3.1 Introduction

In the real life situation supplier offers a price discount to capture the attention of retailer to boost the business sales. Inventory management has a significant role to achieve an optimal inventory model for economic order quantity. This chapter deals with the design of Inventory Control in which the ordering quantity and the delivery time of goods plays a primary part. In earlier research articles two kinds of price reductions are considered, they are unit discounts and accretion discounts. Suppliers suggest a price reduction per unit to raise cash flow or reduce inventories of the few goods over a present spell time of a Manufacturer. In case the sale span proposed is provisional, and then it may non-coincide with usual replenishment spell, and in order to avail the profit of the reduction price, the manufacturer will order a specific quantity in the sale span will increase the Carrying and the Ordering costs. In the earlier periods, the optimum solution is obtained by correlating the two costs of ordering during the specific sale cycle and avoided the sale cycle by ordering the normal EOQ. Tersine and Grasso [72] considered the ordering strategy under a cost raise acknowledged
prior, resulting which Taylor and Bradley [88] discussed similar inventory model in which schedules of increasing costs of the items are declared prior, so as to set an optimal strategy to facilitate with the supplier.

Tersine [73] offered a fundamental single provisional price reduction model which associated the procurement cost in the sales cycle and disregarded the temporary price reduction time. Overall, these inventory models obtained the optimum result by equating the two prices of ordering in the sale cycle, and passed over the sale cycle by ordering the fixed EOQ. Ardalan [5] established an optimum ordering strategy which assumes instant replacement and the ordering policy is achieved under a provisional price reduction. Aull-Hyde [67] obtained optimum ordering strategies for linear review inventory model with permissible backlogs and temporary price reduction was made accessible—this inventory model analysed case with an on-hand or nil inventory.

In this chapter, an optimum ordering strategy is achieved by analysing the price reduction, whether the entire price reduction profit is more than the carrying cost and procurement costs or not for that specific quantity. In addition to this, five various and significant cases of the price reduction sale are analysed and the optimum specific order is achieved in the sale cycle in order to achieve maximum savings.
3.2 Assumptions and Notations

The following assumptions and notations are used to design the proposed optimal Inventory model for deteriorating items. Demand is considered to be deterministic and constant where replenishment is instantaneous. It is assumed that Ordering costs and per-unit Carrying costs do not vary with regard to time whereas regular and specific orders are considered to have same ordering cost. In this model, a sale is given once a year \((S_Q < D_A)\).

3.2.1 Notations

\[
\begin{align*}
D_A & \quad - \quad \text{annual demand rate (units/year)} \\
r & \quad - \quad \text{annual interest rate} \\
\theta & \quad - \quad \text{discounted price (old purchase cost–new purchase cost) (rupees/unit)} \\
F_s & \quad - \quad \text{ordering cost (rupees/order)} \\
P_c & \quad - \quad \text{purchasing cost per item (rupees/unit)} \\
Q^*_L & \quad - \quad \text{normal economic order quantity (units/cycle)} \\
S_Q & \quad - \quad \text{specific order quantity during the sale cycle (units/order)} \\
q_r & \quad - \quad \text{residual on-hand inventory (units)} \\
T_0 & \quad - \quad \text{time at which a specific order is placed}
\end{align*}
\]
3.3 Mathematical Formulation

The optimal order quantity for an EOQ model with respect to uniform demand and instantaneous replenishment is obtained in this chapter. In case if the sale cycle $T_s$ is declared with in the course of the present replenishment cycle, $T_s$ might or might not accord with the replenishment period. The manufacturer has the opportunity to choose the specific quantity $S_Q$ to avail the advantage of the discounted price or otherwise he might place an order for the normal EOQ model if the discounted cost is not gainful. These are the two ordering strategies could be considered by the manufacturer during the sale period. The carrying and the ordering cost will be higher when the manufacturer
places an order for the specific quantity $S_Q$ at time $T_0$ with $q_r$ units on-hand at the same time the inventory level increases to $S_Q + q_r$. The specific quantity must be optimum with the intention to meet the maximum savings so that the higher holding and ordering costs will be enclosed. While the sale cycle is lengthy, the manufacturer will be capable to order a number of specific quantities.

