CHAPTER IV
EQUALIZATION AND MIMO-OFDM CHANNEL ESTIMATION

4.1. INTRODUCTION

There is a terrific increase in usage of wireless mobile communication subscribers all over the world in the recent days. These subscribers exchange high capacity and high data rate multimedia information apart from the voice between the several wireless networks. Also, the availability of the wireless spectrum is shrinking due to the several billions of worldwide users. Hence there is a need for effective utilization of wireless spectrum, high speed data transfer and error-free transmission. For this purpose, several systems with specialized hardware, software and different techniques are adopted. These systems are characterized by various factors such as power allocation, capacity, data transfer speed, SNR, number of antennas, frequency selectivity and fading characteristics.

Most of the times, the received signal is distorted due to the interference between one symbol and the subsequent symbol in wireless communication system. Also, the interference occurs due to the spreading of pulses beyond the allotted time interval. This is identified as Inter Symbol Interference (ISI). This ISI is a critical issue in high speed wireless communication system. In order to minimize the effect of ISI, equalization technique can be utilized.

4.2. EQUALIZATION TECHNIQUES

An equalizer is a type of filter circuit which is connected at the front end of the wireless receiver to compensate the variations in the amplitude and time delay characteristics of the received signal. The equalizer should follow the time varying characteristics of the channel due to the nature of fading. This equalizer can be either time varying or adaptive. An adaptive equalizer has training mode and tracking mode of operations.

4.2.1. Training Mode

A fixed length training sequence is combined with the information and then sent by the transmitter in the training mode of operation. This training sequence may be a fixed length bit pattern or a pseudo-random binary signal. An adaptive filter at the receiver is used to identify the filter coefficients with the aid of training sequences
sent by the transmitter. This equalization is used to compensate the channel variations and recover the original information with less error.

4.2.2. Tracking Mode
In the tracking mode of operation, adaptive algorithms are used to check the variation in the channel characteristics continuously. Also, the filter coefficients are continuously changing based on the channel variations.

4.2.3. Mathematical Representation
The receiver output is represented by the following equation.

\[ y(t) = x(t) * h(t) + n_b(t) \]  \hspace{0.5cm} (4.1)

Where \( x(t) \) is the transmitted signal, \( h(t) \) is the impulse response of the transmitter, channel and receiver, \( n_b(t) \) is the baseband noise of the channel and received signal at the equalizer input, and the symbol ‘\(*\)’ denotes the convolution operation.

If the impulse response of the equalizer is \( h_{eq}(t) \), then the output of the equalizer is the multiplication of \( y(t) \) and \( h_{eq}(t) \) which is expanded as

\[ y_{eq}(t) = x(t) * h(t) * h_{eq}(t) + n_b(t) * h_{eq}(t) \]  \hspace{0.5cm} (4.2)

In order to force the equalizer output \( y_{eq}(t) = x(t) \), \( n_b(t) \) should be zero and also \( h(t) * h_{eq}(t) \) should be \( \delta(t) \) which is the combined impulse response of the transmitter, channel and equalizer. It is described by the equation 4.3.

\[ g(t) = h(t) * h_{eq}(t) = \delta(t) \]  \hspace{0.5cm} (4.3)

The above equation can be described in the frequency domain as follows

\[ H(f) H_{eq}(f) =1 \text{ or } H_{eq}(f) = H^{-1}(f) \]  \hspace{0.5cm} (4.4)

Equation (4.4) clearly shows that the equalizer is a part of an inverse filter of the channel.

4.2.4. Zero Forcing Equalizer
It is a simple form of equalizer in which the equalizer coefficients \( c_n \) force the impulse response of the equalizer as zero. The frequency response of the equalizer is assumed as \( H_{eq}(f) \), the frequency response of the channel is \( H_{ch}(f) \) and the symbol duration is \( T \). The combined channel response with equalizer should satisfy the Nyquist’s criterion. This is mentioned by Equation (4.5).

\[ H_{ch}(f) H_{eq}(f) = 1, f < 1/2T \]  \hspace{0.5cm} (4.5)
This equalizer is the inverse filter of the channel’s frequency response. The main drawback of this equalizer is that it will amplify the noise of highly attenuated folded channel spectrum.

4.2.5. **Adaptive Equalizer**

An Adaptive equalizer is an inverse filter with N+1 taps, N delay elements, and N+1 weights. The adaptive equalizer’s block schematic is shown in Fig.4.1. An adaptive algorithm is used to update the weights continuously to make better equalizer output with fewer errors. This adaptive algorithm is restricted by an error signal $e_k$, where $k$ is the time index. This $e_k$ is the difference between the output of the equalizer and the output of the transmitted signal. This adaptive algorithm uses the error signal $e_k$ and the weight coefficient $w_k$ to make minimum cost function with the help of several iterations.

Let the equalizer input be $x_k$ and the weight coefficient vector be $w_k$, and the equalizer output is the inner product of $x_k$ and $w_k$. It is given by Equation (4.6).

$$y_k = \langle x_k, w_k \rangle$$  \hspace{1cm} (4.6)

The error signal $e_k$ and the Mean Square Error (MSE) are represented by Equations (4.7) and (4.8) respectively.

$$e_k = d_k - y_k$$  \hspace{1cm} (4.7)

Where $d_k$ is the transmitted data.

$$MSE = E[(d_k - y_k)^2]$$  \hspace{1cm} (4.8)

Since $y_k$ depends on the weight function $w_k$, MSE is also related to the weight function $w_k$.

![Fig.4.1 The Basic Structure of Adaptive Equalizer](image)
4.2.6. Adaptive Equalization Algorithms

There are several equalization algorithms used to revise the equalizer coefficients and follow the time varying channel variations. Some of the popular adaptive algorithms are Zero Forcing (ZF) algorithm, Least Square (LS) algorithm, Least Mean Square (LMS) algorithm, Recursive Least square (RLS) algorithm [68] and Minimum Mean Square Error (MMSE) algorithm. The performance of the adaptive algorithm is affected by groups of factors which are shown below.

