4.1 Introduction and background

Penning traps are widely used as devices for confining non-neutral plasmas and exist in many geometries. An abundance of literature exists on Penning traps of different geometries and the relative suitability of these devices in confining non-neutral plasma [79–84]. The effects of individual and collective particle oscillations of the electron plasma have been investigated for coninements in Penning traps [82–84] and for other ions in Paul traps [85, 86].

It is desirable in characterizing the trapped plasma in such traps, to measure the energy distribution of the plasma amongst the different degrees of freedom of motion of the particles, or the nature of the spatial distribution of the plasma within the trap. Measurements through laser fluorescence of trapped ions in a Paul trap [7, 8] reveal for ions under thermal equilibrium achieved from collisions with a buffer gas, a Gaussian form of spatial
distribution of the ions from trap centre. Druyvesteyn et al [87] have used Langmuir probes in the study of the Electron energy distribution functions (EEDF). In a Tokamak edge plasma to measure the EEDF, Tsv K. Popov et al. [4, 5] use Langmuir probe to obtain bi-Maxwellian distributions.

In this chapter, the experiment on measurement of trapped electron energy distribution is discussed and we present a technique, that relies on measuring the number of electrons at the centre of the trap, at different trapping potentials $V$ [70]. This is carried out through monitoring the axial motion of the electrons. This measure of the number of electrons at the centre of the trap as a function of the voltage $V$, results in a graph of the confinement potential versus electron detection signal area. The area under the signal is proportional to the number of electrons [70, 88]. This is similar to the I-V curves obtained using Langmuir probes [87]. The detection of electrons is achieved using detection circuit (discussed in chapter 3). The typical confinement time of electrons in the quadrupole Penning trap in our experiments being in the range of hundreds of milliseconds, is much greater than the time for the electrons entering the trap to achieve thermal equilibrium (about 10 µs).

### 4.2 Theory

As explained in earlier chapter in the section of electron source, the resistively heated thoriated tungsten filament emits electrons that enter the trap volume through a small orifice in one of the end caps. The filament bias is varied between 3 – 5 volts, resulting in electrons entering the trap having different energy distributions centered around the bias potential and trapped subsequently. Since the electrons possess energy of a few eV the electrons temperature $T_e$ is much greater than that of the surrounding neutral gas molecules, the latter being at ambient temperature (300 K). Thus the electron plasma constitutes
a non-thermal plasma under Local Thermodynamic Equilibrium (LTE). For calculating electron-electron collision times we use the analysis as in [73]. Thermal equilibrium by electron-electron collisions is achieved over a time

\[
\tau_{ee} = \frac{16 \pi \varepsilon_0^2 m_e^{\frac{3}{2}} [k_B T_e]^\frac{3}{2}}{\sqrt{3} n_e e^4 \ln \Lambda} \tag{4.1}
\]

where, \( \varepsilon_0 \) is the Permittivity of free space; \( m_e \) is mass of an Electron; \( k_B \) is Boltzmann constant; \( T_e \approx 10^4 \) K is the electron temperature; \( e \) is the charge of an electron; \( n_e \) is electron number density, \( \ln \Lambda \) is the Coulomb logarithm [and is a weak function of temperature and density] wherein \( \Lambda \approx 9N_D \), \( N_D \) being the Plasma parameter that describes the number of particles in the Debye sphere, and is given by

\[
N_D = \frac{4}{3} \pi n_e \lambda_D^3 \tag{4.2}
\]

Number density \( n_e \) in our case is [73],

\[
n_e = \frac{N}{V} = \frac{\text{No. of electrons trapped}}{\text{volume of the trap}}
\]

where, \( N = 6.2 \times 10^6 ; \ V = \frac{4}{3} \pi R^3 ; \ R = 1 \times 10^{-3} \text{ m} \)

Then, \( n_e = 1.4813 \times 10^{15} \text{ m}^{-3} \)