### 3.4 Different cases of sales setups

In this chapter, different sales setups are analysed to obtain the optimum ordering policies based on the nature of the relation between the manufacturer and the supplier. These cases have different intervals of sale cycle as well as the discounted prices.

#### 3.4.1 Sale setup1: Coexistence of the sale cycle with replenishment cycle

Suppose the supplier offers a price discount to the manufacturer at the end of his present inventory period, then the manufacturer must order an optimal specific quantity that increases the yearly profit compared to the normal EOQ. To place an order for a specific quantity in the sale cycle $T_s$, the yearly cost of the optimum specific quantity should be less than the yearly cost of a normal EOQ strategy.
The total yearly cost of the EOQ model is obtained as

$$T_A(Q_L) = P_cD_A + \frac{F_sD_A}{Q_L} + rP_c\frac{Q_L}{2} \quad (3.4.1.1)$$

The total yearly cost of an inventory model with specific quantity is given by

$$T_A(S_0) = P_cD_A - \theta S_0 + F_s\left(1 + \frac{D_A - S_0}{Q_L}\right) + r(P_c - \theta)S_0^2 + rP_c\left(1 - \frac{S_0}{D_A}\right)\frac{Q_L}{2} \quad (3.4.1.2)$$

From the above equations it is clear that $T_A(S_0) < T_A(Q_L)$. Therefore, to achieve the maximum profit $T_A(S_0)$ over $T_A(Q_L)$ the difference between the costs must be greater.

$$\Delta D_u = T_A(Q_L) - T_A(S_0) \quad (3.4.1.3)$$

$$\Delta D_u = \frac{2F_sD_A(S_0 + D_A - Q_L) + Q_L\left[P_c\left(2D_A^2 + rD_AQ_L + rS_0(S_0 - Q_L)\right) - \theta(2D_A + rS_0)\right]}{2D_AQ_L} \quad (3.4.1.4)$$

which is concave in $S_0$.

Thus, the first derivative of $\Delta D_u$ with respect to $S_0$ leads to the maximization of the gain, including the optimum specific quantity, which is

$$S_0^* = \frac{2D_AF_s + 2\theta D_AQ_L + rP_cQ_L^2}{2rQ_L(P_c - \theta)} \quad (3.4.1.5)$$
From the EOQ model, we have $rP_cQ_l^2 = 2D\alpha F_S$.

Therefore the above equation further reduces to

$$S_Q^* = \frac{D\alpha (2F_S + \theta Q_L)}{rQ_L(P_c - \theta)}$$

(3.4.1.6)

It is obvious that when $\theta = 0$, then there is no sale. Hence, the reduced form is $S_Q^* = Q_l$

3.4.2 Sale setup2: Non-Coexistence of the sale cycle with replenishment cycle

Occasionally, the supplier proposes a short-term cost discount which does not accord with any inventory period, i.e., the manufacturer orders a specific quantity $S_Q$ during the time period $T_0$ when the supplier offers a short term sale (sale cycle) $T_s$. The inventory level suddenly rises to $S_Q + q_r$ which will be utilized at rate $D\alpha$. It is prominent to continue the regular ordering at the end of the short sale cycle so that the on-hand inventory is insignificant. The manufacturer shifts back to the original EOQ strategy, once the inventory accumulated at a reduced price is utilized. If he omits the sale period, then there will be an opportunity cost to the manufacturer when he orders normal quantity for not availing the benefit of the reduction.
To place an order for a specific quantity in the period $T_0$, the yearly cost of the optimum specific quantity must be less than the yearly cost for a normal EOQ. The total yearly cost of the EOQ model is obtained in (3.4.1.1)

The total yearly cost of an inventory model with specific quantity is given by

$$T_A(S_0) = P_c D_A - \theta S_0 + F_s \left(1 + \frac{D_A - S_0}{Q_L}\right) + \frac{r(P_c - \theta)S_0^2}{2D_A} + \frac{rP_c q_s S_0}{2D_A} + \frac{rP_c \left(1 - \frac{S_0}{D_A}\right)Q_L}{2} \quad (3.4.2.1)$$

From the above equations, it is clear that $T_A(S_0) < T_A(Q_L)$. Therefore, to achieve the maximum profit $T_A(S_0)$ over $T_A(Q_L)$, the difference between the costs must be greater.

$$\Delta D_A = T_A(Q_L) - T_A(S_0)$$ 

which is concave in $S_Q$. 