**Convergence Rate:** A high-speed rate of convergence is required for the adaptive algorithm to adjust promptly to unknown channel variations.

**Numerical properties:** The stability of the adaptive algorithm is affected by the factors such as Round-off noise and illustration errors.

**Computational Complexity:** Total number of processes used to create a single complete iteration of algorithm.

4.3. MATHEMATICAL MODELING OF LEAST-SQUARE (LS) ALGORITHM

The Least-Square (LS) method is a mathematical procedure to identify the best fit line to the data. It is used to estimate the parameters that fit a function \( f(x) \) for a set of data \( x_1, x_2, \ldots x_n \). For a best fitting of the given data, LS method decreases the sum of squared residuals which is also described as Sum of Squared Errors (SSE). It is given by Equation (4.9).

\[
SSE = \sum_{i=1}^{n} r_i^2 \tag{4.9}
\]

Where \( r_i \) is the residual. It is represented by the following equation.

\[
r_i = y_i - f(x_i) \tag{4.10}
\]

Here, \((x_i, y_i)\) is a pair of data, and \( f(x_i) \) is the estimated function. There may be an error between the real data and the estimated data. This LS method can be either in linear or non-linear mode.

The Linear LS method is one of the most common linear regression methods used to find the best fitting of data as straight line. The estimated function is given by Equation(4.11).

\[
f(x_i) = ax_i + b \tag{4.11}
\]

Where \( a, b \) are the constants. This method is used to minimize the sum of squared errors which are denoted by Equation (4.12).

\[
SSE = \sum_{i=1}^{n} (y_i - (ax_i + b))^2 \tag{4.12}
\]
To minimize this $SSE$ function, the partial derivative of $SSE$ with respect to $a$ and $b$ are considered to be zero. The block diagram of LS estimator is shown in Fig. 4.2.

Fig. 4.2 Block Diagram of LS Estimator

Let it be considered a linear LS estimated function which is represented in the form given in Equation (4.13).

$$y_t = a_1 \times x_1 + a_2 \times x_2 + \cdots + a_m \times x_m + r \quad (4.13)$$

The above function can be described in the matrix format which is given as

$$Y = [X]A + R \quad (4.14)$$

Where $[X]$ is the input matrix of the dataset, $Y$ is the output vector, $A$ is an unknown vector, and $R$ is a residual vector. To minimize the residual vector, the partial derivative of each coefficient of Equation (4.13) is equated to zero. Due to this, a set of normalized equations is obtained which can be represented in the matrix form of Equation (4.15).


The unknown vector $A$ is represented as

$$A = [X]^{-1}Y \quad (4.16)$$

The above equation is used as a basic equation to estimate the channel coefficients in the LS channel estimation of MIMO-OFDM system.

4.4. MATHEMATICAL MODELING OF MINIMUM MEAN SQUARE ERROR (MMSE) ALGORITHM

The MMSE algorithm is used as a mathematical channel estimation model to find channel coefficients. This algorithm minimizes the Mean Square Error (MSE). It can be expressed as quadratic cost function mathematically.

Let $x$ be the (nX1) dimension of input vector and $y$ be the (mX1) dimension of output vector and $\hat{x}$ be the estimated vector. There is an error between the original input and the estimated output. This estimation error is given by Equation(4.17).

$$e = x - \hat{x} \quad (4.17)$$
The MSE is defined as the mean of square of errors. It is described by the trace of error covariance matrix which is shown in Equation (4.18).

\[ \text{MSE} = \text{tr}[E[(\hat{x} - x)(\hat{x} - x)^T)] \]  
(4.18)

If the input vector is a scalar quantity, then the MSE is rewritten as

\[ \text{MSE} = E[(\hat{x} - x)^2] \]  
(4.19)

If the estimated vector has \( n \) predictions, then the MSE is represented by Equation (4.20).

\[ \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (\hat{x} - x)^2 \]  
(4.20)

Also, the MMSE is indentified as the minimum value from the group of mean square errors. It is given as

\[ \text{MMSE} = \arg \min \text{MSE} \]  
(4.21)

The above equations are used as basic equations to estimate the channel coefficients in the MMSE channel estimation of MIMO-OFDM system.

### 4.5. LEAST MEAN SQUARE (LMS) ALGORITHM

LMS algorithm is an uncomplicated algorithm which is used to minimize the MSE. This minimization is carried out with the help of the stochastic gradient algorithm. This equalizer weights are updated by the update Equation (4.22).

\[ w_k(n+1) = w_k(n) + \mu e_k x(n-k) \]  
(4.22)

where \( w_k \) is weight element with \( k \)’th delay, and \( \mu \) is the step size which controls the convergence rate. This algorithm requires only \( 2N+1 \) operation per iteration.

### 4.6. CHANNEL ESTIMATION TECHNIQUES

The wireless systems transmit information as the variation in amplitude and phase of radio waves during transmission. There is a vast change in the amplitude and phase of the wireless signal at the receiver side which affects the receiver performance very much. For the reliable communication, the Channel State Information (CSI) which contains propagation characteristics, multipath fading characteristics and delay profile [108] should be transmitted with original information. This CSI has either instantaneous type or statistical nature. The instantaneous CSI represents the present channel condition. The statistical CSI represents the statistical parameters of the channel such as fading distribution, channel gain, multipath effects and time delay. This CSI can be estimated either at the transmitter or the receiver side with an intention to estimate various channel parameters. For this purpose, several channel
estimation techniques are employed [5] by several researches. This section describes various channel estimation algorithms and their utilization in MIMO-OFDM system. The wireless channels are mostly multipath fading channels. The ISI and Co-Channel Interference (CCI) can occur in the received signal due to this multipath fading. To suppress the CCI, spatial covariance estimation [54] is introduced with the help of cholesky decomposition. To eliminate ISI from the received signal, various detection techniques are utilized at the receiver. The Channel Impulse Response (CIR) of these detectors can be identified by a part of channel estimator. Since the CIR is estimated by the path delay, the Time of Arrivals (TOAs) estimation is a best method to improve accuracy [108]. The channel estimation [22] is based on the well-known series of bits which are exclusive for a particular transmitter, and they frequently appeared in every transmission burst. Thus, the channel estimator is capable of estimating the CIR for each burst independently from the well-known transmitted bits and from the related received samples.

