Chapter 3

Seismic Wave Attenuation

An education isn’t how much you have committed at memory, or even how much you know, it’s being able to differentiate between what you do know and what you don’t.

— James Joseph Sylvester

3.1 Seismic Attenuation

The decrease of amplitude with increasing the distance from its source is referred to as attenuation. It is partly due to geometry of propagation of seismic waves, and partly due to inelastic properties of the material through they travel. The attenuation property is considered to be closely related to inhomogeneity of crust. Seismic attenuation is usually considered to be a combination of two mechanisms, scattering attenuation or scattering loss and intrinsic attenuation or absorption. Measurements of attenuation of direct seismic waves give values for total attenuation. Scattering redistributes wave energy within the medium. Conversely intrinsic absorption refers to the conversion of vibrational energy into heat. Wu (1985) introduced the concept of seismic albedo ($B_0$) as the ratio of seismic loss to the total attenuation. The most important reduction is due to geometric attenuation. Let seismic body waves generated by a seismic source on surface of uniform half space. If there is no energy loss due to friction, the energy ($E_b$) in the wave front at distance $r$ from its source is distributed over the surface of hemisphere with area $2\pi r^2$. The intensity (or energy density $I_b$) of body waves is the
energy per unit area of wave front, so,

\[ I_S(r) = \frac{E_b}{2\pi r^2} \]  \hspace{1cm} (3.1.1)

The surface wave is constricted to spread out laterally. The disturbance affects not only the free surface but extends downwards into the medium to a depth \(d\) (which can be considered to be constant for a given wave). When the wave front of surface waves reaches a distance \(r\) from the source, the initial energy \((E_S)\) for surface wave is distributed over a circular cylindrical surface with area \(2\pi rd\). The intensity of surface wave at distance \(r\) from source is

\[ I_b(r) = \frac{E_S}{2\pi rd} \]  \hspace{1cm} (3.1.2)

Above equations (3.1.1) and (3.1.2) shows that decrease in intensity of body wave is proportional to \(\frac{1}{r^2}\) and the decrease of surface wave intensity is proportional to \(\frac{1}{r}\), respectively. Since \(I \propto A^2\), where \(I =\) Intensity and \(A =\) Amplitude.

So amplitude attenuations of seismic body and Surface waves are proportional to \(\frac{1}{r^2}\) and \(\frac{1}{\sqrt{r}}\) respectively. Aki and Chouet (1975) introduced the parameter \(Q\) (the quality factor) as a measure of decay rate of the coda within the given frequency band, and they showed that this decay rate was independent of recording site and event location, provided that observations were made within approximately 100 \(km\) of epicenter of an event. The parameter \(Q\) defined as the fractional energy loss per cycle (Knopoff, 1964),

\[ \frac{2\pi}{Q} = -\frac{\Delta E}{E}. \]  \hspace{1cm} (3.1.3)

Where \(\Delta E\) is the energy lost in one cycle and \(E\) is the elastic energy stored in the wave.

In other words, the definition of \(Q\) is "If the volume of the material is cycled in stress at frequency \((\omega)\), a dimensionless measure of internal function is given by

\[ \frac{1}{Q} = -\frac{\Delta E}{2\pi E} \quad \text{or} \quad Q = -\frac{2\pi E}{\Delta E}. \]  \hspace{1cm} (3.1.4) (3.1.5)

Where \(E\) is peak strain energy stored in the volume and \((\Delta E)\) is the energy lost in each cycle because of imperfection in the elasticity of the material. This definition is rarely of direct use, but in special experiments.
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In general, $Q$ is a function of frequency, $Q = Q(f)$ and includes loss mechanisms such as intrinsic absorption and scattering attenuation. Intrinsic absorption is due to inelasticity, which includes the loss of elastic energy. Scattering is related to the deflection and/or conversion of seismic energy due to randomly distributed heterogeneities (Aki, 1980).

