzy Einstein weighted averaging operator (IVIFEWA), interval-valued intuitionistic fuzzy Einstein ordered weighted averaging (IVIFEOWA) operator, interval valued intuitionistic fuzzy Einstein weighted geometric (IVIFEWG) operator, interval valued intuitionistic fuzzy Einstein ordered weighted geometric (IVIFEOWG) operator, for aggregating IVIF information. Later, Wang and Liu (2013) proposed the interval valued intuitionistic fuzzy Einstein hybrid weighted averaging (IVIFEHWA) operator and interval valued intuitionistic fuzzy Einstein hybrid weighted geometric (IVIFEHWG) operator, and applied to MCDM problems with IVIF information.

Accordingly, in Chapter V a method to find the weights of attributes based on alternatives performance against each attribute is defined. A decision making model for MAGDM problem when information is given in IVTIF form is constructed by taking into consideration that the attribute weights of each decision maker may be different with
respect to each other. Further, keeping the importance of Einstein aggregation in view, Einstein operation laws and Einstein weighted averaging operator on IVTIFNs is defined. The idempotency, monotonicity, commutativity, boundedness properties of interval valued trapezoidal intuitionistic fuzzy Einstein weighted averaging (IVTIFEW) operator are proved. An illustrative example is provided and comparison is done. Further, a case study: Selection of location to set up Software Company is done using developed method.

5.2. WU AND LIU METHOD OF MAGDM WITH UNKNOWN ATTRIBUTE WEIGHTS

The method of Wu and Liu (2013) for MAGDM when the weights of attribute are completely unknown in advance is presented in step wise.

**Step 1:** Obtain the collective overall preference \( \tilde{t}_{Al} \) of each alternative \( Al \) against each attribute using IVTIFWG operator using Eq. (1.6) and experts weighting vector \( w_{di} \).

**Step 2:** Calculate the weighting vector of IVTIFOWG operator defined in 1.4.4 based on the method given by (Xu, 2005).

**Step 3:** Derive the individual overall preference of each alternatives \( \tilde{A}_i \), \( i = 1, 2, 3, ..., n \), utilizing the information given in the matrix \( \tilde{D} = [\tilde{d}_{ij}]_{m \times n} \), the IVTIFOWG operator and its associated weighting vector \( w = (w_1, w_2, ..., w_n)^T \), \( \tilde{t}_{Al} = IVTIFOWG(\tilde{d}_{i1}, \tilde{d}_{i2}, ..., \tilde{d}_{in}), i = 1, 2, ..., m \)

**Step 4:** Calculate score \( I(S_E(\tilde{t}_{Al})) \) and accurate \( I(H_E(\tilde{t}_{Al})) \) \( (i = 1, 2, 3, ..., n) \) using Eq. (2.2), Eq.(2.3)

**Step 5:** Rank the alternatives \( \tilde{A}_i \) using Eq. (2.4) to select the best alternative.
5.3. **A NEW DECISION MAKING MODEL FOR MAGDM WITH UNKNOWN ATTRIBUTES WEIGHT INFORMATION**

Aggregation of data is a vital step in solving MAGDM problems. In literature it is stated that Einstein operational laws can better represent the union and intersection of fuzzy numbers. Hence, we define the Einstein Operation Laws and Einstein Aggregation Operator for IVTIFNs.

**5.3.1. Einstein Operation laws of IVTIFNs:**

Let \( \tilde{P}_1 = \{(p_1, q_1, r_1, s_1); [\mu^L_{P_1}, \mu^U_{P_1}]; [\nu^L_{P_1}, \nu^U_{P_1}] \} \) and \( \tilde{P}_2 = \{(p_2, q_2, r_2, s_2); [\mu^L_{P_2}, \mu^U_{P_2}]; [\nu^L_{P_2}, \nu^U_{P_2}] \} \) be two IVTIFNs, then

\[
\tilde{P}_1 \oplus_E \tilde{P}_2 = \left[ \frac{p_1 + p_2}{1 + p_1 \cdot p_2}, \frac{q_1 + q_2}{1 + q_1 \cdot q_2}, \frac{r_1 + r_2}{1 + r_1 \cdot r_2}, \frac{s_1 + s_2}{1 + s_1 \cdot s_2} \right] \cdot \left[ \frac{\mu^L_{P_1} + \mu^L_{P_2}}{1 + \mu^L_{P_1} \cdot \mu^L_{P_2}}, \frac{\mu^U_{P_1} + \mu^U_{P_2}}{1 + \mu^U_{P_1} \cdot \mu^U_{P_2}} \right] \cdot
\]

\[
\tilde{P}_1 \otimes_E \tilde{P}_2 = \left[ \frac{p_1 \cdot p_2}{1 + (1 - p_1) \cdot (1 - p_2)}, \frac{q_1 \cdot q_2}{1 + (1 - q_1) \cdot (1 - q_2)}, \frac{r_1 \cdot r_2}{1 + (1 - r_1) \cdot (1 - r_2)}, \frac{s_1 \cdot s_2}{1 + (1 - s_1) \cdot (1 - s_2)} \right] \cdot \left[ \frac{\mu^L_{P_1} \cdot \mu^L_{P_2}}{1 + (1 - \mu^L_{P_1}) \cdot (1 - \mu^L_{P_2})}, \frac{\mu^U_{P_1} \cdot \mu^U_{P_2}}{1 + (1 - \mu^U_{P_1}) \cdot (1 - \mu^U_{P_2})} \right] \cdot
\]

\[
\tilde{P}_1 = \left[ \frac{(1 + p_1)^{\lambda} - (1 - p_1)^{\lambda}}{1 + (1 - p_1)^{\lambda}}, \frac{(1 + q_1)^{\lambda} - (1 - q_1)^{\lambda}}{1 + (1 - q_1)^{\lambda}}, \frac{(1 + r_1)^{\lambda} - (1 - r_1)^{\lambda}}{1 + (1 - r_1)^{\lambda}}, \frac{(1 + s_1)^{\lambda} - (1 - s_1)^{\lambda}}{1 + (1 - s_1)^{\lambda}} \right] \cdot
\]

