In this chapter, some functions are introduced like:

- Totally feebly continuous function
- Slightly feebly continuous function
- Strongly feebly continuous function
- Derivation of some theorems based under this concepts
- Discussion of characterizations of these functions with its related functions

**Definition 3.1:**

Let \((X, \tau)\) be a topological spaces. Any subset \(E\) of \(X\) is called a feebly clopen subset of \(X\) if it is both feebly open and feebly closed.

**Definition 3.2:**

A function \(f : X \rightarrow Y\) is said to be feebly clopen if the image of every open and closed set in \(X\) is respectively feebly open and feebly closed in \(Y\). We have the following diagram.
Theorem 3.3:

Every feebly clopen mapping is feebly closed map and feebly open map.

proof:

Let \( f: X \to Y \) be a feebly clopen mapping. Let \( H \) be any clopen subset of \( X \).

Since \( f \) is feebly clopen mapping , \( f(H) \) is feebly closed and feebly open in \( Y \).

Hence \( f \) is feebly clopen mapping.

The converse of this theorem is trivial. Now we have the following diagram

Theorem 3.4:

A mapping \( f : X \to Y \) is feebly clopen if and only if satisfies the conditions

\[ f(H^0) \subseteq (f(H))^0 \] for every \( H \subseteq X \) and \( \overline{f(H)} \subseteq f(H) \) for every \( H \subseteq X \).

Proof:

Let \( H \) be any open and closed set in \( X \) so that \( H^o=H \) and \( \overline{H}=H \).

Then \( f(H^o)=f(H) \) and \( \overline{f(H)}=f(H) \).

By hypothesis \( f(H^o) \subseteq (f(H))^o \) and \( \overline{f(H)} \subseteq f(H) \).

Therefore, \( f(H) \subseteq (f(H))^0 \) and \( \overline{f(H)} \subseteq f(H) \).
But \((f(H))^o \subset f(H)\) and \(f(\overline{H}) \subset f(H)\).

Hence \((f(H))^o = f(H)\) and \(f(\overline{H}) = f(H)\).

Thus \(f(H)\) is open and closed in \(X\).

Then \(f\) is feebly open and feebly closed. Hence \(f\) is feebly clopen.

**Definition 3.5:**

A function \(f\) from the topological spaces \((X,\tau)\) to \((Y,\sigma)\) is said to be totally feebly continuous if the inverse image of every open subset of \((Y,\sigma)\) is a feebly clopen subset of \((X,\tau)\).

**Definition 3.6:**

A function \(f\) from the topological space \((X,\tau)\) to \((Y,\sigma)\) is said to be Strongly feebly continuous if the inverse image of every subset of \(Y\) is feebly clopen subset of \((X,\tau)\).

**Definition 3.7:**

A function \(f\) from the topological space \((X,\tau)\) to \((Y,\sigma)\) is said to be slightly feebly continuous if the inverse image of every feebly clopen set in \(Y\) is feebly open in \(X\).

**Theorem 3.8:**

Every totally feebly continuous mapping is feebly continuous.

**Proof:**

Let a function \(f\) from the topological space \((X,\tau)\) to \((Y,\sigma)\) be the totally feebly continuous.
Let $H$ be any open set in $Y$.

Since $f$ is totally feebly continuous, $f^{-1}(H)$ is feebly clopen in $X$.

By definition 3.5, $f^{-1}(H)$ is feebly open and feebly closed set in $X$. This implies that $f^{-1}(H)$ is feebly open in $X$.

Hence $f$ is feebly continuous.

**Theorem 3.9:**

A function $f : X \rightarrow Y$ is totally feebly continuous if and only if the inverse image of every closed subset of $Y$ is feebly clopen in $X$.

**Proof:**

Let $B$ be any closed set in $Y$.

Then $Y - B$ is open set in $Y$. By definition 3.5, $f^{-1}(Y - B)$ is feebly clopen in $X$.

That is, $X - f^{-1}(B)$ is feebly clopen in $X$.

This implies that the $f^{-1}(B)$ is feebly clopen in $X$.

On the other hand if $V$ is open in $Y$ then $Y - V$ is closed in $Y$.

By hypothesis $f^{-1}(Y - V) = X - f^{-1}(V)$ is feebly clopen in $X$, which implies $f^{-1}(V)$ is feebly clopen in $X$.

Thus the inverse image of every open set in $Y$ is feebly clopen in $X$. Therefore, $f$ is totally feebly continuous function.

