CHAPTER II
PRELIMINARIES AND LITERATURE REVIEW

This chapter is intended to give more details on the basic terminology and notations [8, 13, 35] required for the research work. Moreover, literature review is carried out briefly.

2.1 BASIC DEFINITIONS IN GRAPHS

Throughout this chapter, the simple, undirected and finite graph with number of vertices ‘p’ and number of edges ‘q’ are considered.

Definition 2.1.1. A graph \( G = (V(G), E(G)) \) has two finite sets: \( V(G) \), the vertex set of the graph, generally denoted by \( V \), which is a nonempty set of elements called vertices, and \( E(G) \), the edge set of the graph, generally denoted by \( E \), which is a possibly empty set of elements called edges, such that each edge \( e \) in \( E \) is assigned an unordered pair of vertices \((u, v)\), called the end vertices of \( e \).

Definition 2.1.2. The order of \( G \) is the cardinality of \( V(G) \), i.e., \( p = |V(G)| \) and the size of \( G \) is the cardinality of \( E(G) \), i.e., \( q = |E(G)| \).

Definition 2.1.3. An edge \( e \) of a graph \( G \) is said to be incident with the vertex \( v \) if \( v \) is an end vertex of \( e \).

Definition 2.1.4. Two edges \( e \) and \( e' \) which are incident with a common vertex \( v \) are said to be adjacent.

Definition 2.1.5. The degree \( d(v) \) or \( d_{G(v)} \) of \( v \) is the number of edges of \( G \) incident with \( v \), counting each loop twice, i.e., it is the number of times \( v \) is an end vertex of an edge.

Definition 2.1.6. Two vertices of a graph which are adjacent are said to be neighbours. The set of all neighbours of a vertex \( v \) of \( G \) is called the neighbourhood set of \( v \). It is denoted by \( N(v) \) or \( N[v] \) and they are respectively
known as open and closed neighbourhood set. 
\[ N(v) = \{ u \in V(G) / u \text{ adjacent to } v \text{ and } u \neq v \} \] and \[ N[v] = N(v) \cup \{v\}. \]

**Definition 2.1.7.** A walk in which all the vertices are distinct is called a **path**. A path on \( n \) vertices is denoted by \( P_n \). The length of \( P_n \) is \( n - 1 \).

**Definition 2.1.8.** A path that begins and ends at the same vertex is called a **cycle**.

**Definition 2.1.9.** A graph \( G \) is called **acyclic** if it has no cycles.

**Definition 2.1.10.** A graph \( G \) is called a **tree** if it is a connected acyclic graph.

**Definition 2.1.11.** An **isomorphism** of graphs \( G \) and \( H \) is a bijection \( f \) between the vertex sets of \( G \) and \( H \). \( f : V(G) \rightarrow V(H) \) such that any two vertices \( u \) and \( v \) of \( G \) are adjacent in \( G \) if and only if \( f(u) \) and \( f(v) \) are adjacent in \( H \). If an isomorphism exists between two graphs, then the graphs are called isomorphic graphs and it is denoted as \( G \cong H \).

**Definition 2.1.12.** If for every vertex \( v \), \( d(v) = k \) for some positive integer \( k \) of the graph \( G \), then \( G \) is called **\( k \)-regular**. A regular graph is one that is \( k \)-regular for some \( k \in N \).

**Definition 2.1.13.** A graph \( G \) is **complete** if every pair of its vertices is adjacent. A complete graph on \( p \) vertices is denoted by \( K_p \).

**Definition 2.1.14.** In the graph \( G \), two vertices \( u \) and \( v \) are called **connected** if \( G \) contains a path from \( u \) to \( v \).

**Definition 2.1.15.** A graph \( G \) is called **connected** if every two of its vertices of \( G \) are connected. A graph that is not connected is called **disconnected**.

**Definition 2.1.16.** The **complement graph** \( G^c \) of the graph \( G = (V(G), E(G)) \) is defined as the graph which has \( V \) as its set of vertices and two vertices are adjacent in \( G^c \) if and only if they are not adjacent in \( G \).
**Definition 2.1.17.** The union of two disjoint graphs $G_1$ and $G_2$ is a graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

**Definition 2.1.18.** If $G_1$ and $G_2$ are two sub graphs of $G$ with at least one vertex in common then the intersection $G_1 \cap G_2$ is given by $V(G_1 \cap G_2) = V(G_1) \cap V(G_2)$ and $E(G_1 \cap G_2) = E(G_1) \cap E(G_2)$.

**Definition 2.1.19.** Let $G_1$ and $G_2$ be two graphs with no vertex in common, the join $G_1 + G_2$ is the graph obtained from the disjoint union $G_1 \cup G_2$ by adding new edges from each vertex in $G_1$ to every vertex in $G_2$.

**Definition 2.1.20.** The Cartesian product graph $G_1 \times G_2$ of graphs $G_1$ and $G_2$ whose vertex set is $V(G_1) \times V(G_2)$ can be defined as follows. Let $u$ be a vertex in $V(G_1)$ and $v$ be a vertex in $V(G_2)$. Then $(u, v)$ is an element of $G_1 \times G_2$ and $(u, v)$ is adjacent to $(u', v')$ if and only if

1. $u = u'$ and edge $vv'$ belongs to $E(G_2)$ or
2. $v = v'$ and edge $uu'$ belongs to $E(G_1)$.

**Definition 2.1.21.** Let $G$ be a graph. If the vertex set $V$ of $G$ can be partitioned into two non-empty subsets $X$ and $Y$ in such a way that each edge of $G$ has one end in $X$ and the other in $Y$ then $G$ is called bipartite. The partitions $X, Y$ are called bipartition of $G$.