\[
\tilde{P}_1 \circ \tilde{P}_2 = \left[ \frac{(1 + \mu^L_{P_1})^{\lambda} - (1 - \mu^L_{P_1})^{\lambda}}{1 + (1 - \mu^L_{P_1})^{\lambda}}, \frac{(1 + \mu^U_{P_1})^{\lambda} - (1 - \mu^U_{P_1})^{\lambda}}{1 + (1 - \mu^U_{P_1})^{\lambda}}, \frac{2(\nu^L_{P_1})^{\lambda}}{2 - (\nu^L_{P_1})^{\lambda}}, \frac{2(\nu^U_{P_1})^{\lambda}}{2 - (\nu^U_{P_1})^{\lambda}} \right] ;
\]

where \( \lambda \) is the weighting exponent.
\[
\begin{pmatrix} 
2(p_i)^2 \\
2(q_i)^2 \\
2(r_i)^2 \\
2(s_i)^2 \\
\end{pmatrix} = \begin{pmatrix} 
\frac{2(p_i)^2}{(2-p_i)^2 + (p_i)^2} & \frac{2(q_i)^2}{(2-q_i)^2 + (q_i)^2} & \frac{2(r_i)^2}{(2-r_i)^2 + (r_i)^2} & \frac{2(s_i)^2}{(2-s_i)^2 + (s_i)^2} \\
\frac{2(\mu^L_{\tilde{P}})^2}{(2-\mu^L_{\tilde{P}})^2 + (\mu^L_{\tilde{P}})^2} & \frac{2(\mu^U_{\tilde{P}})^2}{(2-\mu^U_{\tilde{P}})^2 + (\mu^U_{\tilde{P}})^2} & \frac{(1+\nu^L_{\tilde{P}})^2}{(1-\nu^L_{\tilde{P}})^2} & \frac{(1-\nu^U_{\tilde{P}})^2}{(1+\nu^U_{\tilde{P}})^2} \\
\frac{(1+\nu^L_{\tilde{P}})^2}{(1-\nu^L_{\tilde{P}})^2} & \frac{(1-\nu^U_{\tilde{P}})^2}{(1+\nu^U_{\tilde{P}})^2} & \frac{(1+\nu^L_{\tilde{P}})^2}{(1-\nu^L_{\tilde{P}})^2} & \frac{(1-\nu^U_{\tilde{P}})^2}{(1+\nu^U_{\tilde{P}})^2} \\
\end{pmatrix}
\] (5.4)

Where \( \lambda \) is any scalar with \( \lambda > 0 \).

5.3.2. Einstein Weighted Arithmetic Aggregation Operators of IVTIFNs (IVTIFEWAA)

Let \( \omega \) be the set of IVTIFNs and \( \tilde{P}_i \in \omega, i = 1, 2, 3,...,n \). An interval-valued intuitionistic trapezoidal fuzzy Einstein weighted averaging operator, IVTIFEWAA: \( \omega^w \rightarrow \omega \) is defined as

\[
IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3,...,\tilde{P}_n) = \bigoplus_{i=1}^{n} (w_i \tilde{P}_i)
\]

\[
= \begin{pmatrix} 
\prod_{i=1}^{n} (1+p_i)^{w_i} - \prod_{i=1}^{n} (1-p_i)^{w_i} \\
\prod_{i=1}^{n} (1-p_i)^{w_i} + \prod_{i=1}^{n} (1-p_i)^{w_i} \\
\prod_{i=1}^{n} (1+q_i)^{w_i} - \prod_{i=1}^{n} (1-q_i)^{w_i} \\
\prod_{i=1}^{n} (1-q_i)^{w_i} + \prod_{i=1}^{n} (1-q_i)^{w_i} \\
\prod_{i=1}^{n} (1+r_i)^{w_i} - \prod_{i=1}^{n} (1-r_i)^{w_i} \\
\prod_{i=1}^{n} (1-r_i)^{w_i} + \prod_{i=1}^{n} (1-r_i)^{w_i} \\
\prod_{i=1}^{n} (1+s_i)^{w_i} - \prod_{i=1}^{n} (1-s_i)^{w_i} \\
\prod_{i=1}^{n} (1+s_i)^{w_i} + \prod_{i=1}^{n} (1-s_i)^{w_i} \\
\end{pmatrix}
\]

\[
= \begin{pmatrix} 
\prod_{i=1}^{n} (1+\mu^L_{\tilde{P}})^{w_i} - \prod_{i=1}^{n} (1-\mu^L_{\tilde{P}})^{w_i} \\
\prod_{i=1}^{n} (1-\mu^L_{\tilde{P}})^{w_i} + \prod_{i=1}^{n} (1-\mu^L_{\tilde{P}})^{w_i} \\
\prod_{i=1}^{n} (1+\mu^U_{\tilde{P}})^{w_i} - \prod_{i=1}^{n} (1-\mu^U_{\tilde{P}})^{w_i} \\
\prod_{i=1}^{n} (1-\mu^U_{\tilde{P}})^{w_i} + \prod_{i=1}^{n} (1-\mu^U_{\tilde{P}})^{w_i} \\
\prod_{i=1}^{n} (1+\nu^L_{\tilde{P}})^{w_i} - \prod_{i=1}^{n} (1-\nu^L_{\tilde{P}})^{w_i} \\
\prod_{i=1}^{n} (1-\nu^L_{\tilde{P}})^{w_i} + \prod_{i=1}^{n} (1-\nu^L_{\tilde{P}})^{w_i} \\
\prod_{i=1}^{n} (1+\nu^U_{\tilde{P}})^{w_i} - \prod_{i=1}^{n} (1-\nu^U_{\tilde{P}})^{w_i} \\
\prod_{i=1}^{n} (1-\nu^U_{\tilde{P}})^{w_i} + \prod_{i=1}^{n} (1-\nu^U_{\tilde{P}})^{w_i} \\
\end{pmatrix}
\]

\[
= \begin{pmatrix} 
2 \prod_{i=1}^{n} (\nu^L_{\tilde{P}})^{w_i} \\
2 \prod_{i=1}^{n} (\nu^L_{\tilde{P}})^{w_i} \\
2 \prod_{i=1}^{n} (\nu^L_{\tilde{P}})^{w_i} \\
2 \prod_{i=1}^{n} (\nu^L_{\tilde{P}})^{w_i} \\
2 \prod_{i=1}^{n} (\nu^L_{\tilde{P}})^{w_i} \\
2 \prod_{i=1}^{n} (\nu^L_{\tilde{P}})^{w_i} \\
2 \prod_{i=1}^{n} (\nu^L_{\tilde{P}})^{w_i} \\
2 \prod_{i=1}^{n} (\nu^L_{\tilde{P}})^{w_i} \\
\end{pmatrix}
\]

where \( w = (w_1, w_2,...,w_n)^T \) be the weight vector of \( \tilde{P}_i, i = 1, 2, 3,...,n \), \( w_i \geq 0, \sum_{i=1}^{n} w_i = 1 \).