**Theorem 3.10:**

Every strongly feebly continuous function is totally feebly continuous function.
Proof:

Suppose a function \( f \) from the topological space \((X,\tau)\) to \((Y,\sigma)\) is strongly feebly continuous function.

Let \( E \) be open in \( Y \).

By definition, \( f^{-1}(E) \) is feebly clopen in \( X \). Therefore \( f \) is totally feebly continuous.

Remark 3.11:

The following implication gives the inter-relationship.

\[
\text{Strongly feebly continuous} \quad \Rightarrow \quad \text{Totally feebly continuous} \quad \Rightarrow \quad \text{Feebly continuous}
\]

Theorem 3.12:

Let a function \( f \) from the topological space \((X,\tau)\) to \((Y,\sigma)\). The following are equivalent

(i) \( f \) is totally feebly continuous

(ii) for each point \( p \in X \) and each open set \( M \) in \( Y \) with \( f(p) \in M \), there is a feebly clopen set \( E \) in \( X \) such that \( p \in E \) and \( f(E) \subseteq M \).

Proof:

(i)\(\Rightarrow\)(ii) Suppose a function \( f \) from the topological spaces \((X,\tau)\) to \((Y,\sigma)\) is totally feebly continuous and \( M \) be any open set in \( Y \) containing \( f(x) \), so that \( p \in f^{-1}(M) \).

Since \( f \) is totally feebly continuous, \( f^{-1}(M) \) is feebly clopen in \( X \). Let \( E = f^{-1}(M) \).

Then \( E \) is feebly clopen set in \( X \) and \( p \in E \).

Also \( f(E) = f(f^{-1}(M)) \subseteq M \). This implies that \( f(E) \subseteq M \).
(ii)⇒(i) Let M be open in Y. Let \( p \in f^{-1}(M) \) be any arbitrary point.

This implies that \( f(p) \in M \).

Therefore by (ii) there is a feebly clopen set \( f(J_p) \subseteq X \) containing \( p \) such that \( f(J_p) \subseteq M \), which implies \( J_p \subseteq f^{-1}(M) \), we have \( p \in J_p \subseteq f^{-1}(M) \).

This implies that \( f^{-1}(M) \) is feebly clopen neighbourhood of each of its points.

Hence it is feebly clopen set in X.

Therefore, \( f \) is totally continuous.

**Theorem 3.13:**

If \( f : X \to Y \) is totally feebly continuous and \( A \) is feebly clopen subset of \( X \), then the restriction \( f/A : A \to Y \) is totally feebly continuous.

**Proof:**

Consider the function \( f/A : A \to Y \) and let \( V \) be any open set in \( Y \).

Since \( f \) is totally feebly continuous, \( f^{-1}(V) \) is feebly clopen subset of \( X \).

Since \( A \) is feebly clopen subset of \( X \) and \( (f/A)^{-1}(V) = A \cap f^{-1}(V) \) is feebly clopen in \( A \), it follows \( (f/A)^{-1}(V) \) is feebly clopen in \( A \).

Hence \( f/A \) is totally feebly continuous.

**Definition 3.14:**

A map \( f : (X, \tau) \to (Y, \sigma) \) is said to be feebly irresolute if \( f^{-1}(V) \) is feebly open in \( (X, \tau) \) for each feebly open set \( V \) of \( (Y, \sigma) \).
Definition 3.15:

A function $f$ from the domain $(X,\tau)$ into the co-domain $(Y,\sigma)$, where $X$ and $Y$ are topological spaces is said to be feebly clopen irresolute if $f^{-1}(V)$ is feebly clopen in $(X,\tau)$ for each feebly clopen set $V$ of $(Y,\sigma)$.

Theorem 3.16:

If the functions $f$ and $g$ from $X$ to $Y$ and from $Y$ to $Z$ are slightly feebly continuous and totally feebly continuous respectively, then the function $g \circ f$ from $X$ to $Z$ is feebly continuous.

Proof:

Let $O$ be a open set in $Z$.

Since $g$ is totally feebly continuous, $g^{-1}(O)$ is feebly clopen in $Y$.

Now since $f$ is slightly feebly continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is feebly open in $X$. Hence $g \circ f : X \rightarrow Z$ is feebly continuous.

Theorem 3.17:

If the functions $f$ and $g$ from $X$ to $Y$ and from $Y$ to $Z$ are feebly irresolute function and slightly feebly continuous respectively, then the function $g \circ f$ from $X$ to $Z$ is slightly feebly continuous.