Thus, the first derivative of $\Delta D_A$ with respect to $S_Q$ leads to the maximization of the profit, including the optimum specific quantity, which is

$$S_Q^* = \frac{2D_A F_s + 2Q_L (\theta D_A - rP_c q_s) + rP_c Q_L^2}{2rQ_L (P_c - \theta)} \quad (3.4.2.2)$$

$$S_Q^* = \frac{2F_c D_A + Q_L (\theta D_A - rP_c q_s)}{rQ_L (P_c - \theta)} \quad (3.4.2.3)$$
If the residual inventory $q_r = 0$, then the above equation leads to (3.4.1.4).

### 3.4.3 Sale setup3: Diversified-phase overlapping sale cycle

When the supplier suggests a cost reduction in a lengthy cycle $T_s$, the manufacturer can order $l$ specific quantities $S_Q$ so as to avail the profit of the discount $\theta$. Otherwise the manufacturer can order $k$ regular quantities during $T_s$ by neglecting the sale cycle. Later, the manufacturer shifts to normal EOQ model.

The total yearly cost of an inventory model for ordering one specific quantity $S_Q$ is given by

$$T_s(S_Q) = P_c D_s - l \theta S_Q + F_s \left( 1 + \frac{D_s - l S_Q}{Q_r} \right) + l \frac{r (P_c - \theta) S_Q^2}{2 D_s} + r P_c \left( 1 - \frac{l S_Q}{D_s} \right) \frac{Q_r^*}{2}$$

(3.4.3.1)

where $l$ is calculated as (sale cycle / cycle time for specific order quantity).

In order to achieve the maximum profit $T_s(S_Q)$ over $T_s(Q)$ the difference between the costs must be greater.

Thus, the first derivative of $\Delta D_s$ with respect to $S_Q$ leads to the maximization of the profit, including the optimum specific quantity $S_Q^*$, which is
From the above equation, it is clear that sale setup 3 is similar to sale setup 1. Hence, these two sale setup will have same optimum quantity.

3.4.4 Sale setup4: Reduction cost as a function of specific quantity

During the sale cycle, if the supplier proposes certain money $M_b$ back to inspire the manufacturer to place an order for specific quantity then the entire profit from the discount will be $\theta S_Q$ and $M_b$. The sale cycle will be presumed as in Sale Setup2. The profit of the yearly cost for ordering specific quantity over the price of ordering usual quantity will be specified similar to the profit function $\Delta D_u$ for Sale Setup2. Evidently, the price reduction function will be injected in the place of uniform discount $\theta$. Therefore, the gain function will be

$$\Delta D_u = \frac{2F \alpha D_s (S_o + q_e - Q_e) + S_o (M_o + \theta Q_e)(2D_s + rS_o) + rP_e Q_e [q_e (Q_e - 2S_o) + (Q_e - S_o)S_o]}{2D_s Q_e} \quad (3.4.4.1)$$

The optimum specific quantity is given by

$$S_Q^* = \frac{2D_s F_s + 2D_s (M_o + \theta Q_e) + rP_e Q_e (Q_e - 2q_e)}{2r (Q_e (P_c - \theta) - M_o)} \quad (3.4.4.2)$$
As discussed earlier in the sale setup 1, when there is no on-hand inventory, the above equation reduces to

\[ S_0^* = \frac{2D_s F_s + 2D_s (M_b + \theta Q_L) + rP^2}{2r (Q_L (P_c - \theta) - M_b)} \] (3.4.4.3)

It is evident that the manufacturer must place an order for normal EOQ if the specific quantity is utilized. Hence the above equation can be further reduced by applying the value of \( rP^2 = 2D_s F_s \).

\[ S_0^* = \frac{2D_s F_s + D_s (M_b + \theta Q_L)}{r (Q_L (P_c - \theta) - M_b)} \] (3.4.4.4)

The equation (3.4.4.4) can be reduced to (3.4.1.4) when there is no money back \( M_b = 0 \).