5.3.3. Properties of IVTIFEWAA

In this section, the desirable properties of IVTIFEWAA are proved.
5.3.3.1. Idempotency

If \( w=\left(w_1, w_2, ..., w_n\right)^T \) be the weight vector of \( \tilde{P}_i, i=1, 2, 3, ..., n, \) \( w_i \geq 0, \sum_{i=1}^{n} w_i = 1. \) With all \( \tilde{P}_i = \tilde{P} \) where

\[
\tilde{P} = \left\{ [p, q, r, s]; \left[ \mu^L, \mu^U \right]; [v^L, v^U] \right\}
\]
and \( \tilde{P}_i = \left\{ [p_i, q_i, r_i, s_i]; \left[ \mu^L, \mu^U \right]; [v^L, v^U] \right\} \) then \( IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2, ..., \tilde{P}_n) = \tilde{P} \)

**Proof:**

For \( i=1, 2 \) the idempotency property is stated as \( IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2) = \tilde{P} \)

with \( \tilde{P}_1 = \tilde{P}_2 = \tilde{P} \) where \( \tilde{P} = \left\{ [p, q, r, s]; \left[ \mu^L, \mu^U \right]; [v^L, v^U] \right\} \)

Now \( IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2) = w_1 \tilde{P}_1 \oplus_E w_2 \tilde{P}_2 \)

Then \( w_1 \tilde{P} = \left\lfloor \left( (1+p)^{w_1} - (1-p)^{w_1}, (1+q)^{w_1} - (1-q)^{w_1}, (1+r)^{w_1} - (1-r)^{w_1}, (1+s)^{w_1} - (1-s)^{w_1} \right) \right\rfloor \)

\[
= \left[ \left( (1+p)^{w_1} - (1-p)^{w_1}, (1+q)^{w_1} - (1-q)^{w_1}, (1+r)^{w_1} - (1-r)^{w_1}, (1+s)^{w_1} - (1-s)^{w_1} \right) \cdot \frac{2(v^L)^{w_1}}{(2-v^L)^{w_1} + (v^L)^{w_1}} \cdot \frac{2(v^U)^{w_1}}{(2-v^U)^{w_1} + (v^U)^{w_1}} \right] \]

And

\[
w_2 \tilde{P} = \left\lfloor \left( (1+p)^{w_2} - (1-p)^{w_2}, (1+q)^{w_2} - (1-q)^{w_2}, (1+r)^{w_2} - (1-r)^{w_2}, (1+s)^{w_2} - (1-s)^{w_2} \right) \right\rfloor \]

\[
= \left[ \left( (1+p)^{w_2} - (1-p)^{w_2}, (1+q)^{w_2} - (1-q)^{w_2}, (1+r)^{w_2} - (1-r)^{w_2}, (1+s)^{w_2} - (1-s)^{w_2} \right) \cdot \frac{2(v^L)^{w_2}}{(2-v^L)^{w_2} + (v^L)^{w_2}} \cdot \frac{2(v^U)^{w_2}}{(2-v^U)^{w_2} + (v^U)^{w_2}} \right] \]
$$\begin{align*}
\tilde{w}_1 \tilde{P} \oplus_E \tilde{w}_2 \tilde{P} &= \left[ \left( \frac{(1 + p)^{w_1+w_2} - (1 - p)^{w_1+w_2}}{(1 + p)^{w_1+w_2} + (1 - p)^{w_1+w_2}}, \frac{(1 + q)^{w_1+w_2} - (1 - q)^{w_1+w_2}}{(1 + q)^{w_1+w_2} + (1 - q)^{w_1+w_2}}, \frac{(1 + r)^{w_1+w_2} - (1 - r)^{w_1+w_2}}{(1 + r)^{w_1+w_2} + (1 - r)^{w_1+w_2}}, \frac{(1 + s)^{w_1+w_2} - (1 - s)^{w_1+w_2}}{(1 + s)^{w_1+w_2} + (1 - s)^{w_1+w_2}} \right) \right] \\
&= \left[ \left( \frac{(1 + p) - (1 - p)}{(1 + p) + (1 - p)}, \frac{(1 + q) - (1 - q)}{(1 + q) + (1 - q)}, \frac{(1 + r) - (1 - r)}{(1 + r) + (1 - r)}, \frac{(1 + s) - (1 - s)}{(1 + s) + (1 - s)} \right) \right] \\
&= \left[ \left( \frac{(1 + \mu_L^L - \mu_L^L)}{(1 + \mu_L^L + 1 - \mu_L^L)}, \frac{(1 + \mu_U^L - \mu_U^L)}{(1 + \mu_U^L + 1 - \mu_U^L)} \right) \right] \left[ \frac{4(\nu_L^L)}{4}, \frac{4(\nu_U^L)}{4} \right] \\
&= \left[ p, q, r, s \right] \left[ \mu_L^L, \mu_U^L \right] \left[ \nu_L^L, \nu_U^L \right] \\
&= \tilde{P}
\end{align*}$$
5.3.3.2. Boundedness:

Let \( \tilde{P}_i, i = 1, 2, 3, \ldots, n \) be a collection of IVTIFNs, and let \( \tilde{P}_i^L = \min_i \tilde{P}_i \) and \( \tilde{P}_i^U = \max_i \tilde{P}_i \)

then \( \tilde{P}_i^L \leq IVTIEWAA_w (\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_n) \leq \tilde{P}_i^U \).