4.6.1. Classification of Channel Estimation

The major classification of channel estimation methods for the MIMO-OFDM is shown in Fig. 4.3.

![Fig.4.3 Classification of Channel Estimation](image)

The training-based channel estimation [23], [89] can be carried out by the placement of pilots with original data before the multicarrier modulation at the transmitter side. These pilots are well known to the receiver. At the receiver side, channel parameters are estimated with the help of several pilot-based estimation algorithms. Comb-type, block-type and lattice-type pilots are utilized to estimate the channel. Maximum Likelihood (ML), Least Square (LS) and Minimum Mean Square
Error (MMSE) are some of the most common algorithms used for training-based channel estimation.

4.6.2. Types of Pilots

An arrangement of block-type pilots is shown in Fig. 4.4. In this block-type pilot carrier estimation, pilots are positioned into all frequency bins within the cyclic spaces of OFDM blocks. These pilots are used to estimate the channel in time axis with the help of a time domain interpolation method.

![Fig.4.4 Arrangement of Block-Type Pilots](image)

An arrangement of comb-type pilots [33] is shown in Fig.4.5. In this comb-type pilot carrier estimation, pilot tones are positioned into every OFDM symbol with a particular frequency bin. These pilots are used for the channel estimation in the frequency axis with the help of a frequency domain interpolation method.

![Fig.4.5 Arrangement of Comb-Type Pilots](image)

The block-type pilots are generally matched for frequency selective channels, and the comb-type pilots are appropriate for the fast fading channels. The arrangement of lattice-type pilots is shown in Fig. 4.6. In this lattice-type pilot carrier
estimation, pilot tones are positioned in the time and frequency axes equally with the particular periods. These pilots are used to estimate the channel either in the time domain or in the frequency domain by the interpolation methods.

![Fig. 4.6 Arrangement of Lattice-Type Pilots](image)

**4.7. REVIEWS ON CHANNEL ESTIMATION**

Several researchers contributed to different types of channel estimation techniques in the category of training-based, blind and semi-blind channel estimation techniques to improve the performance of MIMO-OFDM system. These techniques are described in this section. Some of the efficient MIMO encoding algorithms are Alamouti Space Time Block Coding (STBC) and Vertical Bell-Labs Layered STBC (VBLAST-STBC) [69].

**4.7.1. Training-Based Channel Estimation Techniques**

A simple transmit diversity technique is executed by Alamouti[2] with two transmitters and one receiver. This type of diversity also can be extended to two transmitters and several receivers. This scheme is mainly applied to improve diversity at all the distant parts of the wireless system. Also, this technique does not require the response from the receiver to the transmitter. It is an advantage of this technique. The computational complexity of this diversity technique is similar to Maximal-Ratio Receiver Combining (MRRC) technique. One of the drawbacks of this technique is soft failure. Another drawback of this method is the necessity of M times of pilots for M-branch diversity. A Wiener-filter based channel estimation using Space Time Block Coding (STBC) [29] is presented for OFDM system. This estimation has low complexity with accurate estimation. The channel estimation with Minimum Euclidean Distance (MED) decoding [57] is discussed to improve the channel.
capacity and the performance of 4 X 4 MIMO-OFDM system. A combined channel estimation technique for Rician fading channel [7] using LS and MMSE estimation with Singular Value Decomposition (SVD) is described in one approach. This technique increases the estimated channel accuracy with minimum complexity. The BER performance evaluation of Rayleigh, Rician and AWGN channels [9] under different modulations such as QAM and QPSK are discussed in another approach.

The transmission characteristics of 2 X 2 MIMO-OFDM system [14] with Binary Phase Shift Keying (BPSK) is presented with BER and MSE performance. The most common training-based channel estimation techniques such as Least Square (LS) and Minimum Mean Square Error (MMSE) channel estimation are investigated [10], [41] for Single-Input Single-Output (SISO) and MIMO-OFDM systems. The BER and MSE performance of MIMO-OFDM is assessed with the performance of SISO-OFDM. Also, STBC is introduced to improve the performance. The performance evaluation shows that MMSE estimation is superior to LS channel estimation in terms of MSE and BER.

A rapid prototyping framework model [11] is developed by a researcher for wireless LAN MIMO-OFDM system. This approach is very useful to implement MIMO algorithms in VLSI-based hardware. Linear pre-coder design [15] for correlated Rician MIMO channel is described to minimize the pair-wise error probability for non-orthogonal STBC. The optimal space frequency codes [17] for IEEE 802.11-based MIMO-OFDM system with real-time experimentation is developed in the frequency selective fading environment. The joint carrier frequency offset [19] with in-phase and quadrature-phase imbalance estimation is introduced for MIMO-OFDM system. This estimation used a new idea called channel residual energy to minimize the MSE. The decision directed LS channel estimation with subspace tracking [24] is delivered to minimize the error of MIMO-OFDM system. Iterative joint channel estimation [38] for Space Division Multiple Access (SDMA)-MIMO-OFDM system is delivered using genetic algorithm. This technique has the capability to estimate multiple number of users at the transmitter with fewer number of receivers. Recursive MMSE channel estimation [39] is executed to improve the accuracy of data transmission than the MMSE estimation in MIMO-OFDM system. The linear channel estimation of Long Term Evolution (LTE) MIMO-OFDM system is delivered to minimize the MSE [46]. Another linear LS and MMSE channel estimation for LTE-Advanced MIMO-OFDM is investigated [80]. The complexity of
this estimation is reduced by using only the first L number of taps. The power delay profile channel estimation [48] is also described to reduce the distortion in MIMO-OFDM system. The joint time-space-frequency channel estimation technique [120] is introduced for MIMO-OFDM system. This technique has low complexity, and also it has better performance than STBC-MMSE channel estimation.