**Definition 2.1.22.** A complete bipartite graph is a simple bipartite graph $G$, with bipartition $X, Y$, in which every vertex in $X$ is joined to every vertex of $Y$. If $X$ has $m$ vertices and $Y$ has $n$ vertices, such a graph is denoted by $K_{m,n}$.

**Definition 2.1.23.** $P_{n+k}^*$ is a graph obtained by adjoining $k$ pendant vertices to any one of the 2 vertices in bipartition $X$ of $K_{2,n}$, $n \geq 2$.

**Definition 2.1.24.** A rooted graph is a graph in which one vertex is named in a special way so as to distinguish it from other vertices. The special node is called the root of the graph. Let $G$ be a rooted graph. The graph which is obtained by
identifying the roots of $n$ copies of $G$ is called the one-point union of $n$ copies of $G$ and is denoted by $G^{(n)}$.

**Definition 2.1.25.** The depth of a vertex is the length (number of edges) of the path from the vertex to the tree’s root.

**Definition 2.1.26.** The height of a vertex is the length of the longest path between that vertex and a leaf.

**Definition 2.1.27.** The level of a vertex is defined by $1 +$ the number of connection between the vertex and the root.

**Definition 2.1.28.** An $m$-ary tree is a rooted tree in which each vertex has no more than $m$ children.

**Definition 2.1.29.** A full $m$-ary tree is an $m$-ary tree where within each level every vertex has either 0 or $m$ children.

**Definition 2.1.30.** A perfect $m$-ary tree is a full $m$-ary tree in which all leaf vertices are at the same depth. The complete bipartite graph $K_{1,n}$ is called a star graph. In the star graph $K_{1,n}$ the vertex of degree $n$ is called as center vertex and its end vertices are called as apex vertices. The edges are called spokes.

**Definition 2.1.31.** A caterpillar is defined by a graph such that deletion of every vertex of degree one results in a path.

**Definition 2.1.32.** The graph $\langle K_{1,n}, K_{1,m}, w \rangle$ is defined as the graph obtained by joining the center ‘$u$’ of the star $K_{1,n}$ and the center ‘$v$’ of another star $K_{1,m}$ to a new vertex $w$.

**Definition 2.1.33.** A spider is a tree with one vertex of degree at least 3 and all others with degree at most 2.
Definition 2.1.34. A coconut Tree $CT(n,m)$ is the graph obtained from the path $P_n$ by appending $m$ new pendant edges at an end vertex of $P_n$.

Definition 2.1.35. A friendship graph $F_n (n \geq 2)$ is the one-point union of $n$ cycles of length 3.

Definition 2.1.36. The dumbbell graph $D(m,n)$ is a graph obtained by joining a vertex of the cycle $C_m$ to a vertex of the cycle $C_n$ by an edge.

Definition 2.1.37. The corona graph $C_m(K_{1,n})$ is a graph obtained by joining $i^{th}$ vertex of the cycle $C_m$ to the apex vertex of the $i^{th}$ copy of $K_{1,n}$.

Definition 2.1.38. The Dutch windmill graph $D_n(m)$ is the graph obtained by taking $m$ copies of the cycle graph $C_n$ with a vertex in common.

Definition 2.1.39. The shadow graph $D_2(G)$ of a connected graph $G$ is constructed by taking two copies of $G$ say $G_1$ and $G_2$ and join each vertex $v_i$ in $G_1$ to the adjacent vertices of the corresponding vertex $u_i$ in $G_2$.

Definition 2.1.40. For $n \geq 3$, prism $Y_n$ is the Cartesian product $C_n \times K_2$ where $C_n$ is a cycle on $n$-vertices and $K_2$ is the complete graph on 2-vertices.

Definition 2.1.41. A wheel graph $W_n$ is obtained from a cycle $C_n$ by adding a new vertex and joining it to all the vertices of the cycle by an edge. The new edges are called spokes of the wheel.

2.2 GRAPH LABELING

This chapter gives more details on various graphs labeling for graph $G=(V(G),E(G))$ with $p$ vertices and $q$ edges.

Definition 2.2.1. A vertex labeling of a graph $G=(V(G),E(G))$ is an assignment
$f$ of labels from a set of integers to the vertices of $G$ that induces a label for each edge $uv \in E(G)$.

**Definition 2.2.2.** An edge labeling of a graph $G = (V(G), E(G))$ is an assignment $f$ of labels from a set of integers to the edges of $G$ that induces a label for each vertex $v \in V(G)$.

**Definition 2.2.3.** A graceful labeling of a graph $G = (V(G), E(G))$ with $|V(G)| = p$ vertices and $|E(G)| = q$ edges is a one-to-one mapping $f$ of the vertex set $V(G)$ into the set $\{0, 1, 2, \ldots, q\}$ such that when each edge $uv$ is assigned the label $|f(u) - f(v)|$, the resulting edge labels are distinct. A graph which admits the graceful labeling is called a graceful graph.

**Definition 2.2.4.** A graph $G = (V(G), E(G))$ with $p$ vertices and $q$ edges admit an odd graceful labeling if $f : V(G) \to \{0, 1, 2, \ldots, 2q - 1\}$ is injective and the induced function $f^* : E(G) \to \{1, 3, \ldots, 2q - 1\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. A graph which admits odd graceful labeling is called an odd graceful graph.

