CHAPTER 3

ZERO-BACKSCAN MULTIPLE HYPOTHESIS TRACKING ALGORITHM FOR RADAR SURVEILLANCE

This chapter presents the proposed Zero-Backscan Multiple Hypothesis Tracking (ZBMHT) algorithm, the implementation details and its performance analysis.

3.1 INTRODUCTION

The MHT technique originated in 1970’s (Singer et al 1974, Bar-Shalom and Tse 1975, Reid 1979) as a data association technique that evaluates the probabilities of sequences of measurements having originated from various targets. The MHT technique is typically implemented using likelihood-based scores of the individual tracks. The existing tracks are compared to the number of observations in the current scan. The data association is also performed using Bayes rule and Gaussian density (likelihood) function. Based on likelihood calculations, the existing tracks are updated and the tracks which are not updated in successive scans are pruned. The remaining observations are used to initialize new tracks. However, these methods are computationally heavy and have no explicit provision for track initiation.

The computational complexity of MHT is depending on the hypotheses generation process which includes track merging, pruning and number of hypotheses (Cong and Hong 1999). The computational complexity
of the hypothesis generation process of MHT in a given cluster is given by equation (3.1)

\[ O((U + \min(W, U_{\text{hy}})N_{\text{hy}})) \]  

(3.1)

where,  

\( U \) - the number of existing branches  

\( W \) - number of measurements  

\( N_{\text{hy}} \) - Possible hypotheses at the current scan.

### 3.2 JOINT PROBABILISTIC DATA ASSOCIATION

PDA scheme is an important step in tracking, in which posterior association probabilities are computed for all current candidate measurements in a validation gate and used to form a weighted sum of innovations for updating the target’s state (Bar-Shalom and Tse 1975). The basic PDA algorithm assumes that each target is isolated from all other targets: false measurements in a validation gate are modeled as independent clutter points drawn from a Poisson distribution with spatial density \( C \), and detection of a target is an independent event at each sample time with probability of detection \( P_D \).

At each time step, the sensor provides a set of candidate measurements to be associated with the targets. This is done by forming a “validation gate” around the predicted measurement from each target and retaining only those detections that lie within the gate.

The dynamic state and measurement equation are given in equations (3.2) and (3.3) (Dongcai et al 2006).

\[ x_k = F x_{k-1} + G p_{k-1} \]  

(3.2)

\[ Z_k = H x_k + v_k \]  

(3.3)
where, \( x_k \) - State vector at time k (scan k)

\( F \) - State transition matrix

\( G \) - Control input matrix

\( p \) - Process noise

\( Q \) - Process noise covariance

\[ Q_k = E[p_k p_k'] \]

\( p_k' \) is transpose of \( p_k \)

\( Z_k \) - Actual measurement at time k.

\( H \) - Measurement matrix

\( v_k \) - Measurement noise

\( R \) - Measurement noise covariance

\[ R_k = E[v_k v_k'] \]

\( v_k' \) is transpose of \( v_k \)

For example there are \( m \) candidate measurements at time \( k \) and denoted as,

\[ Z_k = \{ z_1, z_2, ..., z_m \} \cup Z_{k-1} \] (3.4)

\( Z_k \) denotes the set of all data vectors.
x is a state vector and the elements are position, velocity and acceleration in x, y and z directions. In the PDA filtering approach, the initial state is assumed to be Gaussian with mean \( \hat{x} \) and covariance Q. The conditional estimated state \( \hat{x} \) is obtained using the combined (weighted) innovation (Fortmann et al 1983). The combined (weighted) innovation becomes

\[
\tilde{z} = \sum_{j=1}^{m} \beta_j \tilde{z}_j
\]  
(3.5)

where, \( \tilde{z}_j \) is the innovation corresponding to measurement \( j \) and

\[
\beta_j = P\{X_j|Z_k\}, \quad j = 0, 1, \ldots, m
\]

is the posterior probability of \( j^{th} \) measurement (or no measurement, for \( j = 0 \)).