Proof:

Let $O$ be feebly clopen in $Z$.

Since $g$ is slightly feebly continuous, $g^{-1}(O)$ is feebly open in $Y$.

Now since $f$ is feebly irresolute function, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is feebly open in $X$. Hence $g \circ f : X \rightarrow Z$ is slightly feebly continuous.
**Theorem 3.18:**

If the functions $f$ and $g$ from $X$ to $Y$ and from $Y$ to $Z$ are feebly clopen irresolute function and totally feebly continuous respectively, then the function $g \circ f$ from $X$ to $Z$ is totally feebly continuous.

**Proof:**

Let $O$ be open in $Z$.

Since $g$ is totally feebly continuous, $g^{-1}(O)$ is feebly clopen in $Y$. Now since $f$ is feebly clopen irresolute function, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is feebly clopen in $X$. Hence $g \circ f : X \to Z$ is totally feebly continuous.

**Theorem 3.19:**

If the functions $f$ and $g$ from $X$ to $Y$ and from $Y$ to $Z$ are is slightly feebly continuous and feebly clopen irresolute function respectively, then the function $g \circ f$ from $X$ to $Z$ is slightly feebly continuous.

**Proof:**

Let $O$ be feebly clopen in $Z$.

Since $g$ is feebly clopen irresolute function, $g^{-1}(O)$ is feebly clopen in $Y$. Now since $f$ is slightly feebly continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is feebly open in $X$. Hence $g \circ f : X \to Z$ is slightly feebly continuous.

**Theorem 3.20:**

If the functions $f$ and $g$ from $X$ to $Y$ and from $Y$ to $Z$ are feebly irresolute function and feebly continuous respectively, then the function $g \circ f$ from $X$ to $Z$ is feebly continuous.
**Proof:**

Let O be open in Z. Since g is feebly continuous, \( g^{-1}(O) \) is feebly open in Y. Now since f is feebly irresolute function, \( f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O) \) is feebly open in X. Hence g \( \circ f : X \rightarrow Z \) is feebly continuous.

**Definition 3.21:**

A space X is said to be feebly clopen- \( T_1 \) if for each pair of distinct points p and q of X, there exist feebly clopen sets E and M containing p and q respectively such that q and p do not belong to E and M.

**Definition 3.22:**

A space X is said to be feebly clopen- \( T_2 \) (feebly clopen Hausdorff) if for each pair of distinct points p and q in X, there exist disjoint feebly clopen sets E and M in X such that q and p belong to E and M.

**Theorem 3.23:**

If a function f from X to Y is a slightly feebly continuous injection and Y is feebly clopen- \( T_1 \) then X is feebly- \( T_1 \).

**Proof:**

Suppose that Y is feebly clopen- \( T_1 \).

For any distinct points p and q in X, there exist feebly clopen set E and M such that f(p) \( \in \) E, f(q) \( \notin \) E and f(p) \( \notin \) M, f(q) \( \in \) M.

Since f is slightly feebly continuous, \( f^{-1}(E) \) and \( f^{-1}(M) \) are feebly open sets of X such that p \( \notin \) \( f^{-1}(E), \) q \( \notin \) \( f^{-1}(E) \) and p \( \notin \) \( f^{-1}(M), \) q \( \in \) \( f^{-1}(M). \)

This shows that X is feebly- \( T_1 \).
**Theorem 3.24:**

If a function $f$ from $X$ to $Y$ is a slightly feebly continuous injection and $Y$ is feebly clopen- $T_2$ then $X$ is feebly -$T_2$.

**Proof:**

For any pair of distinct points $p$ and $q$ in $X$, there exist disjoint feebly clopen sets $E$ and $\text{Min} Y$ such that $f(p) \in E$ and $f(q) \in M$.

Since $f$ is slightly feebly continuous, $f^-(E)$ and $f^-(M)$ are feebly open in $X$ containing $p$ and $q$ respectively.

Therefore $f^-(E) \cap f^-(M) = \varnothing$, because $E \cap M = \varnothing$.

This shows that $X$ is feebly -$T_2$.

**Definition 3.25:**

A function $f: X \to Y$ is said to be

i. almost feebly open (briefly almost f.open) if the image of each F.reg.open set in $X$ is feebly open in $Y$.

ii. almost feebly closed (briefly almost f.closed) if the image of each F.reg.closed set in $X$ is f.closed in $Y$.

iii. almost feebly clopen (briefly almost f.clopen) if the image of each F.reg.clopen set in $X$ is feebly clopen in $Y$.

iv. feebly totally open if the image of each feebly open set in $X$ is feebly clopen in $Y$.

v. feebly totally closed if the image of each feebly closed set in $X$ is feebly clopen in $Y$. 