4.7.2. Pilot Optimization Techniques

Several researchers presented various types of training-based channel estimation algorithms and optimized pilot design methods. A superimposed training-based closed loop method is presented [34]. The effects of quantization and feedback channel coefficients are refined in this technique. Also, optimized 3D pilot design and Square Root-Recursive Least Square (QR-RLS) method is executed to minimize MSE and improve the bandwidth efficiency. Another training-based channel estimation using Least Mean Square (LMS) algorithm [21] is presented. The channel coefficients are estimated with the existing LS channel estimation and then with LMS algorithm. There is an improvement in the functioning of MIMO-OFDM system in terms of minimum amount of MSE and BER. But better performance depends on the proper selection of parameter $\mu$.

The pilot placement is an important factor in the training-based channel estimation. An optimum pilot design using differential evolution algorithm [1] was presented for Least Square (LS) channel estimation in MIMO-OFDM system to minimize the computation time. Multi-cell optimized pilot tone arrangement [42] for LS channel estimation is delivered in another approach to minimize the MSE. The pilot sequence design for the frequency offset as well as frequency selective channel estimators [95] is developed with the aid of Cramer-Rao Bound (CRB) sequence. An optimum pilot sequence design using convex optimization method is discussed [100]. This method is strong against the spatial correlation mismatch at the transmitter. Also, an optimized pilot-aided 3D (time, frequency, space) channel estimation [8] is described for MIMO-OFDM system to improve the mobile performance in the outdoor environments. The optimization of position and power of pilot-tones in LS channel estimation [86], [87] using differential evolution, particle swarm optimization, Lagrange multiplier method [40]-MMSE estimation, optimized time domain channel estimation [66] are delivered to improve the performance. A robust training sequence [18] is interleaved into the MMSE estimator to improve the
performance of MIMO system. This technique outperforms than the optimal estimator. In another approach, the pilot-tone design and placement [12] for LS channel estimation is described. Also, Recursive LS (RLS) algorithm is introduced with tracking factor for better performance results. Co-operative Particle Swarm Optimization (CPSO) [49] is introduced to improve the performance of MMSE channel estimation. The optimum pilot allocation [62] for MIMO-OFDM system under different multiplexing schemes in the time domain and frequency domain are discussed to minimize the MSE. This method is applicable to pilot-data multiplexed schemes. The pilot placement methodology for MIMO-OFDM in wideband indoor communication environment is described [106] to minimize the MSE.

4.7.3. Blind Channel Estimation Techniques
The channel statistical information and specific properties of channel are employed to estimate the channel parameters in blind channel estimation [88]. This estimation does not have any overhead loss. Also, this estimation is suitable for slow time varying channels. The Space time block coding (STBC) and orthogonal space time block coding (OSTBC) algorithms are used as blind channel estimation. A Semi-Definite Relaxation (SDR) technique with OSTBC [84] is discussed for MIMO-OFDM system to estimate the different parameters in time domain. This estimation has better performance than other blind channel estimations.

The performance evaluation of WiMAX MIMO-OFDM is described [3] to evaluate the spectral efficiency of both uplink and downlink system. There is a significant improvement in the spectral efficiency by increasing the number of antennas. A joint channel and frequency offset channel estimation [4] is addressed for MIMO-OFDM system. This estimation is evaluated by Cramer –Rao Bound and MSE. A joint Maximum-Likelihood (ML) blind channel estimation [16] is presented for Single-Input Multi-Output (SIMO) system. The repeated weighted boosting search algorithm and Viterbi algorithm are used to estimate the channel. The main drawback of ML estimation is that it has high complexity. To reduce this complexity, Kalman-filter based CFO channel estimation [81] for MIMO-OFDM is introduced. The joint channel estimations [90, 91] are discussed to estimate the Rayleigh faded channel Complex Amplitude (CA), Complex Gain (CG) and CFO. These methods are more robust to high speed. This algorithm is also better than expectation-maximization (EM) algorithm. Fast Data Protection Method (FDPM) [26] is
described as a subspace tracking method for MIMO-OFDM channel estimation. This method has less complexity than Eigen Value Decomposition (EVD) technique. The MIMO-OFDM system with Narrow Band Interference (NBI) is estimated with the help of compressive sensing theory-based sparsity-aware approach [28]. There is no need of prior information about NBI in this estimation and also it has less complexity. Frequency offset estimator [61] optimum pre-coding scheme [114], Kernal Recursive least Square (KRLS) algorithm [58], bandwidth-efficient channel estimation [50] and optimum block coding scheme are the other types of blind channel estimation techniques for MIMO-OFDM system. Another blind channel estimation called frequency offset estimator for the frequency selective fading channels is described [63]. The main feature of this estimation is that it is not sensitive to power delay variations.

