CHAPTER FIVE

Certain types of non-affine motion in projectively symmetric Finsler space (PS-Fn).
1. Introduction

Infinitesimal affine motions generated by a contra, concurrent, special concircular, recurrent and torse-forming vector field in Riemannian and Finsler spaces have been studied by various authors including K. Takano[113], R. S. Sinha[108].

Projective motion in symmetric Finsler space was introduced by F. M. Meher[42]. Projectively symmetric Finsler space has been studied by R. B. Misra[59]. Misra and Meher[70] have studied contra affine motion in $PS - F_n$. The Purpose of Present chapter is to discuss non-affine motions generated by different vector fields in Projectively symmetric Finsler space($PS - F_n$).

Let $F_n$ be n-dimensional Finsler space equipped with Berwald's connection parameter $G_{jk}^i$, The curvature tensor field $H_{jk}^i$, arising from this connection parameter is homogeneous function of degree zero in $\dot{x}$.

Let $W$ be the projective curvature tensor with components $W_{jk}^i$ and a tensor associated to it be defined in a way similar to
(1.1) \( W_{jk}^i = W_{jkh}^i \dot{x}^j = H_{jk}^i - \frac{\dot{x}^i}{n+1} (H_{jk} - H_{kj}) + \)
\[
\frac{\delta^i_j}{n^2-1} (nH_k + \dot{x}^r H_{kr}) - \frac{\delta^i_k}{n^2-1} (nH_j + \dot{x}^r H_{jr}).
\]

A Finsler space \( F_n (n > 2) \), throughout with projective curvature tensor possess vanishing covariant derivative has been called a projectively symmetric Finsler space [59], such a space is denoted by \( PS - F_n \). It is seen that a \( PS - F_n \) also admits

(1.2) \( W_{jk(i)}^i = 0. \)

Let us consider an infinitesimal transformation

(1.3) \( \bar{x}^i = x^i + \varepsilon \nu^i \)

where \( \nu^i \) is a covariant vector field independent of the directional arguments and \( \varepsilon \) is an infinitesimal constant. It defines an affine motion if and only if the Lie derivative of connection parameter \( G_{jk}^i \) with respect to the said transformation vanishes, that is, if there holds Yano\[118\]

(1.4) \( \varepsilon G_{kh}^i = \nu^i_{(h)(k)} - \nu^i H_{jkh}^i + G_{jkh}^i \nu^j_{(r)} \dot{x}^r = 0. \)

Now, we consider the case of a non-affine motion in which the Lie derivative of the connection parameter \( G_{kh}^i \) satisfies

(1.5) \( \varepsilon G_{kh}^i = \delta^i_j \epsilon_k + \delta^i_k \epsilon_j, \)
where $\epsilon_j(x, \dot{x})$ is a non-zero homogeneous vector and satisfies the following relations.

\begin{equation}
\epsilon_{hj} = \dot{\epsilon}_h\epsilon_j,
\end{equation}

\begin{equation}
\epsilon_j = \epsilon_j \dot{x}^j
\end{equation}

and

\begin{equation}
\epsilon(x, \dot{x}) = \epsilon_j \dot{x}^j
\end{equation}

A vector field $v^i$ is called contra, concurrent, special concircular and torse-forming vector field according as it satisfies

\begin{equation}
v^i_{(i)} = 0,
\end{equation}

\begin{equation}
v^i_{(i)} = c \delta^i_j, \text{ c being a constant}
\end{equation}

\begin{equation}
v^i_{(i)} = \alpha \delta^i_j \text{ } \alpha \text{ is not a constant}
\end{equation}

and

\begin{equation}
v^i_{(i)} = v^i \mu_j + \alpha \delta^i_j
\end{equation}

respectively.

where $\alpha$ is non-zero scalar function and $\mu_j$ being any non null vector field.
Sinha[108] proved that a Finsler space admitting an affine motion also admits

\[(1.11) \quad \mathcal{E}H_{jkh}^i = 0.\]