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vi. almost feebly totally open if the image of each feebly regular open set in \( X \) is feebly clopen in \( Y \).

vii. almost feebly totally closed if the image of each feebly regular closed set in \( X \) is feebly clopen in \( Y \).

viii. almost feebly totally clopen if the image of each feebly regular clopen set in \( X \) is feebly clopen in \( Y \).

**Theorem 3.26:**

Every almost feebly totally closed set is almost feebly closed.

**Proof:**

Let \( X \) and \( Y \) be topological spaces. Let \( f : X \to Y \) be almost feebly totally closed mapping.

Let \( H \) be any feebly regular closed subset of \( X \).

Since \( f \) is almost feebly totally closed mapping, \( f(H) \) is feebly clopen in \( Y \).

This implies that \( f(H) \) is feebly closed in \( Y \).

Therefore \( f \) is almost feebly closed.

**Corollary 3.27:**

Every feebly totally open set is almost feebly open.

**Theorem 3.28:**

If a bijective function \( f : X \to Y \) is almost feebly totally open, then the image of each feebly regular closed set in \( X \) is feebly clopen in \( Y \).
Proof:

Let F be a feebly regular closed set in X. Then X-F is feebly regular open in X.

Since f is almost feebly totally open,  f(X-F) = Y-f(F) is feebly clopen in Y.

This implies that f(F) is feebly clopen in Y.

Theorem 3.29:

A surjective function f : X→Y is almost feebly totally open if and only if for each subset B of Y and for each feebly regular open set U containing \( f^{-1}(B) \), there is a feebly clopen set V of Y such that \( B \subseteq V \) and \( f^{-1}(V) \subseteq U \).

Proof:

Suppose f : X→Y is a surjective almost feebly totally open function and \( B \subseteq Y \).

Let U be feebly regular open set of X such that \( f^{-1}(B) \subseteq U \).

Then \( V = Y - f(X-U) \) is feebly clopen subset of Y containing B such that \( f^{-1}(V) \subseteq U \).

Theorem 3.30:

A map f: (X,τ)→(Y,σ) is almost feebly totally open if and only if for each subset A of (Y,σ) and each feebly regular closed set U containing \( f^{-1}(A) \) there is a feebly clopen set V of Y such that \( A \subseteq V \) and \( f^{-1}(V) \subseteq U \).

Proof:

Suppose f is almost feebly totally open. Let \( A \subseteq Y \) and U be a feebly regular closed set of X such that \( f^{-1}(A) \subseteq U \).

Now X-U is feebly totally open.
Since $f$ is almost feebly totally open, $f(X-U)$ is feebly clopen set in $(Y,\sigma)$. Then 
$V = Y - f(X-U)$ is a feebly clopen set in $(Y,\sigma)$. Note that $f^{-1}(A) \subseteq U$ implies $A \subseteq V$ and 
$f^{-1}(V) = X - f^{-1}(f(X-U)) \subseteq X - (X-U) = U$.

That is $f^{-1}(V) \subseteq U$.

For the converse, let $F$ be a feebly regular open set of $(X,\tau)$.

Then $f^{-1}(f(F)^c) \subseteq F^c$ and $F^c$ is feebly regular closed set in $(X,\tau)$. By hypothesis, 
there exist a feebly clopen set $V$ in $(Y,\sigma)$ such that $f(F) \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so 
$F \subseteq (f^{-1}(V))^c$.

Hence $V^c \subseteq f(F) \subseteq (f^{-1}(V))^c \subseteq V^c$ which implies $f(V) \subseteq V^c$.

Since $V^c$ is feebly clopen, $f(F)$ is feebly clopen.

That is $f(F)$ is feebly clopen in $(Y,\sigma)$. Therefore $f$ is almost feebly totally open.

**Corollary 3.31:**

A map $f : (X,\tau) \rightarrow (Y,\sigma)$ is almost feebly totally closed if and only if for each 
subset $A$ of $(Y,\sigma)$ and each feebly regular open set $U$ containing $f^{-1}(A)$, there is a 
feebly clopen set $V$ of $Y$ such that $A \subseteq V$ and $f^{-1}(V) \subseteq U$.

