CHAPTER 3
EM SIMULATION TECHNIQUES

3.1 INTRODUCTION

The Maxwell’s equations of an electrical system (3.1) - (3.4) solved in order to find the coupling of magnetic and electric field. Maxwell's equations are the combined groups of PDE (Partial Differential Equations) related to EM fields ($\vec{E}, \vec{H}$) to the current and the charge distributions ($\vec{j}, \rho$) and the material characteristics ($\varepsilon, \mu$) in a system.

Maxwell's equations

<table>
<thead>
<tr>
<th>Differential form</th>
<th>Integral form</th>
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<tbody>
<tr>
<td>$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$</td>
<td>$\oint \vec{H} \cdot d\vec{l} = \oint \left( \vec{j} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s}$ (3.1)</td>
</tr>
<tr>
<td>$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$</td>
<td>$\oint \vec{E} \cdot d\vec{l} = -\oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$ (3.2)</td>
</tr>
<tr>
<td>$\nabla \cdot \vec{D} = \rho v$</td>
<td>$\oint \vec{D} \cdot d\vec{s} = \int \rho v , dv$ (3.3)</td>
</tr>
<tr>
<td>$\nabla \cdot \vec{B} = 0$</td>
<td>$\oint \vec{B} \cdot d\vec{s} = 0$ (3.4)</td>
</tr>
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</table>

$\vec{E}$ – Electric flux intensity ($\frac{V}{m}$)

$\vec{H}$ – Magnetic flux intensity ($\frac{A}{m}$)

$\vec{D}$ – Electric flux density ($\frac{C}{m^2}$)
\( \vec{B} \) – Magnetic flux density \( \left( \frac{W}{m^2} \right) \)

\( \rho_v \) - Volume charge density \( \left( \frac{C}{m^3} \right) \)

\( \vec{J} \) - Electric current density \( \left( \frac{A}{m^2} \right) \)

\( \varepsilon \) - Dielectric permittivity of the medium \( \left( \frac{F}{m} \right) \)

\( \mu \) - Dielectric permeability of the medium \( \left( \frac{H}{m} \right) \)

The solving approach of Maxwell’s equations categorized as analytical, numerical or experimental approaches. Among these, the experimental methods are costly and take more time, yet they are most largely used technique and the analytical method includes variable separation and extension of series; however, it does not implement largely. Lastly, the solution from numerical methods for the field issues made possible through accessibility of computers with improved performance. The standard numerical methods are as follows

- Finite Difference Methods (FDM),
- Finite Element Methods (FEM),
- Method of Moments (MoM),
- Partial Element Equivalent Circuit (PEEC) method.

The differences in numerical techniques have their origin in basic mathematics and therefore make one technique more suitable for a specific class of problem compared to others. Distinct problem classes using appropriate modeling approaches mentioned in parenthesis with the EM modeling field and they are:
1. Electrical Interconnect Packaging (EIP) Analysis (PEEC, MoM).
2. PCB Simulations (mixed circuit and EM problem) (PEEC).
3. Coupling Mechanism Characterization (MoM, PEEC).
4. EM Pattern Characterization and Field Strength (MoM).

The problems presented above require different kinds of analysis in terms of domain and variables. The issues stated above need various analysis based on

1. Requested solution domain (frequency and/or time).
2. Requested solution variables
   - Circuit variables (voltages and/or currents).
   - Field variables (magnetic and/or electric fields).

These classifications of EM related to the requested solution set combined with difficulty in equations of Maxwell and then focused on the necessity of implementing correct numerical approach that provides the solution with precision and computing operation. The numerical techniques used for EM simulations, such as FDM, FEM, MoM, and PEEC, classified depending on the formulation of Maxwell's equations, which are solved numerically. The two formulations are displayed parallel (3.1-3.4) as Differential form and Integral form. The main differences between the two formulations are

1. **The discretization of the structure** for differential formulation, the complete structure, including the air needs discretization. For integral formulation, only the materials need discretization. This implies a larger number of cells for differential based techniques and the computational domain needs termination to avoid reaction of outgoing EM waves.
2. **Solution variables** in the differential based techniques where the discretization of the complete computational domain performed, delivers predominantly the solution in field variables. This is suitable
for scattering problems, near field radiation patterns, and EM field excited structures. Post processing of the field variables needed to obtain currents and the voltages in a structure. For integral based techniques, the solution expressed in circuit variables, i.e. currents and voltages. This is suitable for EIP, EMI, and PCB analysis. To convert the system current and voltages to EM field components, post-processing needed.