**Definition 2.2.5.** A graph $G = (V(G), E(G))$ with $p$ vertices and $q$ edges admits an even graceful labeling if $f : V(G) \to \{0, 1, 2, \ldots, 2q\}$ is injective and the induced function $f^* : E(G) \to \{2, 4, 6, \ldots, 2q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. A graph which admits even graceful labeling is called an even graceful graph.

**Definition 2.2.6.** A graph $G = (V(G), E(G))$ with $p$ vertices and $q$ edges is said to be edge graceful if there is a bijection $f : E(G) \to \{1, 2, \ldots, q\}$ such that the induced mapping $f^* : V(G) \to \mathbb{Z}_p$ given by $f^*(v) = \left(\sum f(uv)\right)(\text{mod } p)$, sum taken over all edges incident to $v$, is a bijection.
Definition 2.2.7. A \((p, q)\)-graph \(G\) is said to be edge-odd graceful if there is a bijection \(f\) from \(E(G)\) to the set \(\{1, 3, \ldots, 2q - 1\}\) such that the induced mapping \(f^*\) from \(V(G)\) to the set \(\{0, 1, 2, \ldots, (2k - 1)\}\) defined by \(f^*(v) = (\sum f(uv))(\text{mod } 2k)\), sum taken over all edges incident to \(v\), where \(k = \max\{p, q\}\) makes all edge labels distinct.

Definition 2.2.8. A \((p, q)\)-graph \(G\) is said to be even vertex graceful if there exists an injective map \(f : E(G) \rightarrow \{1, 2, 3, \ldots, 2q\}\) such that the induced mapping \(f^* : V(G) \rightarrow \{0, 2, 4, \ldots, (2k - 2)\}\) defined by \(f^*(v) = (\sum f(uv))(\text{mod } 2k)\), sum taken over all edges incident to \(v\), where \(k = \max\{p, q\}\) makes all edge labels distinct.

Definition 2.2.9. An odd-even graceful labeling of a graph \(G\) with \(p\) vertices and \(q\) edges is an injection \(f\) from \(V(G)\) to \(\{1, 3, \ldots, 2q + 1\}\) such that, when each edge \(uv\) is assigned the label \(f^*(uv) = |f(u) - f(v)|\), the resulting edge labels are distinct. The graph \(G\) with an odd-even graceful labeling is called an odd-even graceful graph.

Definition 2.2.10. A graph \(G\) is said to have \(k\)-equitable labeling if there exists a mapping \(f : V(G) \rightarrow \{0, 1, 2, \ldots, k - 1\}\) such that the induced mapping \(f^* : E(G) \rightarrow \{0, 1, 2, \ldots, k - 1\}\) defined by \(f^*(uv) = |f(u) - f(v)|\) satisfies the conditions: \(v_f(i) - v_f(j) \leq 1\) and \(e_f(i) - e_f(j) \leq 1, 0 \leq i, j \leq k - 1\), where \(v_f(i)\) and \(e_f(i)\) denote the number of vertices and number of edges having label \(i\) under \(f\) and \(f^*\) respectively, \(0 \leq i \leq k - 1\). A graph which admits \(k\)-equitable labeling is called \(k\)-equitable graph.

Definition 2.2.11. A mapping \(f : V(G) \rightarrow \{0, 1\}\) is called binary vertex labeling of \(G\) and \(f(v)\) is called the label of vertex \(v\) of \(G\) under \(f\). The induced edge labeling \(f^* : E(G) \rightarrow \{0, 1\}\) is given by \(f^*(uv) = |f(u) - f(v)|\). Let \(v_f(0), v_f(1)\) be the number of vertices of \(G\) having labels 0 and 1 respectively under \(f\) and let \(e_f(0), e_f(1)\) be the number of vertices of \(G\) having labels 0 and 1 respectively under \(f^*\).
Definition 2.2.12. A binary vertex labeling of $G$ is called cordial labeling if $|v_f(0)−v_f(1)| \leq 1$ and $|e_f(0)−e_f(1)| \leq 1$. A graph $G$ is cordial if it admits cordial labeling.

Definition 2.2.13. Let $G = (V(G), E(G))$ with $p$ vertices and $q$ edges and $f : E(G) \rightarrow \{0, 1\}$. Define $f^*$ on by $f^*(v) = \left(\sum f(uv)\right)(\text{mod} 2)$, sum taken over all edges incident to $v$. The function $f$ is called an $E$-cordial labeling of $G$ if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. A graph that admits $E$-cordial labeling is called $E$-cordial.

Definition 2.2.14. A graph $G$ is said to have totally magic cordial labeling with constant $C$ if there exists a mapping $f : V(G) \cup E(G) \rightarrow \{0, 1\}$ such that $f(u) + f(v) + f(uv) \equiv C \text{ (mod} 2)\text{ for all } uv \in E(G)$ and $|n_f(0)−n_f(1)| \leq 1$, where $n_f(i)(i = 0, 1)$ is the sum of the number of vertices and edges with label $i$.

Definition 2.2.15. A vertex labeling of graph $G$ $f : V(G) \rightarrow \{-1, +1\}$ with induced edge labeling $f^* : E(G) \rightarrow \{-1, +1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $|v_f(+1)−v_f(-1)| \leq 1$ and $|e_f(+1)−e_f(-1)| \leq 1$, where $v_f(+1)$ is the number of vertices labeled with (+1), $v_f(-1)$ is the number of vertices labeled with (−1), $e_f(+1)$ is the number of edges labeled with (+1), and $e_f(-1)$ is the number of edges labeled with (−1). A graph $G$ is signed product cordial if it admits signed product cordial labeling.