More commonly, attenuation can be observed either the temporal decay of amplitude in standing wave at fixed number or the spatial decay in propagating wave at a fixed frequency. In other case for a medium with linear stress-strain relation, wave amplitude is proportional to $E^2$ or $A^2 \propto E$. Hence

$$\frac{1}{Q(\omega)} = - \left( \frac{1}{\pi} \right) \left( \frac{\Delta A}{A} \right)$$

from which amplitude fluctuations due to attenuation can be obtained (Aki and Richards, 2002). Thus in case (1), given that $A = A_0$ and amplitude decreases a fraction $\frac{\pi}{Q}$ at a successive times $\frac{2\pi}{\omega}, \frac{4\pi}{\omega}, \frac{6\pi}{\omega} \ldots \frac{2n\pi}{\omega} \ldots$, then at time $t$ the value of $A(t)$ is

$$A(t) = -A_0 \left( 1 - \frac{\pi}{Q} \right)^n$$

for $t = \frac{2n\pi}{\omega}$, i.e. $t\omega = \frac{2n}{2n}$

Thus,

$$A(t) = A_0 \left[ 1 - \frac{\omega t}{Q} \right]^n$$

$$A(t) = A_0 \left[ 1 - \frac{\omega t}{2nQ} \right]^n$$

$$A(t) = A_0 \left[ 1 - \frac{\omega t}{2Q} + \ldots \right]$$

$$A(t) = A_0 e^{-\frac{\omega t}{2Q}}$$

for large $n$. (3.1.8)

From observations of exponentially decreasing values of $A(t)$, equation (3.1.8) is used to the value of a temporal $Q$. The above result was obtained by using discrete times, since this is the nature of experiments based on case (1). For case (2), however, the derivation of the form $A = A(x)$ for distance $x$ is easier, since particular wave peak can be followed along a distance $dx$ and the gradual spatial decay of $A$ can be observed. Then $\Delta A = \left( \frac{dA}{dx} \right) \lambda$, where, $\lambda$ is wave length given in terms of $\omega$ and phase velocity $c$ by $c = n\lambda$ or $\lambda = \frac{c}{(\frac{2\pi}{\omega})} = \frac{2\pi c}{\omega}$
3.1 Seismic Attenuation

Then equation (1.2.8) becomes

$$A(x) = A_0 e^{\left[\frac{-\omega x^2}{2Q_C}\right]}$$

(3.1.9)

From observations of exponentially decaying values of $A(x)$, equation (3.1.9) used to define the value of spatial $Q$.

### 3.1.1 Types of Seismic Wave Attenuation

Geometrical spreading are simply the energy density decreases that occurs when an elastic wave front expands. In a homogeneous earth of constant velocity and density, the geometric spreading of seismic body waves is proportional to reciprocal of the distance between and source and receiver. Excluding geometrical spreading there are following two types of seismic wave attenuation:

**Intrinsic Attenuation or Absorption**: Intrinsic attenuation or absorption refers to the conversion of vibrational energy into heat for an inelastic medium or the mechanical energy is withdrawn from each passing wave and converted into other forms such as heat. This occurs in large variety of ways. The processes are collectively called intrinsic attenuation or absorption.

![Intrinsic and scattering attenuation of seismic wave energy](Margerin et al.,(1999))
This loss can be accounted for by using the attenuation coefficient ($\alpha$) in the form as:

$$A(t) = A_0 e^{-\alpha r} \quad (3.1.10)$$

where $A_0$ is initial wave amplitude and $A$ is wave amplitude after it has traveled a distance of $r$. Now the concept of quality factor is introduced which is defined as the ratio of total elastic energy ($E$) and energy lost $\Delta E$ in one cycle (Aki and Richards (2002)), i.e.,

$$Q = \frac{2\pi E}{\Delta E} \quad (3.1.11)$$

$Q$ is proportional to the number of cycles of vibration required to dissipate the elastic energy. For more elastic medium, $Q$ is larger. The relation between the energy $E$ and amplitude $A$ of seismic wave is given by

$$E = \frac{1}{2}\rho \omega^2 A^2 \quad (3.1.12)$$

Where $\omega = 2\pi f$ and $f$ is the frequency and $\rho$ is the density. If $A_1$ and $A_2$ are the amplitudes of wave one cycle apart, i.e., one wavelength ($\lambda$) apart in the space then from

$$Q = \frac{2\pi E}{\Delta E} \quad \text{or} \quad 2\pi \frac{Q}{Q} = \frac{\Delta E}{E} = 2\Delta, \quad \text{where}, \quad \Delta = \log\left(\frac{A_1}{A_2}\right) \quad (3.1.13)$$