4.7.4. Semi-Blind Channel Estimation Techniques

The semi-blind channel estimation uses pilot carriers with some normal constraints to estimate channel parameters. It is noted as a hybrid combination of training and blind channel estimation techniques. Maximum Likelihood (ML) joint channel estimation [77] for correlated channels is presented for multiuser MIMO system. The block iterative and recursive algorithms are used in this estimation to improve the spectral efficiency. Semi-blind channel estimation [107] for sparse MIMO-OFDM is described. In this estimation, a blind constraint is created by using the relation between the correlation matrices and Most Significant Taps (MST). This constraint is joined with training-based LS to form a semi-blind estimation. This estimation does not suffer from perturbation error. An iterative method [53] to jointly design the MMSE –MIMO processors is discussed to improve the performance of the MIMO relaying scheme. A low complexity semi-blind Expectation- Maximization (EM)-based frequency domain channel estimation [94] for OFDM system is presented using frequency correlation properties. This estimation has low computation complexity over the other time domain channel estimators.

4.7.5 Other Channel Estimation Techniques

The frequency domain channel estimation technique [71] is investigated for correlated fading channels. In this method, spatial domain Linear MMSE (LMMSE) filtering is used to improve the performance. An enhancement to the MMSE
equalization is introduced with the aid of fuzzy-based Kalman filter algorithm [113]. This method can exactly track the channel parameters of fast multipath fading channels. The channel estimation of MIMO-OFDM system is implemented using Field Programmable Gate Array (FPGA) hardware [72]. The header-based Maximum Likelihood (ML) estimation is used in this approach. An adaptive channel estimation technique using Normalized Least Mean Square (NLMS) and RLS algorithms [75] is explained for the time varying MIMO-OFDM system. The convergence rate of RLS channel estimation is higher than that of NLMS channel estimation. Two-dimensional time-frequency variant channel estimation [78] is investigated with the aid of slepian-basis expansion model for MIMO-OFDM system. This estimator has an advantage of less complexity than other joint channel estimators. MIMO channel modeling using complex Recurrent Neural Network (RNN) is investigated in an approach [83]. This MIMO channel is optimized by Self Organization Map (SOM)-based optimization method. This approach has low BER and low delay. A Space-Alternating Generalized-Expectation-maximization (SAGE) algorithm-based H-infinity channel estimation is developed for MIMO-OFDM system [112]. This estimator is robust against the Non-Gaussian Noise (NGN) channels, and also its performance is better than that of the conventional estimators.

4.8. LS AND MMSE CHANNEL ESTIMATION

The block diagram of the MIMO-OFDM system is shown in Fig. 4.7. The MIMO-OFDM transmitter has $N_T$ parallel paths with $N$ number of sub-carriers and each path performs serial-to-parallel transfer, pilot positioning, $N$-point IFFT and Cyclic Prefix (CP) insertion before the up-conversion of transmitted signal. The channel encoding has also been done per branch. Here, the modulated signals are space-time coded by the Alamouti algorithm before the transmission using multiple numbers of antennas. At the receiver side, the CP is removed and $N$-point FFT is carried out for each receiver branch. After that, the processes like demodulation and decoding are completed. Finally, all the input data are reproduced with certain BER.

In this MIMO signaling technique, $N_T$ number of different signals are transmitted at the same time over $N_T \times N_R$ transmission paths. Each of those $N_R$ received signals is a mixture of all the $N_T$ transmitted signals with distorted noise. This MIMO technique helps for the improvement of the diversity gain with enhanced system capacity. But, the channel estimation as well as the symbol detection has high
complexity due to several channel coefficients. Each data stream of every antenna goes through OFDM Modulation. For the simplicity, MIMO–OFDM system with two transmitters and two receivers is considered.

The Space Time Block Coding (STBC) is used as encoding technique, and the encoding matrix is shown by Equation (4.22). Also, the input vectors $X_1$ and $X_2$ are mentioned by Equations (4.23) and (4.24).

$$X = \begin{bmatrix} X_1 & -X_2^* \\ X_2 & X_1^* \end{bmatrix} \quad (4.22)$$

Where

$$X_1 = (X[0] - X \ast [1]X[2] \ldots \ldots \ldots - X \ast [N - 1]) \quad (4.23)$$

$$X_2 = (X[1]X \ast [0]X[3] \ldots \ldots \ldots X \ast [N - 2]) \quad (4.24)$$

The vectors $X_1$ and $X_2$ are modulated with the help of Inverse Fast Fourier Transform (IFFT) and also a CP is inserted which acts as a Guard Interval (GI). Then, these modulated signals are transmitted by the two different transmitting antennas assuming that the GI is more than the expected delay spread of the multipath channel. The receiver side antenna collects the incoming signal. This signal is a convolution of the channel and the transmitted signal. At the receiver side, the CP is detached first from the received signal. Then, FFT and demodulation are performed. The demodulated signal is mentioned by Equation (4.25).

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_{NR} \end{bmatrix} = \begin{bmatrix} H_{1,1} & \cdots & H_{1,N_T} \\ \vdots & \ddots & \vdots \\ H_{NR,1} & \cdots & H_{NR,N_T} \end{bmatrix} \begin{bmatrix} X_1 \\ \vdots \\ X_{NT} \end{bmatrix} + \begin{bmatrix} W_1 \\ \vdots \\ W_{NT} \end{bmatrix} \quad (4.25)$$

Fig. 4.7 Block Diagram of MIMO-OFDM System
In the above equation, \([W_1, W_2, \ldots, W_{NT}]\) is the Additive White Gaussian Noise (AWGN), and \(H_{m,n}\) is the Single-Input Single-Output (SISO) channel gain among the \(m^{th}\) receiver and \(n^{th}\) transmitter. By knowing the receiver CSI value, Maximum Likelihood (ML) detection method is applied for the decoding of received signal. This can be given by Equations (4.26) and (4.27).

\[
\tilde{s}[2k] = \sum_{l=1}^{N_R} H_{l,1}^* [2k] Y_l[2k] + H_{l,2}[2k] Y_l^* [2k + 1]
\]

\[
\tilde{s}[2k + 1] = \sum_{l=1}^{N_R} H_{l,2}^* [2k + 1] Y_l[2k] - H_{l,1}[2k + 1] Y_l^* [2k + 1] q
\]