Proof: Let

\[
\tilde{P}_i^L = \min_i \tilde{P}_i \\
= \left\{ \left[ \min_i p_i, \min_i q_i, \min_i r_i, \min_i s_i \right] \left[ \min_i \mu_L^{\tilde{P}_i}, \min_i \mu_U^{\tilde{P}_i} \right] \left[ \max_i \nu_L^{\tilde{P}_i}, \max_i \nu_U^{\tilde{P}_i} \right] \right\} \\
\]

and

\[
\tilde{P}_i^U = \max_i \tilde{P}_i \\
= \left\{ \left[ \max_i p_i, \max_i q_i, \max_i r_i, \max_i s_i \right] \left[ \max_i \mu_L^{\tilde{P}_i}, \max_i \mu_U^{\tilde{P}_i} \right] \left[ \min_i \nu_L^{\tilde{P}_i}, \min_i \nu_U^{\tilde{P}_i} \right] \right\} \\
\]

\( IVTIEWAA_w (\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_n) = \bigoplus_{i=1}^n (w_i \tilde{P}_i) \)

\[
= \left[ \begin{array}{cccc}
\prod_{i=1}^n (1 + p_i) \quad \prod_{i=1}^n (1 - p_i) \\
\prod_{i=1}^n (1 + q_i) \quad \prod_{i=1}^n (1 - q_i) \\
\prod_{i=1}^n (1 + r_i) \quad \prod_{i=1}^n (1 - r_i) \\
\prod_{i=1}^n (1 + s_i) \quad \prod_{i=1}^n (1 - s_i)
\end{array} \right] \\
\]

\[
= \left[ \begin{array}{cccc}
\prod_{i=1}^n (1 + \mu_L^{\tilde{P}_i}) \quad \prod_{i=1}^n (1 - \mu_L^{\tilde{P}_i}) \\
\prod_{i=1}^n (1 + \mu_U^{\tilde{P}_i}) \quad \prod_{i=1}^n (1 - \mu_U^{\tilde{P}_i}) \\
\prod_{i=1}^n (2 - \nu_L^{\tilde{P}_i}) \quad \prod_{i=1}^n (2 - \nu_U^{\tilde{P}_i}) \\
\prod_{i=1}^n (\nu_L^{\tilde{P}_i}) \quad \prod_{i=1}^n (\nu_U^{\tilde{P}_i})
\end{array} \right] \\
\]

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using Eq.(2.6), we have

\[
VI\left(I_{VTIEWAA_w}(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_n)\right) = VI\left(\bigoplus_{i=1}^{n} (w_i \tilde{P}_i)\right)
\]

\[
= \left[ \frac{n \left( \prod_{i=1}^{n} (1 + p_i)^{w_i} - \prod_{i=1}^{n} (1 - p_i)^{w_i} \right)}{\prod_{i=1}^{n} (1 + p_i)^{w_i} + \prod_{i=1}^{n} (1 - p_i)^{w_i}} \right] + 2 \cdot \left[ \frac{n \left( \prod_{i=1}^{n} (1 + q_i)^{w_i} - \prod_{i=1}^{n} (1 - q_i)^{w_i} \right)}{\prod_{i=1}^{n} (1 + q_i)^{w_i} + \prod_{i=1}^{n} (1 - q_i)^{w_i}} + \frac{n \left( \prod_{i=1}^{n} (1 + r_i)^{w_i} - \prod_{i=1}^{n} (1 - r_i)^{w_i} \right)}{\prod_{i=1}^{n} (1 + r_i)^{w_i} + \prod_{i=1}^{n} (1 - r_i)^{w_i}} \right] + \left[ \frac{n \left( \prod_{i=1}^{n} (1 + s_i)^{w_i} - \prod_{i=1}^{n} (1 - s_i)^{w_i} \right)}{\prod_{i=1}^{n} (1 + s_i)^{w_i} + \prod_{i=1}^{n} (1 - s_i)^{w_i}} \right]
\]

\[
= \left[ 1 + S_E\left(I_{VTIEWAA_w}(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_n)\right) - H_E\left(I_{VTIEWAA_w}(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_n)\right) \right]
\]
\[
\prod_{i=1}^{n} \left(1 + p_i\right)^{w_i} - \prod_{i=1}^{n} \left(1 - p_i\right)^{w_i} \right) \right] + 2 \cdot \left[ \prod_{i=1}^{n} \left(1 + q_i\right)^{w_i} - \prod_{i=1}^{n} \left(1 - q_i\right)^{w_i} \right] \right) + \prod_{i=1}^{n} \left(1 + r_i\right)^{w_i} - \prod_{i=1}^{n} \left(1 - r_i\right)^{w_i} \right) \right] + 2 \cdot \prod_{i=1}^{n} \left(1 + s_i\right)^{w_i} - \prod_{i=1}^{n} \left(1 - s_i\right)^{w_i} \right) \right]  
\]  

\[
12  
\]  

\[
1 + \left[ \prod_{i=1}^{n} \left(1 + \mu_i^{L}\right)^{w_i} - \prod_{i=1}^{n} \left(1 - \mu_i^{L}\right)^{w_i} \right] + \prod_{i=1}^{n} \left(1 + \mu_i^{U}\right)^{w_i} - \prod_{i=1}^{n} \left(1 - \mu_i^{U}\right)^{w_i} \right] + 2 \cdot \prod_{i=1}^{n} \left(1 + \nu_i^{L}\right)^{w_i} - \prod_{i=1}^{n} \left(1 - \nu_i^{L}\right)^{w_i} \right] + 2 \cdot \prod_{i=1}^{n} \left(1 + \nu_i^{U}\right)^{w_i} - \prod_{i=1}^{n} \left(1 - \nu_i^{U}\right)^{w_i} \right] \right]  
\]  

\[
4  
\]  

\[
4  
\]  