Definition 2.2.16. A graph $G = (V(G), E(G))$ with $p$ vertices is said to be multiplicative if the vertices of $G$ can be labeled with distinct positive integers such that label induced on the edges by the product of labels of end vertices are all distinct.
Definition 2.2.17. A graph $G$ with $p$ vertices is said to be strongly multiplicative if the vertices of $G$ can be labeled with $p$ distinct integers $1, 2, \ldots, p$ such that the labels induced on the edges by the product of labels of the end vertices are all distinct.

Definition 2.2.18. A graph $G = (V(G), E(G))$ with $|V| = p$ is said to have a modular multiplicative divisor labeling if there exists a bijection $f : V(G) \rightarrow \{1, 2, \ldots, p\}$ and the induced function $f^* : E(G) \rightarrow \{0, 1, 2, \ldots, p-1\}$ where $f^*(uv) = (f(u)f(v))(\text{mod} \ p)$ such that $p$ divides the sum of all edge labels of $G$.

Definition 2.2.19. A labeling $f : V(G) \rightarrow \{0, 1, 2, \ldots, |V(G)|\}$ is called an $(a,d)$-edge-antimagic vertex labeling, if the set of edge-weights $\{f(u) + f(v) : uv \in E(G)\}$ forms an arithmetic sequence with the initial term $a$ and the difference $d$, where $a$ and $d$ are two positive integers.

Definition 2.2.20. A graph $G$ with $p$ vertices and $q$ edges is a mean graph if there is an injective function $f$ from the vertices of $G$ to $\{0, 1, 2, \ldots, q\}$ such that when each edge $uv$ is labeled with $\frac{f(u) + f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u) + f(v) + 1}{2}$ if $f(u) + f(v)$ is odd, then the resulting edges are distinct.

Definition 2.2.21. Let $G = (V(G), E(G))$ be a graph with $p$ vertices and $q$ edges, $k$ be an arbitrary natural number larger than 2. If there exists a mapping $f : V(G) \rightarrow \{0, 1, 2, \ldots, q+k-1\}$ such that $f(u) \neq f(v)$ for every $u, v \in V(G)$ and an induced edge mapping $f^*$ defined by $f^*(uv) = |f(u) - f(v)|$, for every $uv \in E(G)$, is a bijection, then a graph is called a $k$-graceful graph, $f$ is called a $k$-graceful labeling, while $f^*$ is called an induced edge’s $k$-graceful labeling.

Definition 2.2.22. Let $G = (V(G), E(G))$ be a graph with $p$ vertices and $q$ edges.
For given positive integer \( k \), a graph \( G \) is \( k \)-edge graceful if there is a bijection \( f : E(G) \to \{k, k + 1, \ldots, k + q - 1\} \) such that the induced mapping \( f^* : V(G) \to \mathbb{Z}_p \) given by \( f^*(v) \equiv (\sum f(uv)) \pmod{p} \), sum taken over all edges incident to \( v \), is a bijection too.

**Definition 2.2.23.** \( k \)-even-edge graceful labeling of a graph \( G = (V(G), E(G)) \) is an injection \( f \) from \( E \) to \( \{2k - 1, 2k, 2k + 1, \ldots, 2k + 2q - 2\} \) such that the induced mapping \( f^* \) defined on \( V \) by \( f^*(v) \equiv (\sum f(uv)) \pmod{2s} \), sum taken over all edges \( uv \) incident to \( v \) are distinct and even where \( s = \max\{p,q\} \) and \( k \) is an integer greater than or equal to 1. A graph \( G \) that admits \( k \)-even-edge graceful labeling is called a \( k \)-even-edge graceful graph.

**Theorem 2.2.21.** [4] A graph \( G \) is a mean graph if it is an \((a,d)\)-edge antimagic vertex graph, \( d \geq 2 \).

**Theorem 2.2.22.** [22] If a \((p,q)\)-graph that is tree or where \( q \geq p \) is super edge-magic, then \( G \) is sequential.

**Theorem 2.2.23.** [22] If a \((p,q)\)-graph \( G \) with \( q \geq p \) is super edge-magic, then \( G \) is harmonious.

**Theorem 2.2.24.** [22] If a tree \( T \) of order is super edge-magic then \( T \) is harmonious.

**Theorem 2.2.25.** [22] A graph \( G \) has an \( \alpha \)-valuation if it is a super edge-magic bipartite -graph with partite sets \( V_1 \) and \( V_2 \), where \( |V_1| = p_1 \) and \( |V_2| = p_2 \), and let \( \{1, 2, 3, \ldots, 2p_1 - 1\} \) be a super edge-magic labeling of \( G \) such that \( f(V_1) = \{1, 2, 3, \ldots, p_1\} \).

**Theorem 2.2.26.** [22] A tree \( T \) is super edge-magic if \( T \) admits an \( \alpha \)-valuation.

**Theorem 2.2.27.** [22] If a graph \( G \) is super edge-magic, then \( G \) is cordial.
Theorem 2.2.28. [9] If a graph $G$ has superior edge-antimagic total labeling, then $G + K_1$ admits edge-antimagic total labeling.

Theorem 2.2.29. [9] If $G$ is superior edge-antimagic then $G^+$ has an edge-antimagic total labeling.

Theorem 2.2.30. [9] If $G$ has superior edge-antimagic total labeling, then $G \ast P_n$ admits edge-antimagic total labeling.

Theorem 2.2.31. [9] If $G$ has superior edge-antimagic total labeling, then $G \ast F_{L_n}$ admits edge-antimagic total labeling.