Where, \(k = 0, 1, 2, \ldots, (N/2)-1\)

The channel gains between two nearby sub-channels are assumed as equal, and these channel gains are characterized by Equations (4.28) and (4.29).

\[
H_{l,1}[2k] = H_{l,1}[2k + 1]
\]

\[
H_{l,2}[2k] = H_{l,2}[2k + 1]
\]

The LS channel estimation of MIMO-OFDM system with \(n^{th}\) transmitter and \(m^{th}\) receiver is represented by Equation (4.30). Also, the input vector \(X\) and the output vector \(Y\) are denoted by Equations (4.31) and (4.32) respectively.

\[
\hat{H}_{LS}^{(n,m)} = (X^{(n)})^{-1} Y^{(m)}
\]

\[
X = \text{diag}(X(0), X(1), \ldots, X(N-1))
\]

\[
Y = [y(0), y(1), \ldots, y(N-1)]^T
\]

The MMSE channel estimation of MIMO-OFDM system with \(n^{th}\) transmitter and \(m^{th}\) receiver is represented by Equation (4.33). Also the covariance \(R_{hY}\), auto-variance \(R_{YY}\) are denoted by Equations (4.34) and (4.35) respectively.

\[
\hat{R}_{MMSE}^{(n,m)} = F R_{hY} R_{YY}^{-1} Y^{(m)}
\]

Where

\[
R_{hY} = R_{hh}^{(m,n)} F^H (X^{(n)})^H
\]

\[
R_{YY} = X^{(n)} F R_{hh}^{(n,m)} F^H (X^{(n)})^H + \sigma^2 I_N
\]

The matrices of \(F\), \(H\) and \(W\) are denoted by the following Equations (4.36), (4.37) and (4.38) respectively.

\[
F = \begin{bmatrix}
W_{N}^{00} & \cdots & W_{N}^{0(N-1)} \\
\vdots & \ddots & \vdots \\
W_{N}^{(N-1)0} & \cdots & W_{N}^{(N-1)(N-1)}
\end{bmatrix}
\]

\[
H = [H(0), H(1), \ldots, H(N-1)]^T
\]

\[
W = [W(0), W(1), \ldots, W(N-1)]^T
\]
Where \( n = 1, 2 \ldots N_T \) and \( m = 1, 2 \ldots N_R \).

\( N_T, N_R \) are the total number of transmitters and receivers present in the MIMO-OFDM system. \( X^{(n)} \) is the diagonal pilots of the \( n \)th transmit antenna with an order of \( N \times N \), and \( Y^{(m)} \) is the received vector of length \( N \) at the receiver side antenna \( m \).

### 4.9. PERFORMANCE EVALUATION OF LS AND MMSE CHANNEL ESTIMATION

The multipath fading channel gains of the 2 X 2 MIMO-OFDM system (two transmitters and two receivers) with sampling interval \( N_s \), are given by Equations (4.39), (4.40), (4.41) and (4.42).

\[
\begin{align*}
  h_{11}(n) &= \delta(n) + \delta(n-0.5N_s) + \delta(n-3.5N_s) \\
  h_{12}(n) &= \delta(n) + \delta(n-0.4N_s) + \delta(n-1.1N_s) \\
  h_{21}(n) &= \delta(n) + \delta(n-0.4N_s) + \delta(n-0.9N_s) \\
  h_{22}(n) &= \delta(n) + \delta(n-0.6N_s) + \delta(n-2.2N_s)
\end{align*}
\]

(4.39) (4.40) (4.41) (4.42)

The functioning of MIMO-OFDM system is assessed with the help of Mean Square Error (MSE) and Bit Error Rate (BER) plots. These plots are shown in Fig. 4.8 and Fig. 4.9. The block and comb type pilots are utilized for the LS and MMSE channel estimations in the MIMO-OFDM system.

The average error in an OFDM block is known as Mean Square Error (MSE). This MSE is mentioned in Equation (4.43).

\[
MSE = \frac{1}{N} \sum_{k=1}^{N} |H(k) - H_e(k)|^2
\]

(4.43)

Where \( k \) is the sub-carrier index, and \( H_e(k) \) is the estimated channel value.

The various parameters of a 2X2 MIMO-OFDM system used in the simulation are represented in Table 4.1.

<table>
<thead>
<tr>
<th><strong>Table 4.1 System Parameters of LS and MMSE Channel Estimation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>System Parameters</strong></td>
</tr>
<tr>
<td>FFT Size</td>
</tr>
<tr>
<td>No. of Sub-carriers</td>
</tr>
<tr>
<td>Guard Interval</td>
</tr>
<tr>
<td>Modulation</td>
</tr>
<tr>
<td>Type of Pilot</td>
</tr>
<tr>
<td>Number of Pilots</td>
</tr>
<tr>
<td>Channel</td>
</tr>
<tr>
<td>Doppler Shift</td>
</tr>
</tbody>
</table>
The transmitter and the receiver are assumed under the proper synchronization. In order to limit the ISI, the value of GI is considered as high. The BER simulation results are shown in Table 4.2 and Fig. 4.8.