The first three, FDM, FEM, and MoM, are the most common techniques used today for simulating EM problems. The fourth technique, the PEEC method is widely used within the Signal Integrity (SI).

3.1.1 FDM

FDM method largely implemented in the modeling of EM due to its simplicity. Moreover, in FDTD approach, the Maxwell equations solved by using differential finite equations in a constraint domain of computation. Hence, in this FDM technique, the entire domain of computations separated into cells in which the solving of equations of Maxwell done. The sizes of cells (volume elements) are resulted based on two major aspects

1. Frequency- The Size of the volume elements is not more than \( \frac{\lambda}{10} \), in which the wavelength is denoted as \( \lambda \) that is analogous to frequency
2. Structure - The volume element size should permit the process of discretizing into micro structures

The cell shapes used in cubical forms and even parallel piped forms used for having side-to-side proportion in the range not above the permitted level for avoiding numerical issues. Some application of FDM are

1. Time-dependent PDEs
2. Seismic wave propagation
3. Geophysical fluid dynamics
4. Solving of Maxwell’s equations
5. Ground penetrating radar

3.1.2 FEM

The finite element method is a powerful numerical technique for managing problems including complicated geometries and assorted media. This method is more complicated than FDTD method, but it is applicable to a wider range of problems. FEM based on differential calculation of equations of Maxwell in which the complete field space discretized. The method is applicable to both time and frequency domains. In the method, partial differential equations (PDEs) solved by transformation to matrix equations.

FEM done by minimizing the energy for a PDE by using mathematical concept functional, where the energy obtained by integrating the (unknown) fields over the structure volume. This procedure commonly explained by considering a PDE described by the function \( u \) with corresponding driving, excitation, function \( f \) as

\[
Lu = f
\]  (3.5)

where, \( L \) is a PDE operator.

3.1.3 MOM

Method of Moments (MoM) created based on integral calculations of equations of Maxwell. This basic characteristic made the discretization with the exclusion of air that surrounds the objects. This technique implemented in the domain of frequency, which is even suitable for time domain issues. Thus, within the MoM (integral equations basis), the distribution of current over the surface or wire is then converted into equations of matrix and solved by inversion matrix (Bert Wong et al. 2012). The Method of Moments describes the
basis of all the electromagnetic analysis techniques. The solutions unknown are then stated as the total of basic functions that are known under which the weighing factors consistent with the basic functions are resolute for inability.

When using MoM for surfaces with a wire-grid approximation, the wire formulation of the problem simplified the calculations and used for far field calculations.

The problem converted into the group of linear equations, expressed in the form of matrix as

$$[Z][I] = [V]$$  \hspace{1cm} (3.6)

Whereas the matrices of $[V]$, $[I]$ and $[Z]$ are the denotations of voltage, current and generalized impedance respectively and for current $I$, the preferred solution is obtained by matrix inversion.

3.1.4 PEEC

PEEC is a method for solving field problems in crosstalk and overshoot. PEEC method, developed by Dr. Albert E. Ruehli, is as if the MoM based on integral calculations of equations of Maxwell by making the technique suitable to free space simulations. The main feature of the PEEC method is the combined circuit and the EM solution performed with the same equivalent circuit between both the domains of frequency and time (A.E.Ruehli, 2008). The initiation of derivation in theoretic form is the whole electric field at observation point $\vec{r}$, stated by using the terms $\vec{A}$ (vector magnetic potential) and $\phi$ (scalar electric potential)

$$\vec{E}(\vec{r}, \omega) = -j\omega\vec{A}(\vec{r}, \omega) - \nabla\phi(\vec{r}, \omega)$$  \hspace{1cm} (3.7)

The vector potential given by,
\[ \vec{A}(\vec{r}, \omega) = \mu \int \nu t \, G(\vec{r}, \vec{r}1) \vec{f}(\vec{r}1, \omega) \, dv \]  
(3.8)

Where, \( \vec{j} \) is the volume current density at a source point \( \vec{r} \),