**Theorem 3.32:**

If $f : X \rightarrow Y$ is almost feebly totally closed and $A$ is feebly regular closed subset 
of $X$ then $f/A : (A,\tau/A) \rightarrow (Y,\sigma)$ is almost feebly totally closed.

**Proof:**

Consider the function $f/A : A \rightarrow Y$ and let $V$ be any feebly clopen set in $Y$. 

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Since f is almost feebly totally closed, \( f^{-1}(V) \) is feebly regular closed subset of X. Since A is feebly regular closed subset of X and \( (f/A)^{-1}(V) = A \cap f^{-1}(V) \) is feebly regular closed in A, it follows \( (f/A)^{-1}(V) \) is feebly regular closed in A. Hence f/A is almost feebly totally closed.

**Remark 3.33:**

Almost feebly totally clopen mapping is almost feebly totally open and almost feebly totally closed map.

**Definition 3.34:**

A map \( f : X \to Y \) is said to be

(i) feebly totally continuous if \( f^{-1}(V) \) is feebly clopen in X for each feebly open set V in Y.

(ii) almost feebly totally clopen continuous if \( f^{-1}(V) \) is feebly clopen in X for each feebly regular clopen set V in Y.

**Theorem 3.35:**

A function \( f : X \to Y \) is almost feebly totally continuous function if the inverse image of every feebly regular open set of Y is feebly clopen in X.

**Proof:**

Let F be any feebly regular closed set in Y. Then \( Y - F \) is feebly regular open set in Y.

By definition \( f^{-1}(Y - F) \) is feebly clopen in X.

That is \( X - f^{-1}(F) \) is feebly clopen in X.

This implies that \( f^{-1}(F) \) is feebly clopen in X.

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**Theorem 3.36:**

Every almost feebly totally continuous function is almost feebly continuous function.

**Proof:**

Suppose \( f : X \to Y \) is almost feebly totally continuous and \( U \) is any feebly regular open subset of \( Y \).

It follows \( f^{-1}(U) \) is feebly clopen in \( X \).

This implies that \( f^{-1}(U) \) is feebly open in \( X \). Therefore the function \( f \) is almost feebly continuous.

**Theorem 3.37:**

Let \( X \) and \( Y \) be topological spaces, \( X = A \cup B \) where \( A \) and \( B \) are feebly clopen subset of \( X \) and let \( f : (X, \tau) \to (Y, \sigma) \) be a function such that \( f/A \) and \( f/B \) are almost feebly totally clopen continuous function. Then \( f \) is almost feebly totally clopen continuous function.

**Proof:**

Let \( U \) be any feebly regular clopen in \( Y \) such that \( f^{-1}(U) \neq \emptyset \).

Then \((f/A)^{-1}(U) \neq \emptyset \) or \((f/B)^{-1}(U) \neq \emptyset \) or both \((f/A)^{-1}(U) \neq \emptyset \) and \((f/B)^{-1}(U) \neq \emptyset \).

Case i : Suppose \((f/A)^{-1}(U) \neq \emptyset \).

Since \( f/A \) is almost feebly totally clopen continuous, there exist a feebly clopenset \( V \) in \( A \) such that \( V \neq \emptyset \) and \( V \subset (f/A)^{-1}(U) \subset f^{-1}(U) \). Since \( V \) is feebly clopen in \( A \) and \( A \) is feebly clopen in \( X \), we have \( V \) is feebly clopen in \( X \).

Thus \( f \) is almost feebly totally clopen continuous function.
Case ii : Suppose \((f/B)^{-1}(U) \neq \emptyset\).

Since \(f/B\) is almost feebly totally clopen continuous function, there exists a feebly clopen set \(V\) in \(B\) such that \(V \neq \emptyset\) and \(V \subseteq (f/B)^{-1}(U) \subseteq f^{-1}(U)\).

Since \(V\) is feebly clopen in \(B\) and \(B\) is feebly clopen in \(X\), \(V\) is feebly clopen in \(X\).

Thus \(f\) is almost feebly totally clopen continuous function.

Case iii : Suppose \((f/A)^{-1}(U) \neq \emptyset\) and \((f/B)^{-1}(U) \neq \emptyset\).

This follows from both the cases (i) and (ii).

Thus \(f\) is almost feebly totally clopen continuous function.

**Theorem 3.38:**

For any bijective map \(f : X \to Y\) the following statements are equivalent:

(i) \(f^{-1} : Y \to X\) is almost feebly totally continuous.

(ii) \(f\) is almost feebly totally open. (iii) \(f\) is almost feebly totally closed.