Equation (1.11) after making use of the Lie differential of the curvature tensor \(H_{jkh}^i\) in view of (1.5) can be written as:

\[(1.12) \quad \mathcal{E}H_{jkh}^i = \delta_j^i \epsilon_{h(k)} - \delta_k^i \epsilon_{h(j)} + \delta_h^i \epsilon_{j(k)} - \delta_h^i \epsilon_{k(j)},\]

\[(1.13) \quad \mathcal{E}H_{jk}^i = \delta_j^i \epsilon_{(k)} - \delta_k^i \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \dot{x}^i \epsilon_{k(j)},\]

\[(1.14) \quad \mathcal{E}H_{kh}^i = (n + 1) \epsilon_{h(k)} - \delta_k^i \epsilon_{h(i)} - \delta_h^i \epsilon_{k(i)},\]

\[(1.15) \quad \mathcal{E}H_k^i = (n + 1) \epsilon_{(k)} - \delta_k^i \epsilon_{(i)} - \dot{x}^i \epsilon_{k(i)}.\]

Let us assume that there exist non-affine motion with a contra, concurrent, special concircular and torse-forming vector field. Such non-affine motion is characterized by (1.7), (1.8), (1.9) and (1.10).

2. Contra non-affine motion in projectively recurrent Finsler space \((WR - F_n)\)

We consider an infinitesimal transformation of the form

\[(2.1) \quad \bar{x}^i = x^i + \epsilon v^i(x), \quad v_{(j)}^i = 0.\]
Where $v^i$ is a contra vector field independent of directional arguments and $\varepsilon$ is an infinitesimal constant and such above transformation is called a contra transformation. It define an affine motion if and only if the lie derivative $\mathcal{L}G^i_{kh}$ of the connection parameters with respect to contra transformation vanishes, that is, if there holds.

$$\mathcal{L}G^i_{kh} = v^i_{(h)(k)} - v^j H^i_{jkh} + G^i_{jkh} v^j_{(r)} \dot{x}^r = 0$$

where $v^i_{(j)} = 0$.

**Definition 2.1:** A Finsler space admits (1.5) as well as (1.7) then the infinitesimal transformation is said to define non-affine motion with a contra vector field (or called contra non-affine motion).

The Lie-derivative of [1-(6.6)] and observing (1.12), (1.13), (1.14) and (1.15), we get

$$(2.2) \mathcal{L}W^i_{jk} = \delta^i_j \varepsilon_{(k)} - \delta_k^i \varepsilon_{(j)} + \dot{x}^i \varepsilon_{j(k)} - \frac{2x^i}{n+1} \varepsilon_{(k<r)} \delta^r_j +$$

$$\frac{\delta^i_j}{n-1} [(n + 1) \varepsilon_{(k)} - \delta_k^r \varepsilon_{(r)} - \dot{x}^r \varepsilon_{k(r)}]$$

$$+ \frac{\delta^i_k}{n-1} [(n + 1) \varepsilon_{(j)} - \delta^r_j \varepsilon_{(r)} - \dot{x}^r \varepsilon_{j(r)}].$$

[69]
In the space admitting non-affine motion. Writing an explicit expression for Lie-derivative of tensor field $W_{jk}^i$, substituting it on the L.H.S. of (2.2), we get [118].

\[
(2.3)v^hW_{jk(h)}^i - v^h_kW_{jk}^i + v^h_jW_{hk}^i + v^h_kW_{jh}^i + (\dot{\gamma}_hW_{jk}^i)v^h_s\dot{x}^s
\]

\[
= \delta^i_j\epsilon_{(k)} - \delta^i_k\epsilon_{(j)} + \dot{x}^i\epsilon_{j(k)} - \frac{2\dot{x}^i}{n + 1}\epsilon_{(k<r)}\delta^r_j
\]

\[
+ \frac{\delta^i_j}{n - 1}[(n + 1)\epsilon_{(k)} - \delta^r_k\epsilon_{(r)} - \dot{x}^r\epsilon_{k(r)}]
\]

\[
+ \frac{\delta^i_k}{n - 1}[(n + 1)\epsilon_{(j)} - \delta^r_j\epsilon_{(r)} - \dot{x}^r\epsilon_{j(r)}].
\]

