CHAPTER - III

NOISE FILTERING AND FETAL ECG SIGNAL SEPERATION

3.1. INTRODUCTION
Any undesirable signal that is being mixed up with the actual signal is known as noise signals. These noise signals can produce error in the results and can degrade the performance of the classifier. So, it is necessary to remove the noise signals present in the signal for better results. This chapter discusses the various noise removal methods that are used in this work to remove the noise signals from the ECG signal obtained from the abdomen of the mother. The signal after noise removal is subjected to fetal ECG signal separation. Various signal separation methods are used in this work to obtain an effective fetal ECG signal for analysis. The techniques used for fetal ECG separation are discussed in latter part of this chapter.

3.2. NOISE FILTERING
The fetal ECG signal obtained from the abdominal areas should be preprocessed for improved classification results. This process includes removal of noise interferences, power line interference, and baseline wandering and electrode contact noise. Several noise removal algorithms are available to remove the undesirable signal from the desired signal. In this work, the response of FIR filter, PCA and the combination of FIR filter and PCA techniques have been implemented. The description of these methods is discussed in the sections given below.

3.3.1 FIR FILTERS
FIR filters are the ones in which the impulse response of the filter is estimated using some finite number of impulse sequences. In this work, band pass filter type is used. This filter allows the range of frequency above and below the upper cut off and lower cut off frequency respectively, and rejects all other frequencies. The magnitude of the signal remains unaltered during this filtering process. The FIR filter used in this work eliminates the noise whose frequency ranges between 1 Hz and 90 Hz. The noises such as baseline wander, EMG noise, and uterine muscle noise are eliminated in this phase. The linear phase can be easily obtained with the use of FIR filters. This linear phase is achieved with the aid of either symmetric \( h(n) = h(N - 1 - n) \) or asymmetric impulse response \( h(n) = -h(N - 1 - n) \). The Z transform of the N Point filter is given by
\[ H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \]  
(3.1)

Where

- **h(n)** - Input parameter to the FIR filter
- **H(z)** – Output parameter to the FIR filter

There are two main stages involved in the design of FIR filters. They are

- Approximation stage
- Realization Stage

In the approximation stage, the transfer function of the filter is estimated for the desired specification. This involves four steps.

- Choose the desired or ideal response in frequency domain
- Choose the class of FIR filter
- Choose the quality of approximation
- Choose an algorithm that estimates the transfer function.

The realization stage involves the design of structure for the derived transfer function in the approximation stage. This can be done by using one of the three methods such as

- Window Method
- Frequency Sampling Technique
- Optimal Filter Design Methods

### 3.2.2. Principal Component Analysis (PCA)

This method of preprocessing reduces the number of input data needed for analysis. This results in faster analysis and classification. The signal analysis using PCA method is the same as the signal analysis using ICA method. This method of analysis assumes the source signals to be Gaussian.

The preprocessing in PCA method is done using the covariance matrix. The covariance matrix is represented as

\[ R_s = \frac{1}{N} \sum_{t=1}^{N} S(t)S^T(t) \]  
(3.2)

Where

- **S(t)** - Samples of input data
- **N** - Total number of samples.

The factors of the covariance matrix are expressed as

\[ R_s = DQD^T \]  
(3.3)
Where,

Q-P x P diagonal matrix

D- Eigen value matrix.

The elements in P x P diagonal matrix are the eigen values of R_

Apart from these techniques, a combination of FIR filter and PCA is used in this work, and the performance is verified with the existing systems. The block diagram of this system is shown in Fig.3.1.

![Block Diagram of Preprocessing Stage](image)

**Fig.3.1. Block Diagram of Preprocessing Stage**

This methodology can be applied for the separation of ECG signals which are obtained by using both direct method and indirect method. Initially, the recorded signal is given to the FIR filter and this filter output is fed as input to the PCA. The output signal from the PCA can be used for further feature extraction process.

The input signal and the response obtained with this combined FIR and PCA are shown in Fig.3.2.