Theorem 2.2.32. [6] If $G$ has an $(a,d)$-edge antimagic vertex labeling, then $G + K_1$ is sequential ($G$ is not a tree).

Theorem 2.2.33. [9] If $G_1$ has superior edge-antimagic total labeling and $G_2$ has super edge-antimagic labeling, then $G_1 \ast G_2$ admits edge-antimagic total labeling.

Theorem 2.2.34. [72] A graph $G$ is super-edge graceful tree of odd order if it is edge graceful.

Theorem 2.2.35. [72] A graph $G$ is edge graceful if $G$ is a super-edge graceful graph and $q \equiv \begin{cases} -1 \pmod{p} & \text{if } q \text{ is even} \\ 0 \pmod{p} & \text{if } q \text{ is odd} \end{cases}$.

2.3 MULTI PROTOCOL COMMUNICATION NETWORKS

Definition 2.3.1. In information technology (IT), a **network** is a group of computers or peripherals which are linked by some sort of communication paths to exchange data or information. Networks can be connected with other networks using cable media or wireless media and it can contain sub networks.

Definition 2.3.2. In a MPLS communication network, a **node** is referred to a connection point, either a redistribution point or a communication end point for the data packet transmissions. Typically, a node has ability to create, receive and transmit information over a communication channel.
Definition 2.3.3. In internet protocol specifications, a network **host** is a computer or other network device connected to a communication network. Each host has its own Internet Protocol (IP) address. When you connect to internet via local internet service provider, the computer gets public IP address from the service provider and it is a host in the internet communication network during that time.

Definition 2.3.4. A **router** is a networking device that assists with sending the data packets from one network device to another based on its contents. A router connects multiple networks together via wired or wireless connections and it performs the routing functions between the networks. Router is defined in layer 3, which provides routing functionality. Some of the newer switches do perform layer 3 routing functions same as the router. Once the data packet reaches the router, it checks the packet for the destination IP address to determine how to route the packet to the adjacent node.

Definition 2.3.5. An **edge router** is referred to a router that routes the traffic between the local area network (LAN) and an external network. It resides at the edge or boundary of a communication network and it is often called as a boundary router. The edge router ensures the connectivity of its network with external networks such as backbone network, wide area network (WAN) or Internet network.

Definition 2.3.6. In a communication network, a **routing switch** is a device that combines the functions of a layer 2 switch, which forwards data by looking at a physical device address (MAC address) and a layer 3 router, which forwards packets by locating a next hop IP address. Because they perform some of the layer 3 functions, routing switches are generally called as the layer-3 devices or switches.

Definition 2.3.7. A **LAN** (Local Area Network) is a network that connects computers and devices within the proximity to each other. It uses a common wired communications media or wireless communication media to connect to a local server or a local gateway. Typically, a LAN includes computers and devices connected to a local gateway within a small geographic area such as an office building, shops, hospitals, residence or school.
Definition 2.3.8. A WAN (Wide Area Network) is a broader network which has two or more LAN networks which are separated by a large geographical area. Typically, WAN bandwidth is either privately owned or taken on rent from the service providers. In a corporate world, the WAN links are needed to connect two or more offices and they are usually established with the leased line circuits in the modern world or satellites in the earlier days.

Definition 2.3.9. In information technology, a protocol is the special set of rules that allows two or more devices to transmit information when they communicate. These are the rules or standards that define the syntax, synchronization of communication and possible error recovery methods that they have to be agreed upon by the communicating devices involved.

Definition 2.3.10. In a communication network, internet Protocol is a protocol that defines format of packets and responsible of delivering data packets from source host to the destination host. Each device has at minimum one public or private IP (layer 3 address) that individually identifies it from rest of the other devices in the Internet network.

Definition 2.3.11. IPv4 (Internet Protocol version 4) is 32 bits network IP address that gets assigned to the each host. It is divided by group of 8 bits called octet. It is needed to route the data traffic over different kinds of communication networks. IPv4 works based on a best effort delivery model as it uses connectionless protocol. This model does not guarantee a packet delivery and it does not also ensure proper sequencing or helps with eliminating the duplicate packet delivery. Typically, these features like data integrity are controlled by transmission control protocol (TCP).

Definition 2.3.12. TCP/IP (Transmission Control Protocol (or) Internet Protocol) is the basic communication protocol that used to connect hosts in the internet public network and also in a private IP network. It makes sure to communicate between the source host and the destination host by defining how the data should be packetized, sent, routed over the network and received.

Definition 2.3.13. A TTL (Time-to-live) field in the data packet determines whether or not the packet should be forwarded to the next node or
discarded. TTL avoids a data packet from circulating indefinitely. In IPV6, the TTL is renamed as the hop count. The sender of the data packet normally determines and sets the TTL value. The value of TTL is from 1 to 255. Each networking device decreases by 1 from the TTL value when it receives the data packet. The network device sends the data packet to the adjacent node whenever the TTL value is greater than 0. Once the TTL value reaches 0, the network device discards the data packet and create an internet control message protocol (ICMP) error message to the sender, that probably need to resend the data packet.

**Definition 2.3.14.** In a telecommunication network, an **ingress** (stepping into) refers to the data traffic entering into the network from the different network.

**Definition 2.3.15.** In a telecommunication network, an **egress** (going out) refers to the data traffic leaving the network to another network.

**Definition 2.3.16.** **QoS (Quality of Service)** refers to broadest of networking techniques designed to guarantee predictable level of applications performance. The data packet communication rates, error rates and other characteristics can be calculated, and then further enhanced, and the performance is guaranteed for the critical applications to certain level. QOS provides prioritization of data packets or network traffic.