On the other hand, from the original definition of absorption coefficient ($\alpha$), $A_1$ and $A_2$ can be expressed as

$$A_1 = A_0 e^{-\alpha r_0}$$
$$A_2 = A_0 e^{-\alpha(r_0+\lambda)}$$

Where $r_0$ is the distance traveled by the wave from the position where the amplitude was $A_0$ to its value is $A_1$ then

$$\frac{A_1}{A_2} = \frac{A_0 e^{-\alpha r_0}}{A_0 e^{-\alpha(r_0+\lambda)}}$$
$$\frac{A_1}{A_2} = e^{-\alpha \lambda \log\left(\frac{A_1}{A_2}\right)} = \alpha \lambda \quad (3.1.14)$$

So, from (3.1.13) and (3.1.14),

$$Q = \frac{2\pi E}{\Delta} = \frac{2\pi}{2\Delta} = \frac{\pi}{\alpha \lambda} \quad (3.1.15)$$
This means that the attenuation coefficient ($\alpha$) represents attenuation or absorption per cycle. This is the relation between quality factor and attenuation coefficient.

**Scattering Attenuation:** Seismologists long ago realized that the classical model of the earth as a body comprising of homogeneous layers can not explain by many observational data (Aki (1969) and Chouet (1975), Sato (1977)). This is especially true when short period waves strongly affected by the shallow crust are used for seismological investigations. If the medium through which the waves are passing is heterogeneous (non uniform) in physical properties, the energy splits into two, the reflected and refracted pulses at each boundary. If many interfaces are closely or irregularly spaced or are irregular in shape, the effect is to produce many subdivisions of the initial pulse traveling in the variety of directions, then the energy is said to be scattered, i.e., the modifications of seismic waves caused by three dimensional heterogeneities is broadly called seismic wave attenuation. Scattering is due to heterogeneities distributed in the earth causes a decrease in the amplitude with travel distance (Aki (1980)), i.e., scattering redistributes wave energy within the medium. Also scattering attenuation associated with heterogeneities. Scattering occurs when seismic waves encounter inhomogeneities (many small obstacles, cracks, wedges, faults, interfaces and variation in density etc.).

Because of scattering the net energy reaching a station becomes less than what would reach there if the scatterers were not present. As a consequence the recorded wave amplitude decreases. Such type of wave attenuation is called scattering attenuation. Scattering tends to spread the energy in time as well as in space.

Although Knopoff and Hudson (1964) showed that P to S and S to P conversions can be neglected, when scattering of high frequencies are considered, complete treatment must be accomplished studying the vector waves and including the possibility of P to S and S to P conversions. However, because of difficulty posed by vector waves, most of the early contributions of this field have considered only scaler waves. Sato (1977) in his isotropic model of coda generation considered P to S and S to P conversions, for coda generations. As in given medium the number of scatterers is numerous and their exact positions are not known. It is normal practice to solve the scattering problem using statical approach. In this approach only small numbers of statical parameters are required to present heterogeneity (Herraiz and Espinosa (1978)). The medium with random fluctuations in property that present the effect of scatterers. In the first case discrete scatterers such as cracks, faults, low or high density and low or high velocity are considered to be uniformly distributed inside a homogeneous medium. Aki (1969), Aki and Chouet (1975) and Dainty (1981), have used this discrete model with random uniform distribution. If the medium is taken as heterogeneous, the scattering field is treated as a continuous medium where a inhomogeneous wave equation should be solved. The problem may become extremely complicated. An approach to this case is
to consider a 'random medium', that is, a space with average characteristics in which deviation from these mean values produce random heterogeneities. The fluctuating parameters may be the velocity density or Lame's parameters. Chernov (1960) used this method to study small scale heterogeneity of the earth.