We consider that the contra non-affine motion be characterized by (2.1) then the equation (2.3) simplifies into:

\[
(2.4) (v^hW_{jk(h)}^i) = \delta^i_j\epsilon_{(k)} - \delta^i_k\epsilon_{(j)} + \dot{x}^i\epsilon_{j(k)}
\]

\[
- \frac{2\dot{x}^i}{n + 1}\epsilon_{(k<r)}\delta^r_j + \frac{\delta^i_j}{n - 1}[(n + 1)\epsilon_{(k)} - \\
\delta^r_k\epsilon_{(r)} - \dot{x}^r\epsilon_{k(r)}] + \frac{\delta^i_k}{n - 1}[(n + 1)\epsilon_{(j)} - \\
\delta^r_j\epsilon_{(r)} - \dot{x}^r\epsilon_{j(r)}].
\]

In view of [1-(7.11)] the equation (2.4) may be written as

\[
(2.5) v^h u_h W_{jk}^i = \delta^i_j\epsilon_{(k)} - \delta^i_k\epsilon_{(j)} + \dot{x}^i\epsilon_{j(k)}
\]
\[-\frac{2\dot{x}^i}{n+1}\delta^{r}_{j}(\varepsilon_{k<r}\delta^{r}_{j}) + \frac{\delta^{i}_{j}}{n-1}[(n+1)\varepsilon_{(k)} - \delta^{r}_{k}\varepsilon_{(r)} - \dot{x}^r\varepsilon_{k(r)}] + \frac{\delta^{i}_{k}}{n-1}[(n+1)\varepsilon_{(j)} - \delta^{r}_{j}\varepsilon_{(r)} - \dot{x}^r\varepsilon_{j(r)}].\]

Where $u_h$ is non zero vector and choose another vector $v^h$ such that

\[(2.6) \quad u_h v^h = 1.\]

Using (2.6) in (2.5) we get

\[(2.7) \quad W_{jk}^i = \delta^{i}_{j}\varepsilon_{(k)} - \delta^{i}_{k}\varepsilon_{(j)} + \dot{x}^i\varepsilon_{j(k)} - \frac{2\dot{x}^i}{n+1}\varepsilon_{(k<r)\delta^{r}_{j}} + \frac{\delta^{i}_{j}}{n-1}[(n+1)\varepsilon_{(k)} - \delta^{r}_{k}\varepsilon_{(r)} - \dot{x}^r\varepsilon_{k(r)}] + \frac{\delta^{i}_{k}}{n-1}[(n+1)\varepsilon_{(j)} - \delta^{r}_{j}\varepsilon_{(r)} - \dot{x}^r\varepsilon_{j(r)}].\]

Thus we can state

**Theorem 2.1:** If $WR - F_n$ admitting contra non-affine motion is necessarily satisfies the equation (2.7) provided that the vector $v^h$ is not orthogonal to vector $u_h$. 

[71]
3. Concurrent non-affine motion in a $PS - F_n$

We consider an infinitesimal transformation of the form

\[(3.1) \quad \bar{x}^i = x^i + \epsilon v^i(x), \quad v^i_{(j)} = c\delta^i_j, \quad c \text{ being a constant.}\]

Where $v^i$ is a concurrent vector field independent of directional arguments and $\epsilon$ is an infinitesimal constant and such above transformation is called a concurrent transformation. It define an affine motion if and only if the lie derivative ($\mathcal{L}_G^i_{kh}$) of the connection parameters with respect to concurrent transformation vanishes, that is, if there holds.

\[\mathcal{L}_G^i_{kh} = v^i_{(h)(k)} - v^j H^i_{jkh} + G^i_{jkh} v^j_{(r)} \bar{x}^r = 0\]

where $v^i_{(j)} = c\delta^i_j, \quad c \text{ being a constant.}$

**Definition 3.1**: A Finsler space admits (1.5) as well as (3.1) then the infinitesimal transformation is said to define non-affine motion with a concurrent vector field (or called concurrent non-affine motion).