Similarly,

\[
VI (\tilde{P}^{L}) = VI (\text{min } \tilde{P}_i)  
\]  

\[
= \left[ \min_{i} p_i + 2 \left( \min_{i} q_i + \min_{i} r_i \right) + \min_{i} s_i \right] \left[ 1 + \left( \min_{i} \mu_i^{L} + \min_{i} \mu_i^{U} - \max_{i} \nu_i^{L} + \max_{i} \nu_i^{U} \right) \right] \left( \min_{i} \mu_i^{L} + \min_{i} \mu_i^{U} - \max_{i} \nu_i^{L} + \max_{i} \nu_i^{U} \right) \right]  
\]  

\[
74  
\]
and

\[ VI(\tilde{P}^U) = VI(\max_i \tilde{P}_i) \]

\[ = \left( \frac{\max_i p_i + 2\left( \max_i q_i + \max_i r_i \right) + \max_i s_i}{12} \right) \times \left[ 1 + \left( \frac{\max_i \mu^L_{\tilde{P}_i} + \max_i \mu^U_{\tilde{P}_i} - \min_i \nu^L_{\tilde{P}_i} + \min_i \nu^U_{\tilde{P}_i}}{4} \right) - \left( \frac{\max_i \mu^L_{\tilde{P}_i} + \max_i \mu^U_{\tilde{P}_i} - \min_i \nu^L_{\tilde{P}_i} + \min_i \nu^U_{\tilde{P}_i}}{4} \right) \right] \]

Since

\[ \min_i \mu^L_{\tilde{P}_i} \leq \frac{\prod_{i=1}^{n}(1 + \mu^L_{\tilde{P}_i}^w) - \prod_{i=1}^{n}(1 - \mu^L_{\tilde{P}_i}^w)}{\prod_{i=1}^{n}(1 + \mu^L_{\tilde{P}_i}^w) + \prod_{i=1}^{n}(1 - \mu^L_{\tilde{P}_i}^w)} \leq \max_i \mu^L_{\tilde{P}_i}, \]

\[ 1 - \frac{2\prod_{i=1}^{n}(\min_i \nu^L_{\tilde{P}_i}^w)}{\prod_{i=1}^{n}(2 - \min_i \nu^L_{\tilde{P}_i}^w)} \leq 1 - \frac{2\prod_{i=1}^{n}(\nu^L_{\tilde{P}_i}^w)}{\prod_{i=1}^{n}(2 - \nu^L_{\tilde{P}_i}^w)} \leq 1 - \frac{2\prod_{i=1}^{n}(\max_i \nu^L_{\tilde{P}_i}^w)}{\prod_{i=1}^{n}(2 - \max_i \nu^L_{\tilde{P}_i}^w) + \prod_{i=1}^{n}(\max_i \nu^L_{\tilde{P}_i}^w)} \]

Hence \( VI(\tilde{P}^L) \leq VI(IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_n)) \leq VI(\tilde{P}^U) \)

According to Eq.(2.8)

\( \tilde{P}^L \leq IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_n) \leq \tilde{P}^U \)
5.3.3.3. Monotonicity:

Let \( \tilde{P}_i, \tilde{Q}_i, i = 1, 2, \ldots, m \) be collection of IVTIFNs. If \( \tilde{P}_i \leq \tilde{Q}_i \), then

\[
IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_m) \leq IVTIFEWAA_w(\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \ldots, \tilde{Q}_m)
\]

Proof:

Let \( IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_m) = \bigoplus_{i=1}^{m} (w_i \tilde{P}_i) \)

And \( IVTIFEWAA_w(\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \ldots, \tilde{Q}_m) = \bigoplus_{i=1}^{m} (w_i \tilde{Q}_i) \)

Since \( \tilde{P}_i \leq \tilde{Q}_i \) for all \( i \) we have

\[
IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_m) \leq IVTIFEWAA_w(\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \ldots, \tilde{Q}_m)
\]

5.3.4. Finding Weights of Attributes using Ambiguity index

In this section, an attribute dependent approach of finding criterion weight is presented. Michael and Raul (2013) stated that the inverse of the variance is the weight attached to the variable. As the ambiguity index represents the variance, in this method the attribute weights are derived based on ambiguity index of alternatives performance.

Definition:

For any criteria \( Cr_j \), the weight of the criteria \( w_{Cr_j} \) is defined as,

\[
w_{Cr_j} = \frac{1}{AI(Cr_j)}, j = 1, 2, \ldots, n \quad \text{where} \quad AI(Cr_j) \text{is the ambiguity index of} \ Cr_j
\]

(5.5)

This method assigns a lesser weight to a variable of high variance and vice versa, which is a desirable property of weights in group decision making.
5.3.5. New Multi Attribute Group Decision Making Model

Let \( A_l, A_{l2}, A_{l3}, \ldots, A_{lm} (i = 1,2,\ldots,m) \) be \( m \) possible alternatives and \( C_{r1}, C_{r2}, C_{r3}, \ldots, C_{rn} (j = 1,2,\ldots,n) \) be \( n \) attributes with which alternatives performance is measured by \( k \) experts \( D_1, D_2, D_3, \ldots, D_k \). Let \( \tilde{P}_{ij}^l \) is the performance of alternative \( i \) with respect to criterion \( j \) under decision maker \( l \) and which is expressed as IVTIFN, represented by

\[
\tilde{P}_{ij}^l = \left[ \left[ p_{ij}^l, q_{ij}^l, r_{ij}^l, s_{ij}^l \right], \left[ \mu_{\tilde{p}_{ij}^l}^L, \mu_{\tilde{p}_{ij}^l}^U \right], \left[ \nu_{\tilde{p}_{ij}^l}^L, \nu_{\tilde{p}_{ij}^l}^U \right] \right].
\]

The developed method is given below in step wise.

**Step 1:** Calculate the ambiguity in each \( \tilde{P}_{ij}^l, AI(\tilde{P}_{ij}^l) \) of each alternative \( i \) with respect to each attribute \( j \) under each decision maker \( l \) using Eq. (2.7)

**Step 2:** Find the average ambiguity of attribute \( C_{rij} = \sum_{i=1}^{m} AI(\tilde{P}_{ij}^l), \ j = 1,2,\ldots,n \) with respect to each alternative \( A_{li} \) for each decision maker \( D_j \). Then determine the weight of each attribute \( w_{Crij} \) with respect to each decision maker \( l \) using Eq. (5.5) and normalize the weights.