![Input and Output signal with the Combination of PCA and FIR](image)

**Fig.3.2. Input and Output signal with the Combination of PCA and FIR**

The results of various noise removal methods used in this work are compared. The parameters such as Peak Signal-to-Noise (PSNR) and MSE are compared for the various methods. The results obtained are given in Table3.1.
Table 3.1: Performance Parameters of Filtering Techniques

<table>
<thead>
<tr>
<th>Filter</th>
<th>PSNR</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIR filter</td>
<td>64.917479</td>
<td>1.431797</td>
</tr>
<tr>
<td>PCA</td>
<td>20.834929</td>
<td>1.17587811</td>
</tr>
<tr>
<td>FIR+PCA</td>
<td>102.922251</td>
<td>0.031947</td>
</tr>
</tbody>
</table>

On comparing the results obtained, it is observed that the combined FIR and PCA techniques produced a promising result with MSE of 0.031947 and PSNR of 102.922251. These methods have done better noise removal process from the fetal ECG signal compared to the other two techniques. Therefore, this combined method is used in this ECG classification work for noise removal.

3.3. SEPARATION OF FETAL ECG FROM ABDOMINAL ECG

The ECG signal after the removal of noise is subjected to the separation of fetal ECG signal from the abdominal ECG signal. The mixture of many signals is known as cocktail signal. Normally on acquiring the fetal ECG signal from the mother’s abdomen, the fetal ECG signal is mixed with the mother’s signal. The separation of fetal ECG signal is a challenging task. The ICA technique has produced many successful results in the separation of cocktail signals. The method is described in the following section.

3.3.1. Independent Component Analysis

The ICA can be applied to solve the BSS problem. This method generates a set of random variables as linear combinations that are statistically independent component variables. This ICA method recovers the unknown source signal from the cocktail signal by making an assumption on the n independent signals associated with the cocktail signal. These n independent signals are given by

\[ S(t) = S_1(t), S_2(t), S_3(t), \ldots, S_n(t) \]  \hspace{1cm} (3.4)

The observed signals are given by

\[ x(t) = x_1(t), x_2(t), x_3(t), \ldots, x_n(t) \]  \hspace{1cm} (3.5)

For the set of

\[ x(t) = As(t) \]  \hspace{1cm} (3.6)

Where

A-Unknown Mixing Matrix
T-Time instance
When ICA method is applied on the mixing matrix, it will unmix the mixed matrix. The mixed matrix contains the combined fetal and mother’s ECG signal obtained from the abdomen. From the unmixed matrix W, the source signals can be easily identified. The output matrix is given by

\[ y(t) = Wx(t) \]  

Where

\( Y(t) \) - Output Matrix \\
\( W \) - Unmixed Matrix

3.3.1.1 Signal Pre-Processing

There are two signal pre-processing techniques applied on the signal before the analysis of the signal is done using ICA method. They are

- Centering
- Whitening

**Centering:** In centering the mean variables are removed, so that the independent components are easily identified. The centered observation vector is given by

\[ x_c = x - m \]  

Where ‘m’ is the mean vector.

The observation matrix \( X \) is subtracted by its mean value to zero to obtain the center the matrix \( X \). This point gives all the observation vectors to be centered. Using the centered data, the unmixed matrix is estimated.

**Whitening:** In this signal processing method, the observation vector is transformed linearly so that the number of parameters to be estimated is reduced. This method removes the correlation in the data.

Let \( X_w \) indicate the whitened vector, it should satisfy the equation

\[ E\{X_w X_w^T\} = I \]  

Where \( E\{X_w X_w^T\} \) is the co-variance matrix of \( X_w \)

The whitening transformation is done by an easy simple method called Eigen value decomposition (EVD). This method decomposes the covariance matrix of ‘x’.

\[ \varepsilon = cov(x) = E[xx^T] \]

\[ \varepsilon = E[ASAS^T] \]

\[ \varepsilon = AA^T \]  

Using Eigen value decomposition,
\( \varepsilon = VDV^T \) \hspace{1cm} (3.11)

where \( V \) denotes the Eigen vector of \( E[xx^T] \)

\( D \) denotes the diagonal matrix of Eigen values.

The transformation used to whiten the observation vector is,
\[
X_w = VD^{-1/2}V^T x
\]

The mixing matrix is transformed into a whitening matrix by whitening process, which is orthogonal.
\[
X_w = VD^{-1/2}V^T x
\]
\[
X_w = A_w S
\] \hspace{1cm} (3.13)

Hence
\[
E\{X_wX_w^T\} = A_w[E\{SS^T\}]A_w^T
\]
\[
= A_wA_w^T
\]
\[
= I
\] \hspace{1cm} (3.14)

The number of parameters to be estimated can be minimized by the whitening process. For example the matrix \( A \) with \( n^2 \) elements is required to be estimated as matrix with \( n(n-1)/2 \) degrees of freedom. From this, one can indicate that half of the ICA problem was solved by the whitening problem.