The predicted measurement for target \( t \) is denoted by \( \tilde{z}_t \) and the \( j^{th} \) measurement is denoted by \( z_j \), the innovation corresponding to measurement \( j \) becomes

\[
\tilde{z}_j = z_j - \tilde{z}_t
\]
(3.6)

and the combined (weighted) innovation for target \( t \) (3.5) becomes

\[
\tilde{z}_t = \sum_{j=1}^{m} \beta^t_j \tilde{z}_j
\]
(3.7)

where \( \beta^t_j \) is the posterior probability that measurement \( j \) originated from target \( t \) and \( \beta^t_0 \) is the probability that none of the measurements originated
from target \( t \) (i.e. it was not detected). The key to the JPDA algorithm is evaluation of the conditional probabilities of the joint events.

The JPDA and PDA approaches utilize the same estimation equations; the difference is in the way the association probabilities are computed. The PDA algorithm computes \( \beta_j^t \), \( j = 0, 1, \ldots, m \), separately for each \( t \), under the assumption that all measurements not associated with target \( t \) are false (i.e., Poisson distributed clutter), the JPDA algorithm on the other hand computes \( \beta_j^t \) jointly across the set of \( N \) targets and clutter.

The PDA algorithm can be extended to correct interfering targets by computing the posterior probabilities jointly across clusters of targets; only the clutter is modeled as Poisson. JPDA is a target-oriented approach, in the sense that a set of established targets is used to form gates in the measurement space and compute posterior probabilities, where each measurement is considered in turn and hypothesized to have come from some established track, a new target, or clutter.

Using Bayes’ rule, the probability of a joint event conditioned on all measurements up to the present time is

\[
P(X|Z_k) = P \left\{ X \bigg| z_1, \ldots, z_m, m, Z_{k-1} \right\} \\
= p(z_1, \ldots, z_m|X, m, Z_{k-1})P(X|m, Z_{k-1})/c \tag{3.8}
\]

The normalization constant \( c = p(z_1, z_2, \ldots, z_m|m, Z_{k-1}) \) is the joint prior density of the measurements, conditioned only on \( m \) (and the past data); it is obtained by summing the numerators over all \( X \).
3.3 MULTIPLE HYPOTHESES TRACKING ALGORITHM

In the MHT approach, a number of hypotheses will be generated and evaluated. When a new data set N is received, for a single observation N+2 tracks are generated. Consider the formulation of alternative hypotheses for the combination of measurement data into tracks. Refer \( y_j(k) \) the jth observation received on time k (scan k). Begin with an observation \( y(1) \) that is not used to update the existing track. The two alternate hypotheses for this observation are false target and new target with probabilities proportional to false and true target density, respectively. Then consider an observation \( y(2) \) received on the next scan, five hypotheses now represent the various combinations of alternatives for the two observations (Blackman 1986, Jitendra et al 2004). These alternative hypothesis and the probability densities are given by equations (3.9) to (3.13).

\( H_{y1}: \) Both observations are false targets

\[
p(H_{y1}) = \beta_{FT} \beta_{FT}
\]

Where, \( \beta_{FT} \) - False target density

\( H_{y2}: \) First observation is a false target; second is a true target denoted new target two T2

\[
p(H_{y2}) = \beta_{FT} \beta_{NT}
\]

\( H_{y3}: \) First observation is a true target T1; second is a false target and the first target T1 is not detected.
\[ p(H_{y3}) = \beta_{NT} (1 - P_D) e^{\frac{-I}{D_E}} \beta_{FT} \quad (3.11) \]

**H\(_{y4}\):** First observation is a true target \( T_1 \) and the observation on the second scan is associated with the first target track \( T_1 \).

\[ p(H_{y4}) = \beta_{NT} (P_D) e^{\frac{-l}{D_E}} g_{11} \quad (3.12) \]

**H\(_{y5}\):** First observation is a true target \( T \); second is another new target \( T_2 \) and the first target is not detected on the second scan.