![Figure 3.2: Simplified relation between obstacles and wavelength size that shows how this parameter affects scattering (Herraiz and Espinosa, 1987).](image)

The seismic wave scattering very much depends on the size of the (figure 3.2) inhomogeneity. When 'a' is the inhomogeneity scale length, 'k' is wave number and 'λ' is wavelength 'ka' constitutes an important parameter. According to Herraiz and Espinosa (1987) if \( ka << 1 \) or \( ka >> 1 \) the waves are not affected by the obstacle and the medium acts like a homogenous body. When \( ka \approx 1 \) (e.g., \( 0.1 < ka < 10 \)), namely the size of heterogeneities are comparable to wavelength of seismic waves the scattering effects are most significant. The incident power is scattered to different directions with large angle to the incident direction. This case is frequently met in many physical situations.

3.2 Methods for Estimation of Seismic Attenuation

The vibrations recorded by a receiver after the direct wave arrival of an earthquake are called the coda. The amplitude of these motions is time and frequency dependent but their decay rate is independent of epicentral distance and source to receiver path.
3.2 Methods for Estimation of Seismic Attenuation

(Herraiz and Espinosa (1987)). Aki and Chouet (1975) considered the Earth as a randomly heterogeneous medium with a uniform statistical property and modeled the displacement envelope $A(\omega, t)$ of coda waves as:

$$A(\omega, t) = C(\omega) t^{-1} e^{-\frac{\omega}{Q_C(\omega)}}$$

(3.2.16)

Where, $C(\omega)$ is the coda source factor at radial frequency $\omega$ and $t$ is the elapsed time measured from the earthquake origin time. $Q_C(\omega)$ is the quality factor which includes the intrinsic and scattering attenuation of the waves. This was the first method to estimate coda $Q$.

Now in the literature there are number of methods available for studying attenuation characteristics of the crust and upper mantel using coda of local earthquake. Some researchers estimate coda $Q$ (Aki’s backscattering model, Sato’s single isotropic scattering model), other use coda waves to estimate direct P and S-wave attenuation characteristics (extended coda normalization method) and some other researchers use coda to determine the intrinsic and scattering attenuation characteristics of heterogeneous earth crust (Wennerberg’s method, multiple lapse time window analysis method). Some of these are discussed in this chapter.

3.2.1 Single Backscattering Model of Aki

The mathematical formulation of this model is based on two assumptions, first is scattering is a weak process and does not produce any secondary scattering. Second assumption is that, the station and source lie at the same point. According to Aki and Chouet (1975), the displacement envelope of coda waves is given by expression (3.2.16). Taking the logarithm of both sides and rearranging terms we obtained

$$\log[A(\omega, t)t] = c - bt$$

(3.2.17)

where $c$ is a constant,

$$b = \frac{\pi (\log_{10} e) f}{Q_C(\omega)}$$

(3.2.18)

The coda source factor $C(\omega)$ in equation (3.2.16) has been replaced by the constant $c$ in equation (3.2.17), since it is dependent only on frequency. $Q_C(\omega)$ is determined from the slope $b$ in equation (3.2.18) by a least-square solution to equation (3.2.17).
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In this model, the coda consists of back-scattered S-waves in a medium where the scattering is weak enough that multiple scattering may be ignored. Many investigators have estimated and interpreted coda $Q_C$ using the single back-scattering model (Aki and Chouet (1975), Sato (1977), Novelo-Casanova and Lee (1991), Pulli (1984)). Aki (1985) suggested that temporal $Q_C$ variations near the source region of an impending earthquake may be an earthquake precursor.

3.2.2 Single Isotropic Scattering Model of Sato

In the model of Sato (1977), the source receiver distance is taken into account. This particularly important when measuring coda waves close to the S-wave arrival. However, in high noisy environments, the coda wave amplitudes from small events are often below the background noise level after twice the S-wave lapse time. Sometimes the digital records may be truncated due to data storage restrictions. Sato (1977) proposed the single isotropic scattering model for non-coincidence source and receiver, thus we can analyze the coda waves just after the S-wave arrival.