3.3.1.2 Problems with ICA techniques

There are two main problems associated with the ICA method analysis. They are

- Variances of the independent components cannot be determined.
- Order of the independent component cannot be determined.

When the ICA techniques are applied after the preprocessing techniques, the fetal ECG signal and the mother’s ECG signal are separated. These signals are assumed as Gaussian and as independent from each other statistically. In order to improve the signal separation technique, an algorithm called FCOMBI is proposed in this work.

3.3.2 Efficient Fast Independent Component Analysis (EFICA)

This method is a modified method of ICA algorithm. The non-Gaussian distributions in the sources are utilized for the separation of signals. In order to separate the signals of each \( d \) source, it requires a set of non-linear functions \( g_k(.)[k = 1,2,\ldots,d]\). It enhances fast ICA by giving a detailed data adaptive choice of non-linearties followed by the refinement step. Let \( S \) contains ‘N’ independent realizations of non-Gaussian.

The ISR matrix has the elements
The applied mother’s signal and the separated fetal ECG signal with EFICA BSS technique are shown in Fig.3.3.

**3.3.3 Weight adjusted second-order blind identification (WASOBI)**

This method is the weighted version of WASOBI algorithm. It falls in the category of second-order statistics of ICA algorithm. The time structure in sources is the main factor that is utilized by this algorithm for the separation of signals. The weighted version of SOBI algorithm is called WASOBI. This algorithm is the member to a family of second-order statistics-based ICA algorithms. This algorithm mainly depends on time structures. Both SOBI and WASOBI depend on Approximate Joint Diagonalization (AJD) of several time lagged determined correlation matrices, which is represented by

\[
R_x(\tau) = \frac{1}{N - \tau} \sum_{n=1}^{N-\tau} x[n]x^T[n + \tau]
\]

where \( x[n] \) is the \( n \)th column of \( x \).

If all sources are Gaussian AR of order M-1, the ISR matrix is given by

\[
ISR_{kl} = \frac{1}{N} \frac{\gamma_k (\gamma_l + \tau^2_l)}{\tau^2_k \gamma_k + \tau^2_k (\gamma_l + \tau^2_l)}
\]

where

\[
\gamma_k = \beta_k - \mu_k^2 \mu_k = E[\varepsilon_k g_k(\varepsilon_k)]
\]

\[
\tau_k = |\mu_k - \rho_k| \rho_k = E[g_k'(\varepsilon_k)]
\]

\[
\beta_k = E[g_k^2(\varepsilon_k)]
\]

where \( E[.\] represents the expectation operator

\( g_k(\cdot) \) represents the derivative of \( g_k(\cdot) \).
\[
ISR_{kl} = \frac{1}{N} \frac{\varphi_{kl} - \sigma_k^2 R_l[0]}{\varphi_{kl} \varphi_{tk} - 1 \sigma_t^2 R_k[0]}
\]  

(3.17)

where

- \(\sigma_k^2\) is the variance of the innovation sequence of the \(k^{th}\) source.

- \(\varphi_{kl} = \frac{1}{\sigma_k^2} \sum_{i,j=0}^{M-1} a_{il} a_{jl} R_k[i-j]\)

(3.18)

where

- \(\{a_{il}\}_{i=0}^{M-1}\) are the AR co-efficient of the \(l^{th}\) source with \(a_{ol} = 1\) for \(k=1,2,\ldots,d\).

- \(R_k[m]\) is the auto-correlation of the \(k^{th}\) source at time lag \(m\).

The applied mother’s signal and the separated fetal ECG signal with WASOBI BSS technique are shown in Fig. 3.4.

![Fig. 3.4 Fetal ECG signal obtained with EFICA algorithm](image)

**3.3.4 MULTI-COMBI Algorithm**

To initialize, take a stack of clusters which are denoted as ‘S’, \(S=\{x\}\) [11]

**Step 1:** Take a cluster from ‘S’ and denote the cluster ‘z’, and it is noted that the cluster ‘z’ will not be singleton.