\[ p(H_{y5}) = \beta_{NT} e^{\frac{-l}{D_E}} (1 - P_D) \beta_{NT} \quad (3.13) \]

where, \( D_E \) is the expected track length

The quantity \( g_{11} \) included in \( p(H_{y4}) \) is the Gaussian likelihood function associated with assigning the first observation received on the second scan to track \( T_1 \). It is defined by the equation

\[ g_{11} = \frac{e^{-d_{11}^2/2}}{(2\pi)^{m/2} \sqrt{|S_1|}} \quad (3.14) \]

where \( d_{11} \) is the normalized distance associated with the observation to track assignment. \( |S_1| \) is the determinant of the residual covariance matrix for track \( T_1 \).
The factors $P_D$ and $(1 - P_D)$ are included in the expressions for the hypothesis probabilities, to represent the probabilities of the target being detected and not detected, respectively on the second scan. Finally the factor $e^{-1/D_E}$ in the last three probabilities represents the probability associated with extending the track for an additional scan.

The Figure 3.1 shows the hypothesis tree representation of the hypothesis (n-BMHT algorithm) formation technique outlined by equations (3.9) to (3.13).

![Figure 3.1 Hypothesis Tree representation of n-BMHT](image)

From the Figure 3.1, it is observed that each node of the tree represents an alternative hypothesis; further branches are added to each node as a new data point is considered. Also, the number of hypotheses (branches) can grow very rapidly unless limiting techniques are applied.
After the hypotheses are constructed the score of \( i^{th} \) hypothesis is calculated for n-BMHT using equation (3.15),

\[
L_{i(nBMHT)} = L_{i(k-1)} + L_{i(k)}
\]  

(3.15)

where,

\[
L_{i}(k) = \ln(1 - P_{D}) \quad \text{no track update}
\]

\[
L_{i}(k) = \ln \left( \frac{P_{D}}{\beta_{FT}(2\pi)^{m/2} \sqrt{|S|}} \right) \cdot \frac{d_{ij}^2}{2} \quad \text{track updated}
\]

Therefore this research proposes a new scheme namely ZBMHT algorithm for Radar surveillance. The performance of the proposed algorithm is compared with the existing algorithms of JPDA and n–Backscan Multiple Hypothesis Tracking (n-BMHT) discussed in Sections 3.2 and 3.3. The main difference in ZBMHT compared with n-BMHT is the instant modification of each observation after the formation of the hypothesis tree.

### 3.4 DEVELOPMENT OF ZBMHT ALGORITHM

For the successful tracking of a moving target it is essential to extract the maximum useful information about the target state from the available observations. The block diagram of the proposed ZBMHT algorithm is shown in Figure 3.2
Figure 3.2 Proposed Zero-backscan MHT algorithm

The proposed block diagram consists of gating, filtering and prediction, hypothesis construction, evaluation and management blocks. The input data is collected from a signal processing unit (perform co-ordination conversion on RADAR data) and applied.

3.4.1 Gating

Gating is the first part of the correlation algorithm for eliminating unlikely ‘observation to track’ pairings. A gate is formed around the predicted target position. If any single observation falls within the gate, it will
be correlated with the track and if more than one return is within the track
gate, further correlation logic is required.

In either of the special cases, where the probability of detection is
unity or there are no extraneous returns, the gate should be infinite for optimal
correlation performance. But the above specified cases are practically
impossible and the purpose of gating is minimizing the observation-to-track
pairings.

A gate \((G)\) is defined such that the correlation is allowed if the
following relationship is satisfied by the normalized distance function \(d^2\)
given by equation (3.16).

\[
d^2 \leq G
\]  (3.16)

A maximum likelihood gate \((G_0)\) can be defined by equation (3.17)
such that as observation falling within that gate is more likely from the track it
may from an extraneous source.

\[
G_0 = 2\ln\left[\frac{P_D}{(1-P_D)\beta_{FT}(2\pi)^{m/2}\sqrt{S}}\right] \tag{3.17}
\]

Hypotheses consist of tracks whose state estimates are updated,
usually with EKF techniques, as new data are received. In prediction part, the
future position is estimated to track the target. The state estimates and
covariance are used to form gates so that when the next data set is received
and to avoid the generation of very unlikely hypotheses. Additional
hypothesis evaluation and management techniques (such as pruning,
combining and clustering) are also required to limit the number of hypotheses.
As the knowledge of information on the target’s kinematics (position, velocity
and acceleration) and sensor characteristics are generally known, it is possible to design a mathematical model of the target.