In Eq. 4.2, \( \lambda_D \) = Debye radius,

\[
\lambda_D = \left( \frac{\varepsilon_0 k_B T_e}{n_e e^2} \right)^{\frac{1}{2}} \tag{4.3}
\]

\[\lambda_D = 1.1219 \times 10^{-4} \text{ m}\]
From Eq. 4.2 we have,

\[ N_D = \frac{4}{3} \pi n_e \lambda_D^3 \]

and,

\[ N_D = 8.7614 \times 10^3 \]

Now,

\[ \Lambda = 9N_D = 7.8853 \times 10^4 \]

Then,

\[ \ln \Lambda = 11.2753 \]

Substituting the values of \( c_0, \pi, m_e, k_B, T_e, n_e, c \) and \( \ln \Lambda \) in Eq. 4.1 we obtain, \( \tau_{ee} \approx 10 \mu s \). This is much smaller than the time over which the measurements are carried out (a few ms) and hence we are probing the regime wherein the electron plasma has achieved LTE.

### 4.2.1 Distribution function

The distribution function may be obtained as follows:

The electron energy distribution function \( f(E) \) specifies the total number of particles that have an energy \( E \). If number density of trapped electrons is known then energy distribution function \( f(E) \) can be measured. The number density is equal to the integration of the distribution function over all energies. The number of electrons \( N \) in the trap with energies up to \( eV \) is given by

\[ N = e \int_0^{eV} f(E) dE, \]

where, \( f(E) \) is the distribution function of the electrons at an energy \( E \)

Similarly, the number of electrons from energy up to voltage is \( V' = V - \Delta V \)

\[ N' = e \int_0^{e(V - \Delta V)} f(E) dE \]
The reduction in number of trapped electrons, when \( V \) changes to \( V' \) is given by

\[
\Delta N = e^{V} \int_0^{e^{V-\Delta V}} f(E) dE - e^{V-\Delta V} \int_0^{e^V} f(E) dE
\]

Taylor expanding around, and considering only first order term, as \( \Delta E \to 0 \) yields

\[
\lim_{\Delta V \to 0} \frac{\Delta N}{\Delta V} = \frac{dN}{dV} = f(E)
\]

A graph of area under the detection signal is plotted against the storage voltage. The area under the detection signal is proportional to the number of trapped electrons [70]. The Maxwell-Boltzmann distribution function can be written in terms of energy as,

\[
f(E) = A e^{-E/kT}
\]

The distribution function in terms of velocity can be written as,

\[
f(v) = A e^{-\frac{1}{2}mv^2/kT}
\]

(4.4)

The constant \( A \) is determined by normalization and by treating Eq. 4.4 as a normalized probability distribution function, so that

\[
\int f(v) d^3v = \int A e^{-\frac{1}{2}mv^2/kT} d^3v = 1
\]

so,

\[
A = \left( \int e^{-\frac{1}{2}mv^2/kT} d^3v \right)^{-1}
\]

Since our measurement is along the z-component of velocity, \( v_z \), from Eq. 4.4, the axial
velocity distribution function can be written as,

$$f(v_z) = \left( \frac{m}{2\pi kT} \right)^{1/2} e\left(-\frac{mv_z^2}{2kT}\right)$$  \hspace{1cm} (4.5)$$

The results that we obtained fits to Eq. 4.5 there by showing the distribution is Maxwellian. The fitting is carried out using a Gaussian function and it is found to fit very well.

### 4.3 Experiment

In this section we describe a LabVIEW based voltage control system that is used for obtaining the distribution function. After loading the trap with electrons of energy distributed around the filament bias voltage that is negative (varied from 3 – 5 V) and which determines the energy of the electrons, the loading is terminated by reducing the filament bias. The ring electrode is at a positive potential while the end caps are grounded.