According to Sato (1977), the mean energy density at angular frequency of scattered waves ($E_{sc}$) caused by inhomogeneities on the surface of an expanding ellipsoid whose foci are the sources and receiver is:

$$E_{sc}(r, \omega) = \frac{n\sigma W(\omega)}{4\pi r^2} K(\alpha) e^{-\frac{\omega t}{Q(\omega)}}$$  \hspace{1cm} (3.2.19)

Where,
- $r =$ source-receiver distance
- $t =$ lapse time from earthquake origin time
- $W(\omega) =$ energy radiated by the source within a unit angular frequency band
- $n =$ scattering per unit volume $\sigma =$ scatter cross-sectional area
- $n =$ effective scattering coefficient, and

$$K(\alpha) = \frac{1}{\alpha} \log \frac{\alpha + 1}{\alpha - 1}$$  \hspace{1cm} (3.2.20)

Where,
- $\alpha = \frac{t}{t_s}$
- $t_s =$ S-wave lapse time
- $Q(\omega) =$ apparent quality factor.
3.2 Methods for Estimation of Seismic Attenuation

3.2.3 Extended Coda Normalization Method

Aki (1980) proposed a technique to determine $Q$ for S-wave by comparing S-wave and coda wave amplitude of events at different hypocentral distances. Yashimoto et al. (1993) extended the coda normalization method of Aki (1980) for the estimation of P wave $Q$.

The fundamental empirical base of the coda normalization is that there is a uniform distribution of coda energy in some volumes surrounding the source at some lapse time. The key observation is support of this method is that coda envelopes have a common decay curve that is independent of the source-receiver distance (Routain and Khalturi (1978), Aki and Chouet (1975), Fehler and Sato (2003). Coda amplitude vary with source size and recording site amplification. This was initially observed by Aki (1969) who saw that the power spectrum of coda waves at a lapse time much larger than the S wave traveled time and is independent of epicentral distance. The uniform distribution of coda energy beyond certain lapse time is also predicted by the single isotropic model (Sato (1977), Aki (1980) proposed a single station method for measuring attenuation. He normalized direct S-wave amplitude by S coda amplitude, measured at a fixed time and at the same frequency. This method corrects for source size and site amplification, thus allowing data to be combined from many earthquakes to find a more stable estimate of attenuation. This technique was applied by different authors (Mayeda et al., (1992), Hoshiba, (1993), Yshimato, (1993)) in several regions of the ward. This method exploit the fact that after some time of earthquake occurrence there is uniform distribution of coda energy (Fehler and Sato (2003)). Result calculated using radiative transfer equations with the assumptions of multiple scattering models, non-isotropic scattering and non-isotropic source radiation also show uniform coda energy distribution in a given volume after a large enough lapse time (Fehler and Sato (2003)).

Let us consider S Coda particle velocity $\hat{u}_{ij}^{S\text{Coda}}(t; f)$ at site $j$ for the $i^{th}$ earthquake, this is the velocity at central frequency $f$ and lapse time $t$ and $\langle .... \rangle_T$ means average over some time window $T$, the average coda power as a convolution of the source, scattering and site amplification show the following relation

$$\langle \hat{u}_{ij}^{S\text{Coda}}(t; f)^2 \rangle_T = \frac{W_i^S(f)g_0(f)N_j^S(f)^2}{2\pi\beta_0^2t^2}e^{-Q\beta_0^2t},$$  \hspace{1cm} (3.2.21)

Where, $W_i^S(f)$ is the S-wave source power and $g_0(f)$ is the total scattering coefficient representing the scattering power per unit volume, $N_j^S(f)$ is the site amplification factor, and $\beta_0$ is the background S-wave velocity.

Now consider direct S-wave particle velocity amplitude at station $j$ and at frequency $f$
3.2 Methods for Estimation of Seismic Attenuation

for local earthquake $i$:

$$
|\hat{u}_{ij}^{S,Direct}(f)| \propto \frac{1}{r_{ij}} \sqrt{W_i^S(f)N_j^S(f)} e^{-Q_{\beta}^{-1}\pi f r_{ij}/\beta_0}, \quad (3.2.22)
$$

where, $r_{ij}$ is the source receiver distance, $\beta_0$ is S-wave velocity and $Q_{\beta}$ is the Q of direct S-waves. Taking the logarithm of the ratio of product of hypocentral distance and the direct S-wave amplitude to the average coda amplitude given by equation (3.2.21), the common site amplification and source terms cancel, we get

$$
\ln r_{ij} |\hat{u}_{ij}^{S,Direct}(f)| \left/ \sqrt{\langle \hat{u}_{ij}^{S,Coda}(t_c; f)^2 \rangle_T}\right. = - (Q_{\beta}^{-1}(f)\pi f / \beta_0) r_{ij} + \text{const.} \quad (3.2.23)
$$

where, the measurements are made over a large enough number of earthquakes, the radiation pattern differences are smooth out. Plotting the left-hand side against hypocentral distance, the the gradient gives the attenuation per travel distance.