We consider that the concurrent non-affine motion characterized by (3.1) then the equation (2.3) may be written as

\[(3.2) v^h W^i_{jk(h)} - c\delta^i_h W^j_{hk} + c\delta^h_j W^i_{hk} + c\delta^h_k W^i_{jh} + (\dot{\alpha}_h W^i_{jk}) c\delta^h_s \bar{x}^s\]
\[
= \delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2\dot{x}^i}{n+1} \epsilon_{(k<r)} \delta^r_j \\
+ \frac{\delta^i_j}{n-1} [(n+1)\epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)}] \\
+ \frac{\delta^i_k}{n-1} [(n+1)\epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)}]
\]

or

(3.3) \[ v^h W^i_{jk(h)} - c W^i_{jk} + c W^i_{jk} + c W^i_{jk} + (\delta^i_h W^i_{jk}) c \dot{x}^h \]

\[
= \delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2\dot{x}^i}{n+1} \epsilon_{(k<r)} \delta^r_j \\
+ \frac{\delta^i_j}{n-1} [(n+1)\epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)}] \\
+ \frac{\delta^i_k}{n-1} [(n+1)\epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)}].
\]

In view of [1-(6.5)] and [1-(6.7)a] the equation (3.3) reduces to

(3.4) \[ v^h W^i_{jk(h)} + 2c W^i_{jk} \]

\[
= \delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2\dot{x}^i}{n+1} \epsilon_{(k<r)} \delta^r_j \\
+ \frac{\delta^i_j}{n-1} [(n+1)\epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)}]
\]

[73]
\[ + \frac{\delta_k^i}{n-1} \left[ (n+1)\varepsilon_{(j)} - \delta^r_{(r)} - \dot{x}^r \varepsilon_{(r)} \right]. \]

Using \([1-(7.8)]\) in equation (3.4), we get

\[(3.5) \]

\[ W_{jk}^i = \frac{1}{2c} \left[ \left( \delta^i_{(k)} - \delta^i_{(j)} + \dot{x}^i \varepsilon_{(j)} - \frac{2x^i}{n+1} \varepsilon_{(k<r} \delta^r_{(j))} \right) \right. \]

\[ + \frac{\delta^i_{(j)}}{n-1} \left. \left[ (n+1)\varepsilon_{(k)} - \delta^r_{(r)} - \dot{x}^r \varepsilon_{(r)} \right] \right. \]

\[ + \frac{\delta^i_{(k)}}{n-1} \left[ (n+1)\varepsilon_{(j)} - \delta^r_{(r)} - \dot{x}^r \varepsilon_{(r)} \right]. \]

Thus we can state

**Theorem 3.1:** If \(PS - F_n\) admitting concurrent non-affine motion then the condition (3.5) necessarily holds.

4. Special concircular non-affine motion in a \(PS - F_n\)

We consider an infinitesimal transformation of the form

\[(4.1) \quad \tilde{x}^i = x^i + \varepsilon v^i(x), \quad v^i_{(j)} = \alpha \delta^i_{(j)}. \]

(\(\alpha\) is not a constant)
Where $v^i$ is a special concircular vector field independent of directional arguments and $\varepsilon$ is an infinitesimal constant and such above transformation is called a special concircular transformation. It define an affine motion if and only if the lie derivative ($\mathcal{L}_G^i_{kh}$) of the connection parameters with respect to special concircular transformation vanishes, that is, if there holds.

$$\mathcal{L}_G^i_{kh} = v^i_{(h)(k)} - v^j H^i_{jhk} + G^i_{jkh} v^j_{(r)} \dot{x}^r = 0$$

where $v^i_{(j)} = \alpha \delta^i_j$, $\alpha$ is not a constant.

**Definition 4.1:** A Finsler space admits (1.5) as well as (4.1) then the infinitesimal transformation is said to define non-affine motion with a special concircular vector field (or called special concircular non-affine motion).