**Step 3:** Construction of decision matrix

Apply IVTIFEWAA operator for assessments of decision makers on alternative with the criterion weighting vectors to obtain a decision matrix.

Then the decision matrix elements are obtained as follows:

\[
\tilde{d}_1^l = Nw_{C_{r1}}^1 \tilde{p}_{11} \oplus_E Nw_{C_{r2}}^1 \tilde{p}_{12} \oplus_E \ldots \oplus_E Nw_{C_{rn}}^1 \tilde{p}_{1n}
\]

\[
\tilde{d}_2^l = Nw_{C_{r1}}^1 \tilde{p}_{21} \oplus_E Nw_{C_{r2}}^1 \tilde{p}_{22} \oplus_E \ldots \oplus_E Nw_{C_{rn}}^1 \tilde{p}_{2n} \text{ and so on}
\]

\[
\tilde{d}_m^l = Nw_{C_{r1}}^1 \tilde{p}_{m1} \oplus_E Nw_{C_{r2}}^1 \tilde{p}_{m2} \oplus_E \ldots \oplus_E Nw_{C_{rn}}^1 \tilde{p}_{mn}
\]

Hence for each decision maker we obtain
\[ \tilde{d}_m^l = Nw_{C_l}^l \tilde{\Pi}_m \oplus_E Nw_{C_2}^l \tilde{\Pi}_m \oplus_E \cdots \oplus_E Nw_{C_n}^l \tilde{\Pi}_m \] where \( l = 1, 2 \ldots k \).

**Step 4:** Calculate overall preference of alternatives with respect to the decision makers weights using IVITFEWAA operator.

If \( Nw_{d_1}, Nw_{d_2}, Nw_{d_3}, \ldots, Nw_{d_l} \) be the weights of decision makers, then the overall preferences of \( \tilde{d}_m \) alternatives can be calculated as follows:

\[ \tilde{\iota}_{A_1} = Nw_{d_1} \tilde{d}_1^1 \oplus_E Nw_{d_2} \tilde{d}_2^1 \oplus_E Nw_{d_3} \tilde{d}_3^1 \oplus_E \cdots \oplus_E Nw_{d_l} \tilde{d}_l^1 \]

\[ \tilde{\iota}_{A_2} = Nw_{d_1} \tilde{d}_1^2 \oplus_E Nw_{d_2} \tilde{d}_2^2 \oplus_E Nw_{d_3} \tilde{d}_3^2 \oplus_E \cdots \oplus_E Nw_{d_l} \tilde{d}_l^2 \]

\[ \cdots \]

\[ \tilde{\iota}_{A_m} = Nw_{d_1} \tilde{d}_m^1 \oplus_E Nw_{d_2} \tilde{d}_m^2 \oplus_E Nw_{d_3} \tilde{d}_m^3 \oplus_E \cdots \oplus_E Nw_{d_l} \tilde{d}_m^l \]

**Step 5:** Compare the alternatives by finding the overall preference of alternatives obtained in Step 4 using the Eq. (2.8) to choose the best alternative.

**5.3.6. Numerical Example**

The proposed method is applied to the practical problem presented by Wu and Liu (2013) discussed in Chapter III, (3.4.1. Numerical Problem 1)

**Step 1:** Ambiguity in each \( \tilde{P}_{ij}^l \) is calculated using Eq. (2.8) and presented in Table 5.1

<table>
<thead>
<tr>
<th></th>
<th>( Al_1 )</th>
<th>( Al_2 )</th>
<th>( Al_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Cr_1 )</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>( Cr_2 )</td>
<td>0.07</td>
<td>0.04</td>
<td>0.07</td>
</tr>
<tr>
<td>( Cr_3 )</td>
<td>0.058</td>
<td>0.067</td>
<td>0.025</td>
</tr>
<tr>
<td>( Cr_4 )</td>
<td>0.058</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>
For $D_2$

<table>
<thead>
<tr>
<th></th>
<th>$Al_1$</th>
<th>$Al_2$</th>
<th>$Al_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Cr_1$</td>
<td>0.058</td>
<td>0.025</td>
<td>0.058</td>
</tr>
<tr>
<td>$Cr_2$</td>
<td>0.053</td>
<td>0.05</td>
<td>0.033</td>
</tr>
<tr>
<td>$Cr_3$</td>
<td>0.025</td>
<td>0.04</td>
<td>0.058</td>
</tr>
<tr>
<td>$Cr_4$</td>
<td>0.067</td>
<td>0.082</td>
<td>0.042</td>
</tr>
</tbody>
</table>

For $D_3$

<table>
<thead>
<tr>
<th></th>
<th>$Al_1$</th>
<th>$Al_2$</th>
<th>$Al_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Cr_1$</td>
<td>0.025</td>
<td>0.07</td>
<td>0.042</td>
</tr>
<tr>
<td>$Cr_2$</td>
<td>0.07</td>
<td>0.04</td>
<td>0.025</td>
</tr>
<tr>
<td>$Cr_3$</td>
<td>0.058</td>
<td>0.058</td>
<td>0.04</td>
</tr>
<tr>
<td>$Cr_4$</td>
<td>0.033</td>
<td>0.058</td>
<td>0.017</td>
</tr>
</tbody>
</table>

**Step 2:** The weights of criteria for each decision maker is calculated using Eq. (5.5) is given below;

For $D_1$, $C_1^l = \sum_{i=1}^{m} Al_i (\bar{P}_i^l) = 0.14$

$w_{c1}^l = 7.14$

Similarly

$w_{c2}^l = 5.56, w_{c3}^l = 6.67, w_{c4}^l = 6.33$

Hence, normalized weights are

$Nw_{c1}^l = 0.265967, Nw_{c2}^l = 0.2179877, Nw_{c3}^l = 0.2835684, Nw_{c4}^l = 0.2324704$