The potential $V$ in the trap is initially set at a higher value at about 10 V. Electrons that enter the trap are confined, partly owing to energy transfer from axial motion to the radial motion through coulomb interactions. The potential is then reduced to a value, $V' = 10 V - \Delta V$, where $\Delta V$ is the reduction of potential in the trap, causing electrons whose energies are larger than $V'$, leave the trap. The time $\tau$ over which this occurs, called the dwell time, is well within the confinement time of the trap for electrons. In our set up, measurement with confinement time of trapped electrons is about 80ms for an external magnetic field of 0.05 T. The energy distribution measurements in the trap are carried out for dwell times that are well within this confinement time. The measurement of confinement time of trapped electrons has been briefly discussed in chapter 3.

The voltage is then ramped down. A schematic of ramp voltage is shown in Fig. 4.1. This results in varying the axial frequency of the trapped electrons. At a certain ramp voltage
Energy distribution measurements

![Graph showing the voltage ramp]

**Figure 4.1:** A schematic of the voltage ramp

the corresponding axial frequency matches the tuned detection circuit frequency resulting in energy transfer from the detection circuit to the electrons, and is registered as a voltage change in the demodulated DC signal from the detection circuit as it is shown in Fig. 4.2. The corresponding signal so obtained is recorded. Measurements are carried out for different $V$.

In our measurements, we fixed the value of $\Delta V$ to 0.5 V for each different values of $V$. As the detection circuit in our setup was tuned to a frequency that corresponded to about 1 V, where resonant energy transfer took place from the circuit to the trapped electrons, we set the minimum $V$ at 2 V. The signal strength is thus recorded yielding the area under the signal.
4.4 Results and Discussion

Fig. 4.3 (a) & (b) are respectively the graphs of area under the detection signal versus storage voltage and the corresponding derivatives of the signal area versus voltage. Fig. 4.3 (a) & (b) represent measurements carried out when electrons are injected from the filament at bias voltages of 5 V and 3 V respectively. The corresponding distribution function for both electron filament bias voltages shows that the maximum of the function is at voltages corresponding to the electron filament bias voltage. Thus, the distribution function of the electrons is a reflection of the total energy distribution, regardless of the fact that we monitor only the axial motion, in these measurements. This is a consequence of equipartition of energy into all degrees of motional freedom. The slope of the curves in
Fig. 4.3 (a) & (b) gives the corresponding distribution functions[89]. The data fits well in each case, to a Gaussian distribution function.

Fig. 4.4, is the distribution functions of the derivatives of signal area versus voltage for filament bias voltage around 5V and for different external magnetic fields. As can be seen from Fig. 4.4, there is no change in the shape of the energy distribution of trapped electrons with varying magnetic field, as expected, since the magnetic field has no role to play on the axial motion of the electrons. The confinement time of the electrons in the trap, however, does depend on the external magnetic field.

In summary, we did a measurement on energy distribution of the trapped electron. The trapped electrons that form a non neutral, non thermal plasma are in Boltzmann equilibrium for an electron gas under low pressures. We confirm in addition to this, that the energy distribution shows a peak that corresponds to the energy of the electrons determined by the filament bias voltage. By varying the filament bias we can see a clear shift in this peak in the distribution function. The electron detection signal that is obtained, following a sequence of steps wherein the storage voltage is set to different values, varies as the storage voltage and this variation does depend on the range of electron energies determined by the filament bias voltage. Moreover, this variation has no dependence on the applied magnetic field. This work thus demonstrates the possibility of a direct measurement of the energy distribution function by monitoring the axial motion that is decoupled from the radial motion of the electrons, through electronic detection, under conditions of LTE of the trapped electrons.
Energy distribution measurements

Figure 4.3: (a) The area under the detection signal versus storage voltage (b) the corresponding derivatives of the signal area versus voltage.

Figure 4.4: Distribution function obtained from derivatives of the signal area versus voltage for electron filament voltage of 5 V and for different external magnetic fields: (a) 0.025 T (b) 0.05 T and (c) 0.1 T.