We consider that the special concircular non-affine motion characterized by (4.1) then the equation (2.3) may be written as

$$(4.2)v^h W^i_{jk(h)} - \alpha \delta^i_h W^i_{jk} + \alpha \delta^h_j W^i_{hk} + \alpha \delta^h_k W^i_{jh} + (\dot{\delta}_j W^i_{jk}) \alpha \delta^h_s \dot{x}^s$$

$$= \delta^i_j \varepsilon_{(k)} - \delta^i_k \varepsilon_{(j)} + \dot{x}^i \varepsilon_{j(k)} - \frac{2 \dot{x}^i}{n + 1} \varepsilon_{(k < r > \delta^i_j)}$$

$$+ \frac{\delta^i_j}{n - 1} [(n + 1) \varepsilon_{(k)} - \delta^i_k \varepsilon_{(r)} - \dot{x}^r \varepsilon_{k(r)}]$$

[75]
\[ + \frac{\delta^i_k}{n-1} \left[ (n+1)\epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)} \right]. \]

In view of [1-(6.5)] and [1-(6.7)a] the equation (4.2) reduces to

(4.3) \[ v^h W^i_{jk(h)} + 2\alpha W^i_{jk} \]

\[ = \delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2\dot{x}^i}{n+1} \epsilon_{(k<r)} \delta^r_j \]

\[ + \frac{\delta^i_j}{n-1} \left[ (n+1)\epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)} \right] \]

\[ + \frac{\delta^i_k}{n-1} \left[ (n+1)\epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)} \right]. \]

Using [1-(7.8)] in equation (4.3), we have

(4.4) \[ W^i_{jk} = \frac{1}{2\alpha} \left[ \left( \delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2\dot{x}^i}{n+1} \epsilon_{(k<r)} \delta^r_j \right) \right] \]

\[ + \frac{\delta^i_j}{n-1} \left[ (n+1)\epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)} \right] \]

\[ + \frac{\delta^i_k}{n-1} \left[ (n+1)\epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)} \right]. \]

Thus we can state:

**Theorem 4.1:** If PS - Fn admitting special concircular non-affine motion then the condition (4.4) necessarily holds.

[76]
5. Torse-forming non-affine motion in a $PS - F_n$

We consider an infinitesimal transformation of the form

$$\tilde{x}^i = x^i + \varepsilon v^i(x), \quad v^i_{(j)} = v^i \mu_j + \alpha \delta^i_j.$$  \hspace{1cm} (5.1)

Where $v^i$ is a torse-forming vector field independent of directional arguments and $\varepsilon$ is an infinitesimal constant and such above transformation is called a torse-forming transformation. It define an affine motion if and only if the lie derivative ($\mathcal{L}G^i_{kh}$) of the connection parameters with respect to torse-forming transformation vanishes, that is, if there holds.

$$\mathcal{L}G^i_{kh} = v^i_{(h)(k)} - v^j H^i_{jkh} + G^i_{jkh} v^j_{(r)} \tilde{x}^r = 0$$

where $v^i_{(j)} = v^i \mu_j + \alpha \delta^i_j$.

**Definition 5.1:** A Finsler space admits (1.5) as well as (5.1) then the infinitesimal transformation is said to define non-affine motion with a torse-forming vector field (or called torse-forming non-affine motion).

We consider that the torse-forming non-affine motion characterized by equation (5.1) then the equation (2.3) may be written as

[77]
\( (5.2) \quad v^h W_{jk(h)}^i - \alpha \delta^i_h W_{jk}^h - v^i \mu_h W_{jk}^h + \alpha \delta^i_j W_{hk}^i \\
+ v^h \mu_j W_{hk}^i + \alpha \delta^i_k W_{jh}^i + v^h \mu_k W_{jh}^i \\
+ (\dot{\delta}_h W_{jk}^i) \alpha \delta^i_s \dot{x}^s + (\dot{\delta}_h W_{jk}^i) v^h \mu_s \dot{x}^s \\
eq \delta^i_j \varepsilon_{(k)} - \delta^i_k \varepsilon_{(j)} + \dot{x}^i \varepsilon_{j(k)} - \frac{2 \dot{x}^i}{n + 1} \varepsilon_{(k < r) \delta^r_j} \\
+ \frac{\delta^i_j}{n-1} [(n+1) \varepsilon_{(k)} - \delta^r_k \varepsilon_{(r)} - \dot{x}^r \varepsilon_{k(r)}] \\
+ \frac{\delta^i_k}{n-1} [(n+1) \varepsilon_{(j)} - \delta^r_j \varepsilon_{(r)} - \dot{x}^r \varepsilon_{j(r)}].
\)