### 3.4.2 Extended Kalman Filter

Many estimation problems are non-linear, but the noise model is assumed as Gaussian. The KF is optimal in minimizing mean square error for linear and Gaussian systems. The observation model in the tracking system is nonlinear as the observations are given in spherical coordinates. For nonlinear problems there is no general analytic expression for the posterior PDF and only approximated estimation algorithms are studied. The EKF is the most popular approach for recursive nonlinear estimation. The main idea is to linearize the system and apply the KF. Equations (3.18) through (3.32) define the EKF. Let us consider that there are N numbers of targets and the target set is denoted by \( T_N = \{1, 2, \ldots, N\} \). The dynamic state and measurement equation are given in equations (3.2) and (3.3).

The states of the target consists of position, velocity and acceleration of the targets given as \((x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z})\) in \(x, y\) and \(z\) coordinates respectively. But the measurement of the target consists of range, azimuth, elevation and range rate as \((r_k, \theta_k, \phi_k, \dot{r}_k)\). The EKF is used as an estimation filter to avoid the nonlinearity problem due to different coordinate system. The block diagram of EKF is shown in Figure 3.3.

The radar sensor measurements \(Z_k\) are four coordinate system gives range, bearing (or azimuth), elevation and range rate. The EKF states are in Cartesian coordinates consists of position, velocity and acceleration of the targets. The sensor measurements are used to form the state by the spherical to Cartesian coordinates and the output at time \(k\) is noted as \(x_k\). The state is used to form measurement matrix. To linearise the measurement
equation the state \( x_k \) is used and \( H \) measurement matrix is constructed. The predicted state given as \( x_{k/\text{k-1}} \) which predict the state of \( k \) at \( k-1 \) scan. The measurement from the predicted state is evaluated by \( H \hat{X}_{k/\text{k-1}} \) and the resultant measurement is denoted as \( Z_{k/\text{k-1}} \). Now the difference between the predicted measurement and the observed measurement at time \( k \) is calculated and the residual and the corresponding covariance are fed as input to compute the Kalman gain \( K_k \). The Kalman gain is used to smooth the predicted state as \( x_{k/\text{k}} \). From the smoothened state the future state of target is predicted as \( x_{k+1/\text{k}} \). For each cycle the difference angle is calculated and the cycle continues.

\[ x_k \]

\[ x_{k/\text{k-1}} \]

\[ Z_{k/\text{k-1}} \]

\[ Z_{k} \]

\[ x_{k/\text{k}} \]

\[ x_{k+1/\text{k}} \]

\[ K_k \]

\[ \hat{X}_{k/\text{k-1}} \]

\[ \hat{X}_{k} \]

\[ \text{Sensor inputs} \]

\[ \text{Spherical to Cartesian coordinates} \]

\[ \text{Measurement Matrix} \]

\[ \text{KF gain calculation} \]

\[ \text{State updating} \]

\[ \text{Cartesian to Spherical coordinates} \]

\[ \text{Current angle} \]

\[ \text{Difference angle} \]

\[ \text{Predicted angle} \]

\[ \text{Prediction of state} \]

\[ \text{Cartesian to Spherical coordinates} \]

\[ \text{Figure 3.3 Block diagram of Extended Kalman Filter} \]
The state transition matrix and control input matrix is given by equations (3.18) and (3.19) respectively.