Table 4.2 Bit Error Rate of the Existing Channel Estimations

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>No Channel Estimation</th>
<th>LS Channel Estimation</th>
<th>MMSE Channel Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.279 X 10^-1</td>
<td>2.465 X 10^-1</td>
<td>2.054 X 10^-1</td>
</tr>
<tr>
<td>5</td>
<td>1.646 X 10^-1</td>
<td>1.395 X 10^-2</td>
<td>1.163 X 10^-2</td>
</tr>
<tr>
<td>10</td>
<td>1.233 X 10^-1</td>
<td>5.800 X 10^-2</td>
<td>4.830 X 10^-2</td>
</tr>
<tr>
<td>15</td>
<td>1.083 X 10^-1</td>
<td>4.000 X 10^-2</td>
<td>3.330 X 10^-2</td>
</tr>
<tr>
<td>20</td>
<td>1.054 X 10^-1</td>
<td>3.650 X 10^-2</td>
<td>3.040 X 10^-2</td>
</tr>
</tbody>
</table>

Fig. 4.8 BER Plot of 2X2 MIMO-OFDM System

The simulation results evidently prove that the BER has very high value without any channel estimation technique, and it has been reduced by the introduction of LS and MMSE channel estimation techniques. Fig. 4.8 illustrates that the BER of MMSE channel estimation is much less than the LS channel estimation.

Table 4.3 MSE of Existing Channel Estimations

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>LS Channel Estimation</th>
<th>MMSE Channel Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8.18 X 10^-2</td>
<td>2.25 X 10^-2</td>
</tr>
<tr>
<td>10</td>
<td>2.46 X 10^-4</td>
<td>9.10 X 10^-3</td>
</tr>
<tr>
<td>15</td>
<td>8.10 X 10^-3</td>
<td>3.62 X 10^-3</td>
</tr>
<tr>
<td>20</td>
<td>2.51 X 10^-3</td>
<td>1.20 X 10^-3</td>
</tr>
</tbody>
</table>
Fig. 4.9 MSE for LS and MMSE Channel Estimators

The simulation results of MSE are exposed in Table 4.3 and Fig. 4.9. It also indicates that the MSE of MMSE channel estimation is less than the LS channel estimation. These results prove that MMSE channel estimation of MIMO-OFDM system has less MSE and BER than the LS channel estimation. So, MMSE channel estimator is superior to LS channel estimator.

4.9.1. Limitations of LS and MMSE Channel Estimation
The LS and MMSE channel estimation techniques are more admired by numerous researchers. But still they have quite a few limitations. The complexity of LS channel estimation is less than MMSE channel estimation, but at the same time, BER and MSE are high. The MMSE estimation has less BER and MSE than LS estimation, but its complexity is more. Also, both estimation techniques cannot minimize the noise levels at the wireless receiver. In order to overcome these limitations, there is a necessity to introduce other algorithms.

4.10. DFT-BASED CHANNEL ESTIMATION
A great demand for high speed data service with superior quality is an important aspect of 4G wireless communication systems. Most of the next generation wireless
systems have null sub-carriers at the end of the extremity. In this context, time domain channel estimation plays a major role.

This section describes Discrete Fourier Transform (DFT) based channel estimation. The already available LS and MMSE channel estimation methods cannot decrease the noise levels at the receiver side. Thus, DFT channel estimation is launched to reduce the noise and MSE [73]. The DFT-based channel estimation operation involves the conversion of the existing LS or MMSE estimated channels from frequency domain to time domain by Inverse Discrete Fourier Transform (IDFT) [116]. By using proper smoothing filter, noise level is minimized. Then Discrete Fourier Transform (DFT) is applied again to convert time domain channels into frequency domain channels. This technique eliminates the effect of noise outside maximum channel delay [23]. The DFT-based channel estimation [32] for the OFDM system is described to improve the performance of LS and MMSE channel estimations. By the introduction of the DFT, the BER of both estimations has been reduced.

Fig. 4.10 shows the block diagram of DFT-based channel estimation. The LS/MMSE estimated channel gains for N elements are transferred to time domain from frequency domain with the help of N-point IDFT. Then the elements with null subcarriers are eliminated and the remaining L-1 elements are again transferred into frequency domain by N-point DFT. The noise level of this output is very less.

![Fig. 4.10 Block Diagram of DFT-Based Channel Estimation](image)

4.11. **DFT CHANNEL ESTIMATION SYSTEM MODEL**

The training symbols for N orthogonal sub-carriers are given by following Equation (4.44).

\[
X = \begin{bmatrix}
X[0] & 0 & \ldots & 0 \\
0 & X[1] & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & X[N-1]
\end{bmatrix}
\]  

(4.44)

Where \(X[k]\) denotes pilot tone of \(K^{th}\) sub-carrier. The channel gain is \(H\), noise vector is \(Z\), and the received training signal is represented as;
The LS channel estimation can be represented as:

\[ \hat{H}_{\text{LS}} = X^{-1}Y \]  

(4.45)

The MMSE channel estimation can be represented as:

\[ \hat{H}_{\text{MMSE}} = W R R^{-1}Y \]  

(4.46)

Where \( W \) is the weight matrix and \( R \) is the autocorrelation.

Let \( \hat{H}[k] \) be the channel gain of \( K^{\text{th}} \) sub-carrier after the estimation which is derived from LS and MMSE channel estimation. The IDFT of this channel estimation is mentioned by Equation (4.48).

\[ \text{IDFT}\{\hat{H}[k]\} = h[n] + z[n], \quad n = 0, 1, 2, ..., N - 1 \]  

(4.48)

\( Z[n] \) is the noise present in the time domain. The coefficients are given as:

\[ \hat{H}_{\text{DFT}}[n] = \begin{cases} h[n] + z[n], & n = 0, 1, ..., L - 1 \\ 0, & \text{otherwise} \end{cases} \]  

(4.49)

Now, the DFT transform is taken for the left behind \( L \) elements in frequency domain. It is mentioned in Equation (4.50).

\[ \hat{H}_{\text{DFT}}[k] = \text{DFT}\{\hat{H}_{\text{DFT}}[n]\} \]  

(4.50)

The instantaneous MSE within an OFDM block is represented as:

\[ \text{MSE} = \frac{1}{N} \sum_{k=1}^{N} |H(k) - H_e(k)|^2 \]  

(4.51)

Where \( H(k) \) is the DFT-channel matrix, and \( H_e(k) \) is the estimated DFT-channel matrix. The MSE is calculated for both LS and MMSE estimations. Also, it is calculated for the DFT-based channel estimation.