For $D_2$, we get

$Nw_{c1}^2 = 0.2647044, Nw_{c2}^2 = 0.2505864, Nw_{c3}^2 = 0.2971124, Nw_{c4}^2 = 0.1875974$
For $D_3$, 
\[ Nw^3_{C1} = 0.2498717, \ Nw^3_{C2} = 0.2497024, \ Nw^3_{C3} = 0.1899364, \ Nw^3_{C4} = 0.31049 \]

**Step 3:** Applying IVTIFEWAA operator the decision matrix is obtained and presented below. For Example,

\[
d^1_i = Nw^1_{C1} \tilde{P}^1_{11} \oplus_E Nw^1_{C2} \tilde{P}^1_{12} \oplus_E Nw^1_{C3} \tilde{P}^1_{13} \oplus_E Nw^1_{C4} \tilde{P}^1_{14} \\
= (0.265967) < [0.2,0.3,0.4,0.5];[0.3,0.6],[0.1,0.3] > \oplus_E \\
(0.2179877) < [0.4,0.6,0.7,0.8];[0.4,0.7],[0.1,0.2] > \oplus_E \\
(0.2835684) < [0.3,0.5,0.6,0.8];[0.3,0.4],[0.2,0.3] > \oplus_E \\
(0.23247040) < [0.1,0.3,0.4,0.6];[0.5,0.7],[0.2,0.3] > \\
= <[0.26,0.43,0.52,0.68];[0.41,0.61],[0.15,0.28]> \\
\]

**Table 5.2: Decision matrix of example 5.3.6**

<table>
<thead>
<tr>
<th>AI_1</th>
<th>AI_2</th>
<th>AI_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>$D_2$</td>
<td>$D_3$</td>
</tr>
<tr>
<td>&lt;[0.26,0.43,0.52,0.68];[0.41,0.61],[0.15,0.28]&gt;</td>
<td>&lt;[0.23,0.44,0.56,1];[0.4,0.56],[0.2,0.4]&gt;</td>
<td>&lt;[0.2,0.33,0.44,0.55];[0.34,0.5],[0.19,0.34]&gt;</td>
</tr>
<tr>
<td>&lt;[0.3,0.39,0.55,0.66];[0.48,0.62],[0.21,0.33]&gt;</td>
<td>&lt;[0.36,0.48,0.58,0.74];[0.43,0.58],[0.22,0.34]&gt;</td>
<td>&lt;[0.35,0.46,0.59,1];[0.51,0.71],[0.15,0.29]&gt;</td>
</tr>
<tr>
<td>&lt;[0.45,0.54,0.65,0.77];[0.47,0.59],[0.2,0.33]&gt;</td>
<td>&lt;([0.44,0.53,0.68,0.81];[0.41,0.58],[0.14,0.26]&gt;</td>
<td>&lt;[0.45,0.59,0.71,1];[0.44,0.54],[0.21,0.46]&gt;</td>
</tr>
</tbody>
</table>

**Step 4:** The overall preferences of alternatives are calculated using the defined IVTIFEWAA operator with the given decision maker’s weights which are given below:

Where $Nw_{d1} = 0.2, Nw_{d2} = 0.3, Nw_{d3} = 0.5$

\[
\hat{\tilde{a}}_i = Nw_{d1} \tilde{a}^1_1 \oplus_E Nw_{d2} \tilde{a}^2_1 \oplus_E Nw_{d3} \tilde{a}^3_1 \\
= (0.2)([0.26,0.43,0.52,0.68];[0.41,0.61],[0.15,0.28]) \oplus_E \\
(0.3)([0.3,0.39,0.55,0.66];[0.48,0.62],[0.21,0.33]) \oplus_E \\
(0.5)([0.45,0.54,0.65,0.77];[0.47,0.59],[0.2,0.33]) \\
\]
Similarly,

\[
\tilde{r}_{Al_1} = <[0.37, 0.47, 0.59, 0.72]; [0.44, 0.6]; [0.19, 0.32]>
\]

\[
\tilde{r}_{Al_2} = <[0.38, 0.5, 0.64, 1]; [0.41, 0.64]; [0.17, 0.31]>
\]

\[
\tilde{r}_{Al_3} = <[0.38, 0.51, 0.63, 1]; [0.45, 0.59]; [0.19, 0.38]>
\]

**Step 5:** The value index of overall alternatives preferences obtained in step 4 calculated using Eq. (2.6) and compared using the Eq.(2.8)

The value index of \(\tilde{r}_{Al_1}, \tilde{r}_{Al_2}, \tilde{r}_{Al_3}\) are obtained as

\[
VI(\tilde{r}_{Al_1}) = 0.131075, \quad VI(\tilde{r}_{Al_2}) = 0.1586, \quad VI(\tilde{r}_{Al_3}) = 0.13115
\]

The results shown that, \(Al_2\) is the best alternative followed by \(Al_3\) then \(Al_1\).

On comparing the result with Wu and Liu (2013), it is observed that, even if \(Al_2\) is the best alternative in both the methods the proposed method can strictly order the alternatives when compared to Wu and Liu (2013) in the case of expert risk is neutral.

**5.4. CASE STUDY: SELECTION OF LOCATION TO SET UP SOFTWARE COMPANY**

Location selection for a software company is the process of finding an appropriate location to establish company and has a strong influence on the success of company with the goal to maximize the use of resources and minimize cost. The attrition levels play a vital role in location selection. Along with this the other criteria that should be taken care of is resource availability, investment cost, and employee recruitment. After primary evaluation, the locations namely Tirupathi (\(Al_1\)), Hyderabad (\(Al_2\)) and Bengaluru (\(Al_3\)) are evaluated based on four criteria: Resource availability (\(Cr_1\)), Attrition levels (\(Cr_2\)), Employee recruitment (\(Cr_3\)) and Costing (\(Cr_4\)) by two experts, \(D_1\) and \(D_2\). The weighting vector of the decision maker weights are given by \(W_{(D_1, D_2)} = \{w_{d_1}, w_{d_2}\} = \{0.5, 0.5\}\) . The information given by DMs is presented in Table 5.3 and Table 5.4.
Table 5.3: Assessment by $D_1$ for case study 5.4