**Definition 2.3.17.** In a communication network, **layering** is nothing but splitting different kind of tasks of computer communication (ex: packetization, transmission, routing) needed into different distinct logical functions or layers which works together in some consecutive and hierarchical way, with each layer typically having an interface only to the upper layer above it and the lower layer below it.

**Definition 2.3.18.** **Layer 2** is called as the Data Link layer, which is the second layer in the open systems interconnection (OSI) model. The data link layer regulates the flow of data across the physical media and it provides a well defined interface to the network layer. The switch is generally falls into the data link layer. It operates at layer 2 level and switches data to the adjacent network device by using the destination media access control (MAC) address table.
**Definition 2.3.19. Layer 3** is called as the network layer, which is the third layer of the OSI model. The router falls into the network layer. It operates at the layer 3 and builds the routing table with the network addresses of the adjacent network devices in the local area and the wide area network, then choosing the best routes or paths, quality of service and routing the data packets to the adjacent network device.

**Definition 2.3.20.** A **packet** is the piece of information or data which is routed between a source host and a destination host over the Internet network or any other packet-switched communication network. Once any data or information (http or https web pages, chat messages, voice or video messages, files, Email messages etc) is sent from source to the destination over the Internet network, TCP layer divides the data or information into small sizes to route the packets across the network. Each and every packet is uniquely numbered and it consists of the source and the destination IP address. Each data packet can travel via different route or path in the network before it reaches the destination. Once all the data packets are arrived at the destination, they are reassembled to get the original message.

**Definition 2.3.21. Packet-switching** describes the type of traffic communications method where each packet is checked for its destination IP address and it is routed across the different network. Because each packet is processed and sent individually, it is possible that the packets can even follow different paths or routes to reach its destination host. This type of communication method between sender and receiver is connectionless communication (rather than dedicated connection). Most data traffic over the Internet uses packet switching communication method and the Internet is basically a connectionless network.

**Definition 2.3.22.** A **Forwarding Equivalence Class (FEC)** is a term used in the MPLS network to describe a group of packets with identical characteristics which probably forwarded in the identical method, over the same path and with the same forwarding process. This means that they probably bound to the identical MPLS label.

**Definition 2.3.23.** **Router ID** is a 32-bit number that individually recognizes the router in the communication network. The router ID is assigned by selecting the highest IP address allocated to the same router interfaces which are active.
2.4 LITERATURE REVIEW

Various types of graph labeling like graceful labeling, cordial labeling, total magic cordial labeling, equitable labeling, strongly multiplicative labeling, complementary labeling etc., have been studied in the recent years.

In the year 1991, Gnanajothi [29] pioneered the concept of ‘odd graceful graph’.

After that more than hundred papers have been published on odd graceful labeling. A few important results are as follows. (i) Every graph with \( \alpha \) -labeling has an odd graceful labeling. (ii) Every graph with an odd cycle is not odd graceful.

(iii) The cycle graph \( C_n \) is odd graceful if and only if \( n \) is even. (iv) The comb graph \( C_n \Theta K_1 \) is odd graceful if and only if \( n \) is even. (v) The comb graph \( P_n \Theta K_1 \), the complete bipartite graph \( K_{m,n} \), the disjoint union of copies of \( C_4 \), the book graph, the one-point union of copies of \( C_4 \) and the path graph \( P_n \) are all odd graceful. (vi) The Cartesian product graph \( C_n \times K_2 \) is odd graceful if and only if \( n \) is even. (vii) Rooted trees of height 2 are odd graceful. (viii) The graphs obtained from \( P_n \) (\( n \geq 3 \)) by adding exactly two leaves at each vertex of degree 2 of \( P_n \) are odd graceful. (viii) The graphs obtained from a star by adjoining to each end vertex the path \( P_3 \) or by adjoining to each end vertex the path \( P_4 \) are odd graceful graphs [7, 19, 32, 79, 85, 91].

Nirmala Gnamam Pricilla (2008), [56] established the concept of even graceful labeling. It has been proved that the well known graphs such that the grid graph \( P_m \times P_n \forall m,n \), the one vertex union of ‘\( t \)’ isomorphic copies of any path graph \( P'_n \forall n > 1,t \geq 1 \), the comb graph \( P_n \Theta K_1 \forall n \), the complete bipartite graph \( K_{m,n} \forall m,n \) and the multiple fan graph \( P_n + K_1 \forall n > 1,t \geq 1 \) are even graceful.

Lo S.P (1985), [46] defined edge graceful labeling and generalized the necessary condition for a graph with \( p \) vertices and \( q \) edges to be edge graceful is that \( q(q + 1) \equiv \left[ p(p + 1)/2 \right] \ (\text{mod} \ p) \) [28, 71, 72, 73].
Solairaju and Chithra (2008), [76] defined the concept of edge-odd graceful labeling and developed edge-odd graceful labeling of various graphs related to paths, the comb graph $P_n \Theta K_1$, where $n \geq 2$, the bistar graph $B_{n,n}$, when $n$ is odd, the graph $\langle K_{1,n} : 2 \rangle$ and the double star graph $K_{1,n,n}$, when $n$ is even [36, 52, 70, 77, 78].

Gayathri et al., (2007), [25] found the concept of even-edge graceful labeling and have proved the following results. (i) Cycles are even-edge graceful if and only if the cycles are odd. (ii) Even cycles with one pendant edge are even-edge graceful. (iii) Wheels and fans $P_n + K_1$ are even-edge graceful. (iv) $C_4 \cup P_m \forall m$ are even-edge graceful.