### 4.12. PERFORMANCE EVALUATION OF DFT ESTIMATION

The different types of system parameters for the simulation of DFT-based channel estimation of 2X2 MIMO-OFDM system are represented in Table 4.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT Size</td>
<td>32</td>
</tr>
<tr>
<td>No. of Sub-carriers</td>
<td>128</td>
</tr>
<tr>
<td>Guard Interval</td>
<td>4</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Number of Pilots</td>
<td>8</td>
</tr>
<tr>
<td>Pilot Spacing</td>
<td>4</td>
</tr>
<tr>
<td>Channel</td>
<td>AWGN</td>
</tr>
<tr>
<td>Sampling Frequency</td>
<td>10 MHz</td>
</tr>
</tbody>
</table>
This simulation has been done under the assumption that proper synchronization is retained between the transmitter and the receiver.

The comparison of LS channel estimation and LS-DFT channel estimation in terms of MSE is shown in Table 4.5. It evidently indicates that LS-DFT channel estimation has 89.2% less average MSE than the LS channel estimation.

Table 4.5 MSE Values of LS and LS-DFT Channel Estimations

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>LS Channel Estimation</th>
<th>LS-DFT Channel Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9.658 X 10^{-1}</td>
<td>1.190 X 10^{-1}</td>
</tr>
<tr>
<td>4</td>
<td>9.527 X 10^{-1}</td>
<td>7.410 X 10^{-2}</td>
</tr>
<tr>
<td>6</td>
<td>9.423 X 10^{-1}</td>
<td>4.608 X 10^{-2}</td>
</tr>
<tr>
<td>8</td>
<td>9.341 X 10^{-1}</td>
<td>2.854 X 10^{-2}</td>
</tr>
<tr>
<td>10</td>
<td>9.276 X 10^{-1}</td>
<td>1.759 X 10^{-2}</td>
</tr>
<tr>
<td>12</td>
<td>9.225 X 10^{-1}</td>
<td>1.077 X 10^{-2}</td>
</tr>
<tr>
<td>14</td>
<td>9.184 X 10^{-1}</td>
<td>6.540 X 10^{-3}</td>
</tr>
<tr>
<td>16</td>
<td>9.152 X 10^{-1}</td>
<td>3.920 X 10^{-3}</td>
</tr>
<tr>
<td>18</td>
<td>9.126 X 10^{-1}</td>
<td>2.310 X 10^{-3}</td>
</tr>
<tr>
<td>20</td>
<td>9.106 X 10^{-1}</td>
<td>1.340 X 10^{-3}</td>
</tr>
</tbody>
</table>

Similarly, the comparison of MMSE channel estimation and MMSE-DFT channel estimation in terms of MSE is shown in Table 4.6.

Table 4.6 MSE Values of MMSE and MMSE-DFT Channel Estimations

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>MMSE Channel Estimation</th>
<th>MMSE-DFT Channel Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9.598 X 10^{-1}</td>
<td>3.390 X 10^{-1}</td>
</tr>
<tr>
<td>4</td>
<td>9.479 X 10^{-1}</td>
<td>2.084 X 10^{-1}</td>
</tr>
<tr>
<td>6</td>
<td>9.379 X 10^{-1}</td>
<td>1.229 X 10^{-1}</td>
</tr>
<tr>
<td>8</td>
<td>9.301 X 10^{-1}</td>
<td>7.055 X 10^{-2}</td>
</tr>
<tr>
<td>10</td>
<td>9.244 X 10^{-1}</td>
<td>3.996 X 10^{-2}</td>
</tr>
<tr>
<td>12</td>
<td>9.202 X 10^{-1}</td>
<td>2.255 X 10^{-2}</td>
</tr>
<tr>
<td>14</td>
<td>9.174 X 10^{-1}</td>
<td>1.277 X 10^{-2}</td>
</tr>
<tr>
<td>16</td>
<td>9.155 X 10^{-1}</td>
<td>7.270 X 10^{-3}</td>
</tr>
<tr>
<td>18</td>
<td>9.143 X 10^{-1}</td>
<td>4.170 X 10^{-3}</td>
</tr>
<tr>
<td>20</td>
<td>9.139 X 10^{-1}</td>
<td>2.410 X 10^{-3}</td>
</tr>
</tbody>
</table>
Fig. 4.11 The MSE Performance of LS-DFT and MMSE-DFT Channel Estimation

Table 4.6 shows that MMSE-DFT channel estimation has 84.5% less average MSE than the MMSE channel estimation. The simulated graph of DFT-based channel estimation for LS and MMSE algorithms in MIMO-OFDM system is shown in Fig. 4.11.

Fig. 4.12 The Performance of LS and MMSE Channel Estimation with and without DFT
Fig. 4.12 demonstrates the comparison of average MSE for LS and MMSE channel estimations with and without the DFT estimation technique. By the introduction of DFT technique, the average MSE of both estimations is reduced very much without a doubt. Thus, this DFT-based estimation is considered as an efficient technique to minimize the MSE. It also increases the efficiency. This clearly proves that the received signal has less noise interference.

4.13. SUMMARY

The basic concepts of MIMO system, MIMO-OFDM system and various equalization techniques have been discussed in the different sections of this chapter. The significance of adaptive equalization with its mathematical representation, a variety of adaptive equalization algorithms such as LS and MMSE with their mathematical modeling are illustrated in this chapter. Also, a variety of channel estimation techniques of several researchers are reviewed. The performance of the existing MIMO-OFDM system with LS and MMSE channel estimation has been evaluated in terms of BER and MSE. The DFT-based channel estimation analysis and simulation have been done to overcome the limitations of LS and MMSE algorithms.