**Step 2:** Apply EFICA and WASOBI on the cluster \(z\), and the corresponding ISR matrices \(ISR^{EF}\) and \(ISR^{WA}\) are calculated using the given formula

\[
ISR_{kl} = \frac{1}{N} \frac{\gamma_k (\gamma_l + \tau_k^2)}{\tau_k^2 \gamma_k + \tau_k^2 (\gamma_l + \tau_k^2)}
\]

(3.19)

\[
ISR_{kl} = \frac{1}{N} \frac{\varphi_{kl} - \sigma_k^2 R_l[0]}{\varphi_{kl} \varphi_{tk} - 1 \sigma_t^2 R_k[0]}
\]

(3.20)
**Step 3:** Construct a set ‘C’ of possible clusters from \( I \in \{1, \ldots, \text{dim}(z)\} \), i.e. if ‘z’ contains 3 signals, then the possible ‘C’ will be
\[
C = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}
\]

**Step 4:** Compute \( \hat{\mathcal{I}}R^{EF}(I) \) and \( \hat{\mathcal{I}}R^{WA}(I) \) for \( I \in C \)

**Step 5:** Assume \( E = \min_I \) and \( W = \min_I \hat{\mathcal{I}}R^{WA}(I) \)

**Step 6:** If \( E < W \), then pick up the set of best EFICA separated clusters,
\[
I_1 = \arg\min_I \hat{\mathcal{I}}R^{EF}(I) \quad (3.21)
\]
Continue for \( k = 1, 2 \ldots \) until the \( \hat{\mathcal{I}}R^{EF}(I_{k+1}) > W \)
This procedure picks up the lowest ISR. EFICA separated the clusters one by one.

If \( E > W \), extract \( S_1, S_2, \ldots, S_M \) from \( S^{WA} \) using \( \hat{\mathcal{I}}R^{WA} \)

**Step 7:** Update ‘S’ by substituting ‘z’ with \( S_1, S_2, \ldots, S_M \)

**Step 8:** All the clusters in ‘S’ are singleton, then stop. Otherwise return and start from Step 1.

The applied mothers signal and the separated fetal ECG signal with MULTI-COMBI a BSS technique are shown in Fig.3.5.

![Fig.3.5 Fetal ECG signal obtained with MULTI-COMBI algorithm](image)

### 3.3.5 FCOMBI algorithm

This method is used to separate the fetal ECG signal from the abdominal ECG signal. This method is a modified version of the MULTI-COMBI algorithm. By incorporating the strengths of EFICA and WASOBI, the FCOMBI algorithm is developed. These methods are described below.

**3.3.5.1 Steps in FCOMBI**

Step 1: Apply WASOBI on the input data.
Step 2: EFICA is applied on the output of WASOBI.
Step 3: Run WASOBI on the cluster of unresolved components in the output of EFICA.

The applied mother’s signal and the separated fetal ECG signal with FCOMBI a BSS technique are shown in Fig.3.6.

![Fetal ECG signal obtained with FCOMBI algorithm](image)

**Fig.3.6 Fetal ECG signal obtained with FCOMBI algorithm**

### 3.4 RESULTS

The results obtained with the various fetal ECG separation methods used in this work are given in Table 3.2.

<table>
<thead>
<tr>
<th>Method</th>
<th>SIR</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFICA</td>
<td>0.918949</td>
<td>4.039379</td>
</tr>
<tr>
<td>WASOBI</td>
<td>3.553334</td>
<td>1.067269</td>
</tr>
<tr>
<td>MULTI-COMBI</td>
<td>1.760368</td>
<td>5.376125</td>
</tr>
<tr>
<td>FCOMBI</td>
<td>6.544849</td>
<td>7.927071</td>
</tr>
</tbody>
</table>

From Table 3.2., it is observed that the proposed BSS method known as FCOMBI algorithm has produced better results among the other techniques. This is confirmed by observing the SIR and PSNR of the separated signal. The proposed algorithm for the classification of fetal ECG disorders is carried out using the entire above BSS techniques. Morphological features are extracted from the fetal ECG signals obtained using the above methods. Classifiers are employed to classify the
extracted features to their relevant class of disorders. The proposed method for the classification of fetal ECG disorders is discussed in Chapter 4 of this thesis.

3.5 SUMMARY
In this chapter, the undesirable noise signals from the abdominal ECG signal were removed using different noise removal techniques, and the results were compared. It also discussed various separation techniques for the separation of fetal ECG signal, and the effectiveness of the proposed algorithm was clarified using SIR and PSNR values.