Free-space Green's function denoted as \( G \)

\[ G(\vec{r}, \vec{r}1) = \frac{e^{-j\beta R}}{4\pi R} \]  
(3.9)

Whereas \( R \) is expanded as \( R = |\vec{r} - \vec{r}1| \). The scalar potential given by,

\[ \phi(\vec{r}, \omega) = \frac{1}{\epsilon} \int \nu t \, G(\vec{r}, \vec{r}1) q(\vec{r}1, \omega) \, dv \]  
(3.10)

Where, \( \nu t \) is the volume of the conductor and \( q \) is the charge density at the conductor. If equations 3.8 and 3.9 substituted into 3.10, an electric field integral equation with the unknown variables \( \vec{j} \) and \( q \) obtained as

\[ \vec{E}(\vec{r}, \omega) = -j\omega \mu \int \nu t \, G(\vec{r}, \vec{r}1) \vec{f}(\vec{r}1, \omega) \, dv - \frac{\nu}{\epsilon} \int \nu t \, G(\vec{r}, \vec{r}1) q(\vec{r}1, \omega) \, dv \]  
(3.11)

Equation 3.11 solved by expanding each unknown \( \vec{j} \) and \( q \), into a series of pulse basis functions with known amplitude (G. Antonini, 2007).

3.1.4.1 Introduction

Significant contributions in the development of the PEEC method comprise

- Dielectric insertion.
- Demonstration of Equivalent circuit with the potential factors.
- Equivalent circuit demonstration with started partial element.
- Inclusion of scattering formulation and incident fields with the PEEC models
- PEECs in a non-orthogonal form.
The attempt for research analysis of PEEC approach has increased gradually from the past years. The motive is the enormous demand for the combinational circuit, simulations of EM and the improved performance of computers is accessing the huge EM simulations.

3.1.4.2 Basic PEEC theory

3.1.4.2.1 PEEC Current Density Expansion

The total current density $\vec{j}$, is expanded in PEEC formulation to include conduction current density, $\vec{j}_c$, due to the losses in material and a polarization current density, $\vec{j}_p$, due to dielectric material properties resulting in

$$\vec{j} = \vec{j}_c + \vec{j}_p$$  \hspace{1cm} (3.12)

3.1.4.2.2 PEEC Charge Density Expansion

In (3.13) the charge density is denoted $q^T$ to indicate the combination of free, $q^F$, and bound, $q^B$, charge density.

$$q^T = q^F + q^B$$  \hspace{1cm} (3.13)

This allows in modeling of displacement current owing to bound charging of dielectrics having permeability ($\varepsilon_r$) greater than one discretely from conducting currents due to unbound charges. For the precise electric conductors, the total charge density $q^T$ reduced to $q^F$. While for perfect dielectrics, the total charge density reduces to $q^B$ (Ruehli 2008).

3.1.4.3 EM Modeling using PEEC Method

The PEEC technique has show better appropriateness with combinational circuit and EM-analysis (Hee Won Kang et al. 2014). Moreover,
this technique effectively implemented in various domains of EM modeling such as

- Radiation of EM emitted from PCB.
- Modelling of Transmission Line.
- Modelling of Noise effect.
- Evaluations of Inductance.
- Scattering Issues.
- Power Electronic Devices.
- Analysis of Antenna.
- Lightning-Induced Effects.
- Lightning Protection Devices.
- Effects of inductance in Chip Design.

The applications of PEEC technique has increased more due to constant improvement in application of the technique. The following section describes the basic steps involved in creating PEEC models by considering each part in a PEEC based EM simulation tool.
Partial elements evaluation is the subsequent stage in flowchart demonstrating the common simulation tool of EM for PEEC technique. The partial elements calculated based on mathematical formulations. These calculations performed,

- Partial inductances.
- Partial potential factors.
- Resistances of volume cell.
• Surface- as well as volume cell delays.
• Surplus capacitance of dielectric cells.

**Partial Self-Coefficient of Potential:** The precise definition of partial self and mutual-coefficient of potential is defined as the second set of partial elements needed for PEEC. The computation of these coefficients is fundamental for the capacitance calculations and PEEC equivalent circuits. It is required to define the building blocks and basic geometries for attaining better accurateness and rapid calculation of partial potential factors.