\[
F = \begin{bmatrix}
1 & 0 & 0 & T & 0 & 0 & T^2/2 & 0 & 0 \\
0 & 1 & 0 & 0 & T & 0 & 0 & T^2/2 & 0 \\
0 & 0 & 1 & 0 & 0 & T & 0 & 0 & T^2/2 \\
0 & 0 & 0 & 1 & 0 & 0 & T & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & T & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(3.18)

\[
G = \begin{bmatrix}
T^2/2 \\
T \\
1 \\
T^2/2 \\
T \\
1 \\
T^2/2 \\
T \\
1
\end{bmatrix}
\]

(3.19)

where, \( T \) is the sampling time interval. The main idea of the EKF is first-order linearization of the estimation problem and the posterior PDF is assumed to be Gaussian. In tracking system, EKF linearises the measurement matrix where the observations are in spherical coordinates and states are in Cartesian coordinates. First order linearization is done by measuring Jacobian integrals of variables in measurement equations (Greg Welch and Gary
Bishop, 2006). The measurement matrices of the tracking system having observations in spherical coordinates are shown in equation (3.20)

\[
H = \begin{bmatrix}
    r_k & \theta_k & \phi_k & \dot{r}_k
\end{bmatrix}^T
\]  
(3.20)

The measurements represented by Cartesian coordinates are given by equation (3.21)

\[
x = r \cos \phi \sin \theta
\]
\[
y = r \sin \phi \sin \theta
\]
\[
z = r \cos \theta
\]  
(3.21)

\[
H = \begin{bmatrix}
    \partial r/\partial x & \partial r/\partial y & \partial r/\partial z & \partial r/\partial \phi & \partial r/\partial \theta & \partial r/\partial \phi & \partial r/\partial \theta
\end{bmatrix}
\]

The range \( r_k \), azimuth angle \( \theta_k \), elevation angle \( \phi_k \) and range rate \( \dot{r}_k \) of spherical coordinates at time \( k \) are given by equations (3.22), (3.23), (3.24) and (3.25) respectively.

\[
r_k = \sqrt{x_k^2 + y_k^2 + z_k^2}
\]  
(3.22)

Azimuth angle of spherical coordinates

\[
\theta_k = \tan^{-1}\left( \frac{z_k}{\sqrt{x_k^2 + y_k^2}} \right)
\]  
(3.23)

Elevation angle is given by,
\[ \phi_k = \tan^{-1}\left( \frac{y_k}{x_k} \right) \]  

(3.24)

Range rate of spherical coordinates

\[ \dot{r}_k = \left( x_k \dot{x}_k + y_k \dot{y}_k + z_k \dot{z}_k \right) \sqrt{x_k^2 + y_k^2 + z_k^2} \]  

(3.25)

The measurement noise is shown in equation (3.26)

\[ v_k = \begin{bmatrix} \tilde{r}_k \ 	ilde{\phi}_k \ 	ilde{\theta}_k \end{bmatrix}^T \]  

(3.26)

\( \tilde{r}_k, \tilde{\phi}_k, \tilde{\theta}_k \) and \( \tilde{\phi}_k \) are the corresponding measurement noises range, azimuth angle, elevation angle and range rate are assumed to be zero mean Gaussian noises with known variances \( \sigma_r^2 \), \( \sigma_{\phi}^2 \), \( \sigma_{\theta}^2 \) and \( \sigma_r^2 \) respectively. Jacobian integral is used to linearise the measurement equation given in equation (3.2).

The state dynamics of EKF uses target acceleration model for random target motion. The acceleration model and measurement matrix are used to continue the KF operations (Singer 1970).

The predicted state vector is given by equation (3.27).

\[ x_{k/k-1} = Fx_{k-1/k-1} + Gp_{k-1} \]  

(3.27)

The predicted error covariance and measurement residual error are given by equations (3.28) and (3.29) respectively.

\[ P_{k/k-1} = FP_{k-1/k-1}F^T + Q_{k-1} \]  

(3.28)
The Innovation (or residual) covariance and optimal Kalman gain are provided by equations (3.30) and (3.31) respectively:

\[ S_k = H P_{k|k-1} H^T + R_k \]  

\[ K_k = P_{k|k-1} H^T S_k^{-1} \]  

Filtered state estimate is calculated using equation (3.32):

\[ x_{k|k} = x_{k|k-1} + K_k e_k \]  

The equation (3.33) provides estimated filtered error covariance:

\[ P_{k|k} = (I - K_k H) P_{k|k-1} \]  

The equations (3.16) to (3.33) are used as recursive to perform smoothening of observations and predicting the position of targets.