Aki (1980) first applied this method to seismogram of local earthquakes recorded in Kanto, Japan. Later Yoshimoto et. al. (1993) extended the conventional coda normalization method to measure $Q_{\alpha}^{-1}$. He assume that earthquakes within the small magnitude range have the same spectral ratio of P to S-wave radiation within a narrow frequency range $f \mp \Delta f$ as

$$
\frac{S_P(f)}{S_S(f)} = \text{Const.}(f) \quad (3.2.24)
$$

For lapse time greater than roughly twice the direct S-wave travel time, the spectral amplitude of the data at lapse time $t_c$, $A_C(f, t_c)$ is independent of hypocentral distance $r$ in the regional distance range, and is written (e.g., Aki, 1980) as

$$
A_C(f, t_c) = S_S(f)P(f, t_c)G(f)I(f) \quad (3.2.25)
$$

where, $f$ is the frequency, $S_S(f)$ is the source spectral amplitude od S-waves, $P(f, t_c)$ is the coda expectation factor and $I(f)$ denotes the instrumental response. By normalizing the source spectral amplitude of S-wave by the spectral amplitude of coda waves, we get the following simple equation

$$
\langle \ln |\frac{A_S(f, r)}{A_C(f, t_c)}|^r \rangle_{r \pm \Delta r} = - \left( \frac{\pi f}{Q_{\beta}(f) \beta} \right)^r + \text{const.} \quad (3.2.26)
$$
where, \( r^r \) denotes the geometrical spreading exponents \( \beta \) represents the average S-wave velocity. \( Q_S(f) \) may be estimated from a linear regression of \( \langle \ln[\frac{A_S(f, r)r^r}{A_C(f, t_c)}]\rangle_{r \pm \Delta r} \) versus \( r \) by means of the least squares method.

From equation (3.2.24) and (3.2.25) we can easily derive the following relation:

\[
A_C(f, t_c) \propto S_S(f) \propto S_P(f)
\]

This equation means that we may use the spectral amplitude of the S coda wave for the normalization of P-wave \( Q_\alpha(f) \) is the quality factor of P wave, and \( \alpha \) represents the average P-wave velocity. Now we get the equation:

\[
\langle \ln[\frac{A_P(f, r)r^r}{A_C(f, t_c)}]\rangle_{r \pm \Delta r} = -\frac{\pi f}{Q_\alpha(f)\alpha} r + \text{const.}
\] (3.2.27)

where, \( A_P(f, r) \) is the spectral amplitude of the direct P-wave. \( Q_\alpha(f) \) may be estimated from a linear regression of \( \langle \ln[\frac{A_P(f, r)r^r}{A_C(f, t_c)}]\rangle_{r \pm \Delta r} \) versus \( r \) by means of the least squares method. Using extended coda normalization method we can simultaneously estimate \( Q_\beta \) and \( Q_\alpha \) from a data set obtain at a single station.

### 3.2.4 Wennerberg’s Method of Estimation of \( Q_i^{-1} \) and \( Q_S^{-1} \) from Coda \( Q^{-1} \)

Scattering models have been developed in order to infer physical properties of the lithosphere from observations of seismic codas. The two parameters generally sought are the average scattering strength and the energy absorption due to intrinsic attenuation. Most of the methods available for separation of these two parameters are based on the assumption of isotropic distribution of scatters and constant \( Q_i \) in the same earth volume (Zeng 1991, Fehler et. al. 1992). It is generally accepted that heterogeneity of the earth decreases with increasing depth. So the above assumption may be unrealistic. The lapse time dependence of \( Q_C \) is considered to be a point indicating the fact that medium properties are not uniform. The lapse time dependence of \( Q_C \) can be explained in terms of decreasing \( Q_i \) with increasing depth (Wennerberg (1993)).