**Partial Mutual Coefficients of Potential:** Effective calculation routines for partial mutual coefficients of potential are, as for partial inductances, more important than for partial self-coefficients of potential due to mutual capacitive/electric field coupling all surface cells in discretization.

**Matrix formulation:** The calculated values of all partial elements are stored in matrices to facilitate in formulation of circuit equations.

**Matrix solution:** For frequency domain full wave PEEC models, the orthodiagonal terms in L and P matrices are complex to account for retardation. Thus, a complex solution in terms of node voltages and/or volume cell currents obtained. The simpler admittance method creates a dense system matrix resulting in a more time consuming solution process.

**Discretization:** The discretization performed at this stage forms the basic volume and surface cells from which the partial elements are calculated. Fine grain discretization requires with the memory of 2GB RAM and minimum 4 GB of Hard disk. The stages in the discretization process is the surface and volume cell size is determined by using

1. **The maximum frequency proposed by PEEC technique** - A thumb law for EMI/EIP modeling is that the surface and the volume cell
sizes would not be more than 120\textsuperscript{th} of the shorter wavelength for exact demonstration of the real waveforms.

2. **Conductors shape** - The proportion of length, width and thickness focused for making accurate calculation of values of partial element. The excess value ratio in dimension of the cell may then generate certain numerical concerns with calculations of partial element while using equations in a closed form and numeric integrals.

3. **Proximity with another conductor** - The overlapping, closely spaced, perpendicular or parallel conductors or the discretization in certain circumstances needs to be proportionate.

4. **Memory and Time confines** - The partial elements quantity in the PEEC technique is directly proportionate to discretization process. Then the partial elements evaluation period and the absolute system’s solution period increased extremely for over-discretized concerns.

The discretization initiated by nodes assignment within the structure. The flowing of current is among the nodes i.e., with volume cells, when every charging node is estimated with individual surface cell.

### 3.1.4.4 Formation of PEEC Model

The theoretical derivation is the summation of electric field, \( \vec{E} \), in a multi conductor system and is termed by using the \( \vec{A} \) (vector magnetic potential), \( \phi \) (scalar electric potential)

\[
\vec{E} (\vec{r},t) = \frac{\partial}{\partial t} \vec{A} (\vec{r},t) - \nabla \phi (\vec{r},t) \tag{3.14}
\]

With the device having \( k \) conductors, the free-space Green's function electromagnetic potentials specified by using


\[ \vec{A}(\vec{r}, t) = \sum_{k=1}^{k} \frac{\mu}{4\pi} \int \frac{\vec{j}(\vec{r}, t)}{||\vec{r} - \vec{r}||} d\vec{r} \]  

(3.15)

\[ \phi(\vec{r}, t) = \sum_{k=1}^{k} \frac{1}{4\pi} \int \frac{q(\vec{r}, t)}{||\vec{r} - \vec{r}||} d\vec{r} \]  

(3.16)

Where,

\[ t_1 = t - \frac{||\vec{r} - \vec{r}||}{v} \]

Indicates the retardation period of the medium having the speed of propagation \( v \). Charge density ‘q’ includes the bound charges and the confined charges in a dielectric medium.

3.1.4.5 Simplified PEEC Model

The PEEC method is an accessible EM simulation technique requiring only a general circuit. The PEEC method can now be used for facilitate in understanding of EM theory or as an aid in a design process. Simplified PEEC methods are used for modeling single layer PCBs, half wave dipoles and simple log periodic antennas. The approach (Thierry Le Gouguec et al. 2012) is to discretize the PCB structures into parallel rectangular surface and rectangular parallel piped volume cells, which are required for closed form equations.

The proposed method to generate simplified PEEC models for PCB structures have been verified by comparing simplified PEEC model simulations against measurements for several prototype PCBs (Ruehli 1972). The agreement is very good for investigated single layer PCB. However, the integral equation method causes restricted amount of freedom than differential equation method, it may also create dense matrix. The robust lessening of quantity of degrees of
freedom may then provide a rapid technique. Moreover, while the distribution of current within the conductor needs to be determined in large aspect or there is a great dielectric medium, then the quantity of degree of freedom is increased by exceeding the stage in which the integral equation technique could be more rapid than the differential equation technique.