**Proof:**

(i) \(\Rightarrow\) (ii): Let \(U\) be a feebly regular open set of \((X,\tau)\).

By assumption, \((f^{-1})^{-1}(U) = f(U)\) is feebly clopen in \((Y,\sigma)\) and so \(f\) is almost feebly totally open.

(ii) \(\Rightarrow\) (iii): Let \(F\) be a feebly regular closed set of \((X,\tau)\).

Then \(F^c\) is feebly regular open set in \((X,\tau)\).

By assumption \(f(F^c)\) is feebly clopen in \((Y,\sigma)\).

Hence \(f\) is almost feebly totally closed.
(iii)$\Rightarrow$(i) : Let $F$ be a feebly regular closed set of $(X, \tau)$.

By assumption, $f(F)$ is feebly clopen set in $(Y, \sigma)$.

But $f(F) = (f^{-1})^{-1}(F)$ and therefore $f^{-1}$ is almost feebly totally continuous.

**Definition 3.39:**

A map $f : X \to Y$ is said to be super feebly clopen continuous if for each $x \in X$ and for each feebly clopen set $V$ containing $f(x)$ in $Y$, there exist a feebly regular open set $U$ containing $x$ such that $f(U) \subseteq V$.

**Theorem 3.40:**

Let $f : X \to Y$ be almost feebly totally open.

Then $f$ is super feebly clopen continuous if $f(x)$ is feebly clopen in $Y$.

**Proof:**

Let $G$ be feebly clopen in $Y$.

Now $f^{-1}(G)$ is feebly regular open in $X$, $f(f^{-1}(G)) = G \cap f(x)$ is feebly clopen in $Y$, since the intersection of feebly clopen set is feebly clopen.

Therefore by the definition 3.25 (vi), $f^{-1}(G)$ is feebly regular open in $X$.

Hence $f$ is super feebly clopen continuous function.

**Theorem 3.41:**

If $f : X \to Y$ is surjective and almost feebly totally open, then $f$ is super feebly clopen continuous.
Proof:

Let $G$ be feebly clopen in $Y$. Take $A = f^{-1}(G)$.

Since $f(A) = G$ is feebly clopen in $Y$, by theorem 3.40, $A$ is feebly regular open in $X$. Therefore $f$ is super feebly clopen continuous.

Theorem 3.42:

Let $(X,\tau)$, $(Y,\sigma)$ and $(Z,\eta)$ be topological spaces.

Then the composition $g \circ f : (X,\tau) \to (Z,\eta)$ is super feebly clopen continuous function where $f : (X,\tau) \to (Y,\sigma)$ is super feebly clopen continuous function and $g : (Y,\sigma) \to (Z,\eta)$ is feebly clopen irresolute function.

Proof:

Let $A$ be a feebly regular closed set of $(X,\tau)$.

Since $f$ is super feebly clopen continuous, $f(A)$ is feebly clopen in $(Y,\sigma)$. Then by hypothesis $f(A)$ is feebly clopen set.

Since $g$ is feebly clopen irresolute, $g(f(A)) = (g \circ f)(A)$. Therefore $g \circ f$ is super feebly clopen continuous.

Theorem 3.43:

Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ be two mappings such that their composition $g \circ f : X \to Z$ be almost feebly totally closed mapping then the following statements are true.

(i) If $f$ is super feebly clopen continuous and surjective, then $g$ is feebly clopen irresolute function.

(ii) If $g$ is feebly clopen irresolute function and injective, then $f$ is almost feebly totally closed function.
Proof:

(i) Let A be a feebly clopen set of \((Y,\sigma)\).

Since \(f\) is super feebly clopen continuous, \(f^{-1}(A)\) is feebly regular closed in \(X\).

Since \((g\cdot f)(f^{-1}(A))\) is feebly clopen in \((Z,\eta)\).

That is \(g(A)\) is feebly clopen in \((Z,\eta)\), since \(f\) is surjective. Therefore \(g\) is feebly clopen irresolute function.

(ii) Let B be feebly regular closed set of \((X,\tau)\).

Since \(g\cdot f\) is almost feebly totally closed, \((g\cdot f)(B)\) is feebly clopen set in \((Z,\eta)\), since \(g\) is feebly clopen irresolute function.

Now \(g^{-1}((g\cdot f)(B))\) is feebly clopen set in \((Y,\sigma)\).

That is \(f(B)\) is feebly clopen set in \((Y,\sigma)\).

Since \(f\) is injective and therefore \(f\) is almost feebly closed function.