In order to estimate both intrinsic \( (Q_i^{-1}) \) and scattering \( (Q_S^{-1}) \) attenuation, Wennerberg (1993) proposed a method which takes into account the numerical correction of the \( Q_C \) parameters estimated using Aki and Chouet’s (1975) single back scattering hypothesis for the multiple scattering formulation of Zeng (1991). The total energy
attenuation parameters due to propagation $\eta_d$ is combination of scattering attenuation ($\eta_s$) and intrinsic attenuation ($\eta_i$), thus

$$\eta_d = \eta_s + \eta_i$$  \hspace{1cm} (3.2.28)

using the relation $\eta = \frac{\omega}{Qv}$, where, $v$ is the average medium velocity and $\omega$ is angular frequency

$$\frac{1}{Q_d} = \frac{1}{Q_S} + \frac{1}{Q_i}$$  \hspace{1cm} (3.2.29)

The coda amplitude models considered here include intrinsic attenuation effect as common overall exponential factor, $e^{-\frac{\omega t}{2Q_i}}$, coda models may be write in the units of mean free time which is a function of dimensionless parameters $\tau$ where,

$$t = \frac{\tau}{\eta_Sv} = \tau \frac{Q_S}{\omega}$$

with the help of numerical calculations that compare coda shapes of the single backscattering and Zeng’s model, Wennerrberg (1993) shown that the observed $Q_c$ is related to the intrinsic and scattering attenuation as follows:

$$\frac{1}{Q_C(\tau)} = \frac{1}{Q_i} + \frac{1 - 2\delta(\tau)}{Q_C}$$  \hspace{1cm} (3.2.30)

where, $1 - 2\delta(\tau) = \frac{1}{(4.44 + 0.732\tau)}$ and $\tau = \frac{\omega t}{Q_S}$ is the mean free time, $t$ is the lapse time and $\omega$ is the angular frequency. Using equation (1.4.33) and (1.4.34) $Q_S$ and $Q_i$ can be expressed as

$$\frac{1}{Q_s} = \frac{1}{2\delta(\tau)} \left( \frac{1}{Q_d} - \frac{1}{Q_C(\tau)} \right)$$   \hspace{1cm} (3.2.31)

$$\frac{1}{Q_i} = \frac{1}{2\delta(\tau)} \left( \frac{1}{Q_C(\tau)} + \frac{2\delta(\tau) - 1}{Q_d} \right)$$   \hspace{1cm} (3.2.32)

Combining the above four equations, $Q_S$ is given as the positive root of the following equations

$$(Q_C - Q_d)aQ_S^2 + Q_S[b\omega t(Q_C - Q_d) - cQ_dQ_C] - b\omega tQ_dQ_C = 0$$  \hspace{1cm} (3.2.33)
where, \( a, b \) and \( c \) are constant and values of these are 4.44, 0.738 and 5.44 respectively.

To estimate the contribution of intrinsic and scattering components of the attenuation it is necessary to compute \( Q_C \) and \( Q_d \). Then \( Q_i \) may be calculated with the help of equation (3.2.24) and \( Q_S \) by equation (3.2.25). This method provides numerically simple technique to calculate \( Q_S \) and \( Q_i \). In this method it is assumed that the source and the receiver are co-located, care has to be exercised to choose the combination of \( Q_C \) and \( Q_d \) that can be used for estimation of \( Q_S \) and \( Q_i \). This can be achieved by using \( Q_d \) values estimated from S-waves of earthquakes recorded at hypocentral distances whose maximum values should be comparable to the maximum distance from which coda reach a station, so that this coda is used for \( Q_S \), \( Q_S \) and \( Q_i \) estimation (Wennerberg (1993)). A simple way of checking this is to use \( Q_C \) values estimated with coda whose lapse time to the end of the coda window is approximately twice the maximum travel time of S-wave used for estimation of \( Q_d \).

Most of the methods of estimation of \( Q_S \) and \( Q_i \), including, this one assume isotropic model. However, \( Q_C \) shows lapse time dependence and \( \tau \) also depends on lapse time, an attempt can be made to see if lapse time dependence of \( Q_S \) and \( Q_i \) can be estimated. The lapse time dependence of \( Q_C \) can be related to its variations with depth.