CHAPTER 6

RESULT ANALYSIS
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Result and Discussion

6.1 STEADY STATE SOLUTION

By applying Analytical solution derived in chapter 3, the interior steady-state temperature distribution of the One-dimensional cylindrical living tissue in the resting state can easily and accurately obtained. In the following calculations, the typical values for tissues properties and other parameters are used and presented through Table-1.
Table 1: Thermal properties of tissue

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat of Blood</td>
<td>3850 J/Kg.°C</td>
</tr>
<tr>
<td>Temperature at Infinity</td>
<td>298 °K</td>
</tr>
<tr>
<td>Arterial blood temperature</td>
<td>310 °K</td>
</tr>
<tr>
<td>Thermal conductivity of tissue</td>
<td>0.48 W/m.°C</td>
</tr>
<tr>
<td>Blood perfusion</td>
<td>3 Kg/s.m³</td>
</tr>
<tr>
<td>Metabolic heat generation</td>
<td>1085 W/m³</td>
</tr>
<tr>
<td>Heat convection coefficient</td>
<td>10.023 W/m².°C</td>
</tr>
</tbody>
</table>

The calculation results of the influences of the thermal conductivity, the blood perfusion, the metabolic heat generation, and the coefficient of heat transfer on temperature distribution are shown in following figures.

Fig.1 Effects of the thermal conductivity

![Graph showing the effects of thermal conductivity on temperature distribution.](image)
Figure 1 shows that within a certain distance, the higher the thermal conductivity is, the more sharply the temperature drops in the radial direction, as the higher thermal conductivity has the better capacity of heat transfer. This result is caused by the effect of the convection boundary condition and is not very notable due to comparatively small thermal conductivity of biological materials.

The effect due to different blood perfusion rates on the temperature distribution is shown by figure 2, where in comparison the existence of blood perfusion strongly affect the temperature distribution in living tissue. The curves in the figure indicate that the gradient of the temperature variation in radial direction decreases with increasing blood perfusion because of the higher rate of heat distribution caused by the blood perfusion.
In comparison to above two discussion, changes in the values of the metabolic heat generation are found to have a very small effect on the temperature distribution.

Figure 4 concludes that, higher the coefficient of heat transfer, the lower the temperature near the boundary of the body.

Thus, the influence of the blood perfusion rate is more significant than that of the other thermal parameters for the central temperature of
the body, the changes of the coefficient of heat transfer mainly results in the different temperature variations in the neighborhood of the surface of the living tissue and the effect of metabolic heat generation is inconsequential and can be omitted.

6.2 TRANSIENT SOLUTION

By applying Analytical solution derived in chapter 5, in the following calculations, the typical values for tissues properties and other parameters are used and presented through Table-2. As demonstrated in earlier works by Liu and Xu 1999, the interior tissue temperature usually tends to a constant within a short distance, such as 2-3 cm, therefore L=3 cm was used in this study. For some particular issue, such as deeper heating, or more intense source of heat, the distance between skin surface and the body core will exceed this depth. In this case, new bounded core large enough to neglect the influence of the surface heating showed be in corporate into the calculations. Although any heating style for $Q_r(x,t)$ can be dealt with present analysis, only the most popular and simple specific absorption rate (SAR) expression for $Q_r(x,t)$ will be particularly analyzed. For the plane wave heating of laser or microwave, the heat absorption in the muscle tissue can simply be approximated by Beer’s law as:
\[
Q_x(x, t) = - \frac{\partial q_x}{\partial x} = \eta P_0(t) \exp(-\eta x)
\]  

(6.2.1)

where, \(P_0(t)\) is the time-dependent heating power on skin surface, and \(\eta\) is the scattering coefficient. Since the heating power and the scattering coefficients may vary from one heating equipment to another, it is of great importance to study the influence of these parameters upon the behavior of the tissue temperature. Such heating pattern can be encountered in tissue heating by laser, microwave or ultrasound; etc. Our study is expected to be very useful in a large variety of bio thermal process.

**Table 2: Thermal properties of tissue**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat of tissue</td>
<td>4200 J/Kg.(^0)C</td>
</tr>
<tr>
<td>Density of tissue</td>
<td>1000 Kg/m(^3)</td>
</tr>
<tr>
<td>Arterial blood temperature</td>
<td>37 (^0)C</td>
</tr>
<tr>
<td>Body core temperature</td>
<td>37 (^0)C</td>
</tr>
<tr>
<td>Thermal conductivity of tissue</td>
<td>0.5 W/m.(^0)C</td>
</tr>
<tr>
<td>Blood perfusion</td>
<td>0.0005 ml/s/ml</td>
</tr>
<tr>
<td>Metabolic heat generation</td>
<td>33800 W/m(^3)</td>
</tr>
<tr>
<td>Heat convection coefficient</td>
<td>10 W/m(^2).(^0)C</td>
</tr>
<tr>
<td>Surrounding air temperature</td>
<td>25 (^0)C</td>
</tr>
<tr>
<td>Density of blood</td>
<td>1000 Kg/m(^3)</td>
</tr>
<tr>
<td>Specific heat of blood</td>
<td>4200 J/Kg.(^0)C</td>
</tr>
</tbody>
</table>
6.2.1 Surface Adiabatic Condition and Spatial Heating

Figure 1 shows that the transient temperature at different times of skin surface when biological tissues subject to two different spatial heating and skin surface was treated as adiabatic (namely, $f_i(t) = 0$) (where $\eta = 200 \text{ m}^{-1}$ for laser-tissue interaction) Figure 1(a) depicts the case of constant heating, and Fig. 1(b) the case of sinusoidal heating. The former one reflects the situation where the human skin was heated by a laser, while the later case can be found in the perfusion estimation. Obviously, in both cases, the tissue temperature at the early stage of heating decreases from the body core to the skin surface. However, it will
gradually be improved due to the spatial heating. Moreover, there is an intercross for temperature curves at different times in Fig. 1(b) which indicates the temperature oscillation inside the tissue when subjected to the sinusoidal heating.