Sridevi et al., (2012), [80] derived the concept of odd-even graceful labeling. They proved that some well known graphs namely the path graph $P_n$, the comb graph $P_n \Theta K_1$, the star graph $K_{1,n}$, the graph $K_{1,2,n}$, the complete bipartite graph $K_{m,n}$ and the bistar graph $B_{m,n}$ are odd-even graceful.

Cahit (1990), [15] proposed the notion of $k$-equitable labeling of graphs. The author has shown the following: $C_n$ is 3-equitable if and only if $n \equiv 3 \bmod 6$, the friendship graph $C_n^{(n)}$ is 3-equitable if and only if $n$ is even, an Eulerian graph with $q \equiv 3 \bmod 6$ edges is not 3-equitable and all caterpillars are 3-equitable.

Szaniszlo (1994), [82] revealed that $K_n$ is not $k$-equitable for $3 \leq k < n$, and $C_n$ is $k$-equitable if and only if $k$ meets all of the following conditions: $n \neq k$; if $k \equiv 2,3 \bmod 4$, then $n \neq k - 1$; if $k \equiv 2,3 \bmod 4$, then $n \neq k \bmod 2k$ [81, 90, 95].

Jayapal Baskar Babujee and Shobana Loganathan (2011), [10] introduced the new concept of labeling called the signed product cordial labeling and investigated this labeling for path graph, cycle graphs, star $K_{1,n}$, Bistar $B_{n,n}$, $P_n^+ n \geq 3$ and $C_n^+$ [59, 67].

Cahit (1987), [14] initiated a variation of both graceful and harmonious labeling. Let $f$ be a function from the vertices of $G$ to $\{0,1\}$ and for each edge $xy$
assign the label \( |f(x) - f(y)| \). Call \( f \) a cordial labeling of \( G \) if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. A graph is called cordial if it admits cordial labeling and he conjectured that every tree is cordial, \( K_n \) is cordial if \( n \leq 3 \), \( K_{m,n} \) is cordial for all \( m \) and \( n \), the friendship graph \( C^{(t)}_3 \) is cordial if and only if \( t \neq 2 \text{mod} 4 \), all fans are cordial and the wheel graph \( W_n \) is cordial if and only if \( n \equiv 3 \text{mod} 4 \).

**Cahit** and **Yilmaz** (2000), [16] identified a graph as \( E_k \) cordial if it is possible to label the edges with the numbers from the set \( \{0, 1, 2, \ldots, k-1\} \) in such a way that, at each vertex \( v \), the sum of the labels on the edges incident with \( v \) modulo \( k \) satisfies the inequalities \( |v(i) - v(j)| \leq 1 \) and \( |e(i) - e(j)| \leq 1 \), where \( v(s) \) and \( e(t) \) are respectively, the number of vertices labeled with \( s \) and the number of edges labeled with \( t \) [44, 86, 92 and 94].

**Cahit** (2002), [17] developed another two well known graph labeling namely total sequential cordial labeling and totally magic cordial labeling based on cordial labeling. He indicated a graph \( G \) to be totally magic cordial provided that there is a mapping \( f \) from \( V(G) \cup E(G) \) to \( \{0, 1\} \) such that \( f(a) + f(b) + f(ab) \) is a constant modulo 2 for all edges \( ab \in E(G) \) and \( |f(0) - f(1)| \leq 1 \) where \( f(0) \) denotes the number of vertices labeled with 0 and number of edges labeled with 0 and \( f(1) \) denotes the number of vertices labeled with 1 and the number of edges labeled with 1[34, 58, 68, 93].

**Cahit** stated a graph \( G \) is totally sequential cordial if there is a mapping \( f \) from \( V(G) \cup E(G) \) to \( \{0, 1\} \) such that for each edge \( e = ab \) with \( f(e) = |f(a) - f(b)| \) it is true that \( |f(0) - f(1)| \leq 1 \) where \( f(0) \) denotes the number of vertices labeled with 0 and the number of edges labeled with 0 and \( f(1) \) denotes the number of vertices labeled with 1 and the number of edges labeled with 1. The author exhibited that the following graphs have a total magic cordial labeling: \( K_{m,n} \) \((m, n > 1)\), trees, cordial graphs and \( K_n \) if and only if \( n = 2, 3, 5, \) or 6. He also proved that the
following graphs have a total sequential cordial labeling: trees, cycles, complete bipartite graphs, friendship graphs, cordial graphs, cubic graphs other than $K_4$, wheels $W_n (n > 3)$, $K_{4k+1}$ if and only if $k \geq 1$ and $\sqrt{k}$ is an integer, $K_{4k+2}$ if and only if $\sqrt{4k+1}$ is an integer, $K_{4k}$ if and only if $\sqrt{4k+1}$ is an integer and $K_{4k+3}$ if and only if $\sqrt{k+1}$ is an integer.

Beineke and Hegde (2001), [11] defined the concepts of multiplicative labeling and strongly multiplicative labeling and attained the following results [53, 87]. (i) Every cycle $C_n$ is strongly multiplicative. (ii) Every wheel $W_n$ is strongly multiplicative. (iii) Complete graph $K_n$ is strongly multiplicative if and only if $n \leq 5$. (iv) Complete bipartite graph $K_{n,n}$ is strongly multiplicative if and only if $n \leq 4$. (v) Every spanning subgraph of a strongly multiplicative graph is strongly multiplicative. (vi) Every graph is an induced subgraph of a strongly multiplicative graph [1, 53, 87, 88].