### 3.4.4 Sale setup5: Incremental price reduction

When the supplier suggests a short term price reduction in the sale period, subject to the quantity ordered, i.e., higher price reduction for larger order. Thus the price reduction function is

\[
\theta(S_Q) = \begin{cases} 
\theta_1, & 0 \leq S_Q \leq S_Q^1, \\
\theta_2, & 0 \leq S_Q \leq S_Q^2, \\
\vdots & \vdots, \quad \text{where } \theta_1 \leq \theta_2 \leq \ldots \leq \theta_n, \\
\vdots & \vdots \\
\theta_n, & S_Q^{n-1} \leq S_Q 
\end{cases}
\]
It is clear that while the quantity is optimum for the greatest price reduction the highest total savings can be achieved. The optimum specific quantity for the greater price reduction $\theta_i$ will yield the maximum price reduction. Furthermore, the optimum quantity must satisfy the supplier’s conditions to conclude it as a feasible. The following procedure will provide the optimum specific quantity for the discussed sale setup.

Procedure:

Step 1: Assign $i = n$ and $\theta = \theta_i$.

Step 2: Calculate the specific ordering quantity $S_Q$ for sale setup1 using equation (3.4.1.4)

Step 3: Calculate the specific ordering quantity $S_Q$ for sale setup2 using equation (3.4.2.3)

Step 4: If $S_{Q_i} \leq S_{Q} \leq S_{Q_i}$, then $S_{Q} = S_{Q_i}$. Stop the procedure. Or else, fix $n = n-1$ go to step 1.

3.5 Comparative analysis of different price reduction setups

It is necessary to have a comparative analysis on different sale setups in order to acquire a summary of the two various parameters, sale cycle and the price reduction. The specific ordering quantity differs from one setup to another as various parameters are involved. Setup1 is the essential setup for all other setup. By inserting or deducting from the
specific quantity of Setup 1, the optimum specific quantity for all other setups can be achieved. $T_s$ is offered at the end of the present inventory cycle in a shortened period for all setups except setup 3 whereas in setup 3 it is observed in an elongated form. The discount price is constant for setups 1 to 3. In Setup 4, it is treated in the form of $\theta \frac{M_s}{S_Q}$.

For setup 5, $\theta(S_Q) = \left( \theta_i, S_Q^{t-1} \leq S_Q \leq S_Q^t, S_Q^0 = 0 \right)$.

Associatively, the residual on hand inventory $q_r$ plays a main part in the assessment of the optimum specific result $S_Q$. When there is no residual on hand inventory it is observed that the optimum specific quantity and the yearly profit are larger, but as the residual on hand inventory increases the other parameters optimum specific quantity and the yearly profit are decreasing. Hence it is always worthy to place an order if the residual inventory is minimal. It is understood that for greater price reduction total yearly gain is maximized. In this model, the target of availing the higher discounted, by ordering a specific quantity in the sale cycle is achieved. As an outcome of this model, the manufacturer will possibly increase the efficiency of his inventory system by ordering a specific quantity on this sale cycle.
3.6 Conclusion:

This chapter is designed to accomplish the optimum ordering strategy for various sale setups based on the nature of the relation between the manufacturer and the supplier. The optimum principle utilized was to attain the maximum yearly profit of the specific ordering quantity over the fixed EOQ strategy. The results attained for the optimum specific quantities and the total gain fluctuates from those previously obtained results. The yearly profit attained is linearly associated to the price discount and the on-hand residual inventory. Comparative study exposed a remarkable development in the usefulness of the inventory structure once a specific order is placed in the sale cycle. Elite sale openings are a common prodigy and it is significant to comprise them in inventory model. Hence the proposed inventory model has the practical application in the manufacturing world.