6.2.2 Effect of the Scattering Coefficient

![Graphs showing effect of scattering coefficient on temperature response at skin surface.](image)

**Fig 2(a)** Effect of scattering coefficient on Temperature(°C) response at skin surface

**Fig 2(b)** Effect of scattering coefficient on Temperature(°C) response at skin surface

When $f_1(t) = 0$

$$P_0(t) = 250 \text{ W/m}^2$$  

When $f_1(t) = 0$

$$P_0(t) = 250 + 200\cos(0.02t) \text{ W/m}^2$$

Figure 2 depicts that the effect of the scattering coefficient on the temperature transient at the skin surface. It shows that the larger coefficient, the higher temperature increases. For the other heating frequencies, the corresponding thermal responses can also be studied using the same way. Since different heating apparatus such as laser or
microwave may have different power and scattering coefficient (Li and Liang 1989), calculations as performed above are expected to be useful for the heating dose planning during hyperthermia treatment or parameters estimation. For example, performance of the different heating apparatus with specific power and decay coefficient can be evaluated using the present model to predict the temperature response thus induces in the tissue.

6.2.3 Effect of Surface Constant Heating

It is observed from figure 3 that the temperature response at skin surface, a, B and C are the transient temperature of tissues subject to the constant surface heating $\Phi_1(t) = 1000 \, W/m^2$, $\Phi_2(t) = 500 \, W/m^2$, $\Phi_3(t) = 200 \, W/m^2$ respectively. Obviously, the larger surface heating, the higher temperature increases. Such information is also very important for thermal comfort evaluation.
6.2.4 **Effect of Step Heating at Skin Surface**

![Graphs showing transient temperatures at three positions when step heating is applied](image)

Fig. 4(a) Transient temperatures at three positions when step heating is applied

(spatial heating $Q_r = 0$)

(a) $\omega_b = 0.0005 \text{ ml/s/ml}$  
(b) $\omega_b = 0.004 \text{ ml/s/ml}$

It is noted from figure 4 that the highest temperature increase appears at the time when the surface heating was stopped. For the case of blood perfusion $b_w = 0.004 \text{ ml/s/ml}$, at the end of the step heating, the tissue temperature will quickly reach a steady state. The result can help to better understand the temperature history of living tissues subject to step heating and thus benefit the thermal injury analysis.

6.3 **CONCLUSIONS**

The temperature distribution with spatial or transient heating on skin surface and inside the biological bodies is considered. The exact solution of the Pennes bioheat transfer equation with the time-dependent boundary heating condition on the skin surface is derived using Green’s
function method. The analytical expression obtained is useful for describing the tissue temperature distribution in one dimension domain, and especially it is more accurate that the results of the previous results in initial period of time domain. The time dependent heating condition, which can be dealt with by the present solution included most of the possible external heating style such as constant, sinusoidal, step heating both in volume and at boundary. Moreover, based on the requirement for the tumor killing temperature and the skin burn threshold, an approach to optimize the cancer hyperthermia parameters can be obtained using the current solution. Therefore the present investigation has more capability to deal many practical bioheat transfer problems than quite a few existing analytical solutions.
SCOPE OF THE STUDY
The objective of present work is to provide a mathematical modeling approach for the understanding of complex phenomena such as skin cancer in humans. The use of mathematical techniques and tools for the understanding of such bio-heat transfer problems are widely acceptable these days and thus present work highlights the problem of current research interest.

In my study, I have developed a model, based on basic physics of heat flow in cancerous part of human skin and after it implementation, analyzed different flow properties. The process of constructing a model is carefully considered and presented in full details. Generally, the complexity of bioheat transfer equations makes it difficult to obtain their analytical solution, even though, analytical solutions to these equations, if attainable, are of important significance in the study of bioheat transfer because, they can not only accurately reflect the actual physical feature of equation but also be used as standards to verify the corresponding results of numerical calculations.

In present work, analytical solution to bioheat transfer problems with space or transient heating on skin surface or inside biological bodies are obtained using Green’s function method, as it is independent of source term and can be flexibly used to calculate the
temperature distribution for various spatial source profiles. The Green’s function method is capable of dealing with the transient or space dependent boundary conditions, which is an asset when one requires obtained solutions to be capable of solving bioheat problems with transient boundary conditions. The obtained solutions will incorporate relatively complete situations such as space dependent boundary conditions and volumetric heatings. The solution form is very straightforward and easy to be utilized and are expected to be very useful in a variety of bio-thermal practices such as therapeutic and surgical protocols, Hyperthermia cancer treatment, thermal comfort analysis, tissue property measurement, evaluation of the thermal injury, radiation therapy etc.

Thus in my small attempt, I tried to give the brief idea about the involvement and impact of mathematics in biological phenomena and the role of the Mathematics in treatment of Cancer. A quantitative and qualitative understanding of cancer can be facilitated using a mathematical framework that describes the principles that governs tumor initiation and progression. I hope my small effort to relate Mathematics to the complicated biological phenomena of cancer may provide some useful information for future understanding of such
biological processes and the solution to the well known Pennes bio-
heat equation may throw a light to the future cancer treatments.

I will consider this as a moment of great pleasure and pride if my
small effort will become helpful for the future treatments of Cancer,
and I will be responsible for saving someone from cancer in my own
way.

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