Revathi and Rajeswari (2012), [62, 63] accomplished the notion of modular multiplicative divisor graph and derived that the complete graph $K_n$, for all prime number $n > 3$, complete bipartite graph $K_{m,n}$, cycle graph $C_n$, $n \equiv 1,2 \pmod{3}$ are modular multiplicative divisor graphs. Also they proved split graph of cycle $C_n$, helm graph $H_n$, flower graph $f_{心境}$, cycle cactus $C_{4(n)}$, extended triplicate graph of a path are modular multiplicative divisor graphs.

Ali Ahmad et al., (2012), [4] established the relation between mean labeling and $(a,d)$-edge-antimagic vertex labeling. Moreover, they have found the mean labeling of two classes of caterpillars.

Baca and Youssef (2007), [6] examined the relationship between the sequential graphs and the graphs having an $(a,d)$-edge-antimagic vertex labeling. They have proved the following results. (i) Let $G$ be a connected $(p,q)$-graph which is not a tree. $G + K_1$ is sequential if $G$ has an $(a,d)$-edge-antimagic vertex labeling. (ii) Let $G$ be a connected graph which is not a tree, such that the degree of every
vertex is odd, $p$ is even, and $p + q \equiv 2 \pmod{4}$. Then there is no $(a,d)$-edge-antimagic vertex labeling of $G$. (iii) Let $G$ be an Eulerian graph with $q$ even. If $G$ has an $(a,d)$-edge-antimagic vertex labeling then $q \equiv 0 \pmod{4}$. (iv) Let $G$ be an odd degree $(p,q)$-graph. If $p \equiv 2 \pmod{4}$ and $q \equiv 0 \pmod{4}$ then $G$ is not $(a,d)$-edge-antimagic total for every $d$. If $p \equiv 0 \pmod{4}$ and $q \equiv 2 \pmod{4}$ then $G$ is not $(a,d)$-edge-antimagic total for every even $d$. (v) Let $G$ be an odd degree $(p,q)$-graph. If $p \equiv 2 \pmod{4}$ and $q \equiv 0 \pmod{2}$ then $G$ is not $(a,d)$-edge-antimagic total for every odd $d$. (vi) A graph $G$ is not super $(a,d)$-edge-antimagic total for every even $d$ and $q \equiv 2 \pmod{4}$.

Baskar Babujee and Babitha (2010), [9] discussed the following relation between the different labeling. (i) $G + K$ admits edge-antimagic total labeling if $G$ has superior edge-antimagic total labeling. (ii) If $G$ is superior edge-antimagic then $G^+$ has an edge-antimagic total labeling. (iii) If $G$ has superior edge-antimagic total labeling then $G \hat{o} P_n$ admits edge-antimagic total labeling. (iv) $G \hat{o} K_{1,n}$ is edge-antimagic for any arbitrary superior edge-antimagic graph $G$. (v) $G \hat{o} F_{1,n}$ admits edge-antimagic total labeling if $G$ has superior edge-antimagic total labeling. (vi) $G_1 \hat{o} G_2$ admits edge antimagic total labeling if $G_1$ has superior edge-antimagic total labeling and $G_2$ has super edge-antimagic labeling.

Figueroa Centeno et al., (2001), [22] distinguished the relation between super edge-magic labeling and other classes of labeling. (i) A $(p,q)$-graph $G$ that is a tree or where $q \geq p$ is sequential if $G$ is super edge-magic. (ii) $G$ is harmonious if $G$ with $q \geq p$ is super edge-magic. (iii) A tree $T$ of order $p$ is harmonious if $T$ is super edge-magic. (iv) A tree $T$ is super edge-magic if $T$ has an $\alpha$-valuation. (v) A graph $G$ is cordial if $G$ is super edge-magic [54].

Slater (1982), [75] introduced $k$-graceful graphs, which is the generalization of graceful graphs. A graph $G$ with $q$ edges is $k$-graceful if there is a labeling $f$ from the vertices of $G$ to $\{0,1,2,\ldots,q+k-1\}$ such that the set of edge labels
induced by the absolute value of the difference of the labels of adjacent vertices is \( \{ k, k + 1, k + 2, \ldots , q + k - 1 \} \) [18, 20, 48].

Gayathri and Kousalya Devi (2011), [26] defined \( k \)-even-edge graceful labeling and proved that (i) Cycles with a pendant edge, cycles with a cord and crowns \( C_n \Theta K_1 \), paths, stars, bistars, are \( k \)-even-edge graceful. (ii) Graphs obtained from \( P_n \) by replacing each edge by a fixed number of parallel edges and sparklers are \( k \)-even-edge graceful.

Lakshmi Prasanna, Sravanthiand and Nagalla Sudhakar (2014), [45] explored the role of graph labeling in communication field such as network security, network addressing, channel assignment process, social networks.

Alexandru Vladoi (2010), [3] used algorithm based labeling for an efficient communication between the wireless broadcast systems. In order to achieve the efficient communication, the distance between two broadcast systems needs to be spaced out at maximum. The radio communications has been modeled using graphs such as star graph, complete graph, wheel graph, complete bipartite graph, and cycle or chain graph.


2.5 CONCLUSION

This chapter gives some basic definitions and theorems on graphs, graph labeling, and multiprotocol label switching communication networks necessary for the development of the topic.

The next chapter is focused on odd graceful labeling, even graceful labeling and the new concept even-even graceful labeling of graphs.