Using [1-(6.5)], [1-(6.7)a] and [2-(2.5)] in equation (5.2), we have

\( (5.3) \quad v^h W_{jk(h)}^i - v^i \mu_h W_{jk}^h + v^h \mu_j W_{hk}^i \\
+ 2 \alpha W_{jk}^i + v^h \mu_k W_{jh}^i + (\dot{\delta}_h W_{jk}^i) v^h \mu \\
eq \delta^i_j \varepsilon_{(k)} - \delta^i_k \varepsilon_{(j)} + \dot{x}^i \varepsilon_{j(k)} - \frac{2 \dot{x}^i}{n + 1} \varepsilon_{(k < r) \delta^r_j} \\
+ \frac{\delta^i_j}{n-1} [(n+1) \varepsilon_{(k)} - \delta^r_k \varepsilon_{(r)} - \dot{x}^r \varepsilon_{k(r)}] \\
+ \frac{\delta^i_k}{n-1} [(n+1) \varepsilon_{(j)} - \delta^r_j \varepsilon_{(r)} - \dot{x}^r \varepsilon_{j(r)}].
\)

In view of [1-(7.8)] the equation (5.3) may be written as

[78]
(5.4) $\nu^i \mu_h W^h_{jk} + \nu^h \mu_j W^i_{hk} + 2 \alpha W^i_{jk} + \nu^h \mu_k W^i_{jh} + W^i_{hk} \nu^h \mu$

$$= \delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2 \dot{x}^i}{n + 1} \epsilon_{(k<r)} \delta^r_j$$

$$+ \frac{\delta^i_j}{n - 1} [(n + 1) \epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)}]$$

$$+ \frac{\delta^i_k}{n - 1} [(n + 1) \epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)}].$$

Thus we can state

**Theorem 5.1:** If $PS - F_n$ admitting torse-forming non-affine motion then then the condition (5.4) necessarily holds.

At this stage if we assume that the space under consideration is projectively flat then (5.4) is immediately reduces

(5.5) $\delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2 \dot{x}^i}{n + 1} \epsilon_{(k<r)} \delta^r_j$

$$+ \frac{\delta^i_j}{n - 1} [(n + 1) \epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)}]$$

$$+ \frac{\delta^i_k}{n - 1} [(n + 1) \epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)}] = 0.$$
\[(5.4) \nu^i \mu_h W^h_{jk} + \nu^h \mu_j W^i_{hk} + 2\alpha W^i_{jk} + \nu^h \mu_k W^i_{jh} + W^i_{hjk} \nu^h \mu
\]

\[= \delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2\dot{x}^i}{n+1} \epsilon_{(k<r)} \delta^r_j \]

\[+ \frac{\delta^i_j}{n-1} [(n+1)\epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)}] \]

\[+ \frac{\delta^i_k}{n-1} [(n+1)\epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)}]. \]

Thus we can state

**Theorem 5.1:** If PS - \(F_n\) admitting tose-forming non-affine motion then then the condition (5.4) necessarily holds.

At this stage if we assume that the space under consideration is projectively flat then (5.4) is immediately reduces

\[(5.5) \delta^i_j \epsilon_{(k)} - \delta^i_k \epsilon_{(j)} + \dot{x}^i \epsilon_{j(k)} - \frac{2\dot{x}^i}{n+1} \epsilon_{(k<r)} \delta^r_j \]

\[+ \frac{\delta^i_j}{n-1} [(n+1)\epsilon_{(k)} - \delta^r_k \epsilon_{(r)} - \dot{x}^r \epsilon_{k(r)}] \]

\[+ \frac{\delta^i_k}{n-1} [(n+1)\epsilon_{(j)} - \delta^r_j \epsilon_{(r)} - \dot{x}^r \epsilon_{j(r)}] = 0. \]
Theorem 5.2: If a projectively symmetric flat Finsler space admitting torse-forming non-affine motion then the condition (5.5) necessarily holds.