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Al_1$</td>
</tr>
<tr>
<td>$Cr_1$</td>
<td>[0.2,0.25,0.3,0.35];[0.03,0.05]</td>
</tr>
<tr>
<td>$Cr_2$</td>
<td>[0.45,0.5,0.6,0.7]; [0.9,0.96]; [0.03,0.05]</td>
</tr>
<tr>
<td>$Cr_3$</td>
<td>[0.4,0.5,0.7,0.8]; [0.9,0.96]; [0.02,0.03]</td>
</tr>
<tr>
<td>$Cr_4$</td>
<td>[0.4,0.5,0.6,0.7]; [0.93,0.95]; [0.02,0.03]</td>
</tr>
</tbody>
</table>

Table 5.4: Assessment by $D_2$ for case study 5.4

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Al_1$</td>
</tr>
<tr>
<td>$Cr_1$</td>
<td>[0.1,0.2,0.3,0.4]; [0.8,0.9]; [0.05,0.1]</td>
</tr>
<tr>
<td>$Cr_2$</td>
<td>[0.3,0.4,0.5,0.6]; [0.9,0.91]; [0.02,0.03]</td>
</tr>
<tr>
<td>$Cr_3$</td>
<td>[0.6,0.7,0.8,0.9]; [0.8,0.9]; [0.05,0.1]</td>
</tr>
<tr>
<td>$Cr_4$</td>
<td>[0.3,0.4,0.5,0.6]; [0.7,0.8]; [0.1,0.2]</td>
</tr>
</tbody>
</table>

Step 1: Ambiguity index of each alternative performance for each decision maker are found using Eq. (2.7) and presented in Table 5.5.

Table 5.5: Ambiguity index of alternatives performance

<table>
<thead>
<tr>
<th></th>
<th>$Al_1$</th>
<th>$Al_2$</th>
<th>$Al_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Cr_1$</td>
<td>0.038</td>
<td>0.057</td>
<td>0.056</td>
</tr>
<tr>
<td>$Cr_2$</td>
<td>0.069</td>
<td>0.065</td>
<td>0.046</td>
</tr>
<tr>
<td>$Cr_3$</td>
<td>0.127</td>
<td>0.078</td>
<td>0.069</td>
</tr>
<tr>
<td>$Cr_4$</td>
<td>0.079</td>
<td>0.037</td>
<td>0.039</td>
</tr>
</tbody>
</table>
For $D_2$

<table>
<thead>
<tr>
<th></th>
<th>$A_{l_1}$</th>
<th>$A_{l_2}$</th>
<th>$A_{l_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{r_1}$</td>
<td>0.071</td>
<td>0.042</td>
<td>0.063</td>
</tr>
<tr>
<td>$C_{r_2}$</td>
<td>0.079</td>
<td>0.067</td>
<td>0.081</td>
</tr>
<tr>
<td>$C_{r_3}$</td>
<td>0.071</td>
<td>0.054</td>
<td>0.042</td>
</tr>
<tr>
<td>$C_{r_4}$</td>
<td>0.058</td>
<td>0.062</td>
<td>0.08</td>
</tr>
</tbody>
</table>

**Step 2:** The weights of criteria are obtained as below.

For $D_1$

\[
Nw_{C_1}^1 = 0.29774, \quad Nw_{C_2}^1 = 0.239985, \quad Nw_{C_3}^1 = 0.162012, \quad Nw_{C_4}^1 = 0.300259
\]

For $D_2$

\[
Nw_{C_1}^2 = 0.27226, \quad Nw_{C_2}^2 = 0.20415, \quad Nw_{C_3}^2 = 0.28823, \quad Nw_{C_4}^2 = 0.23535
\]

**Step 3:** Applying IVTIFEWAA operator the decision matrix is obtained and presented below.

<table>
<thead>
<tr>
<th></th>
<th>$A_{l_1}$</th>
<th>$A_{l_2}$</th>
<th>$A_{l_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>&lt;[0.37,0.44,0.54,0.64]&gt;</td>
<td>&lt;[0.18,0.26,0.33,0.4]&gt;</td>
<td>&lt;[0.91,0.94]&gt;</td>
</tr>
<tr>
<td>$D_2$</td>
<td>&lt;[0.35,0.46,0.57,0.69]&gt;</td>
<td>&lt;[0.81,0.88]&gt;</td>
<td>([0.05,0.01] )</td>
</tr>
</tbody>
</table>

Step 4: The overall preferences of alternatives using are given below:

\[
\tilde{t}_{A_{l_1}} = \{[0.36,0.45,0.56,0.67]; [0.86,0.92]; [0.04,0.06]\}
\]

\[
\tilde{t}_{A_{l_2}} = \{[0.27,0.36,0.45,0.54]; [0.84,0.89]; [0.05,0.07]\}
\]

\[
\tilde{t}_{A_{l_3}} = \{[0.25,0.35,0.44,0.55]; [0.88,0.9]; [0.02,0.05]\}
\]
Step 5: The values index of overall preferences of alternatives using Eq. (2.6) is,

\[ VI(\tilde{r}_{Al_1}) = 0.22875, \quad VI(\tilde{r}_{Al_2}) = 0.1782, \quad VI(\tilde{r}_{Al_3}) = 0.18445 \]

Hence by Eq. (2.8), \( Al_1 > Al_3 > Al_2 \) and the best alternative is \( Al_1 \).

This shows that \( Al_1 \) is the best alternative followed by \( Al_3 \) then \( Al_2 \). This agrees with the human intuition as the alternative \( Al_1 \)'s attrition level is more preferred than other two.

5.5. CONCLUSIONS

This Chapter addresses an approach to solve MAGDM problems with unknown criteria information when the information of decision maker is given in IVTIF in a view that different decision makers have different perception on weights of criterion. Einstein operation laws on IVTIFNs are defined. The weight of criteria is found using ambiguity in the performance of alternatives. The problem is solved by make use of IVTIFEWAA operator. An illustrative example is provided and comparison is done. Comparison reveals that the proposed method can strictly rank the alternatives compared to existing method. Finally, the proposed method is applied to a real world problem: Selection of location to set up Software Company.