ZBMHT is evolved for reducing the response time of data association compared to existing correlation logics. The n-backscan has exponential increase in response time when number of targets increase. Another association logic JPDA forming track for all the possible combination of targets shows higher response time. The ZBMHT not required any memory to store the previous data like n-BMHT. The overview of ZBMHT is same as n-BMHT which consists of gating, filtering and prediction and hypotheses construction, evaluation and management.
3.4.3 Hypothesis Tree Construction

The ZBMHT uses the structure branch algorithm for the construction of the hypotheses tree. After an observation is received, the hypotheses are constructed for all the possible existing tracks. From the evaluated values of hypotheses, the maximum valued hypothesis is taken as the root for next session. The construction for ZBMHT is shown in Figure 3.4.

![Figure 3.4 Hypotheses tree construction for ZBMHT](image)

From Figure 3.4, if track 2 has maximum likelihood value, observation 1 is correlated with track 2. During observation 2, the new track has maximum likelihood value, and then observation 2 is associated with the new track. If a false track has maximum likelihood value then observation 3 is pruned.
3.4.4 Hypothesis Evaluation

Evaluation of hypotheses is done by calculating the likelihood of the observations with all the existing and updated tracks. Prediction of tracks is done by gating to narrow down the field of view for targets. The observations are received and used to update all the predicted tracks, and also to calculate the residual and its covariance of the hypotheses. After the hypotheses are constructed the score of \(i^{th}\) hypothesis is calculated by Bayesian track scoring, which is given by Equation (3.34).

\[
L_i(\text{ZMHT}) = \ln \left( \frac{P_D}{\beta_{FT}(2\pi)^{m/2}\sqrt{|S|}} \right) \cdot \frac{d_{ij}^2}{2}
\]  

(3.34)

where
- \(L_i\) - log likelihood score of \(i^{th}\) hypotheses
- \(P_D\) - estimated probability of detection
- \(\beta_{FT}\) - false target density
- \(m\) - Measurement dimensionality.
- \(S\) - Residual covariance matrix
- \(d_{ij}^2\) - normalized statistical distance function

3.4.5 Bayesian Track Scoring

A relatively simple sequential technique for track scoring can be developed by applying the Bayes’ rule. This technique goes beyond the method SPRT (Sequential Probability Ratio Test) because prior probabilities and track update residual information are readily included. Also, the same method will be applied for track deletion. Using Bayes’ rule, the probability of true track correlate with measurement data D is (Blackman 1986), given by equation (3.35)
where, \( p\left(\frac{D_T}{T}\right) \) is the probability of receiving the measurement data \( D \) given that a true target is present.

\[ p_0(T) \] is the prior probability if a true target is appearing within the scan volume. \( p(D) \) is the probability of receiving the data \( D \), which is given by equation (3.36)

\[
p(D) = p\left(\frac{D_T}{T}\right)p_0(T) + p\left(\frac{D_F}{F}\right)p_0(F)
\]  \hspace{1cm} (3.36)

where \( p\left(\frac{D_T}{T}\right) \) and \( p_0(F) \) are defined as false target in the same manner that \( p\left(\frac{D_T}{T}\right) \) and \( p_0(T) \) are defined for true targets. Noting that \( P_0(T)=1-P_0(F) \), combining (3.35) and (3.36), and dividing numerator and denominator by \( p\left(\frac{D_F}{F}\right) \) gives

\[
p\left(\frac{T}{D}\right) = \frac{L(D)p_0(T)}{L(D)p_0(T)+1-P_0(T)}
\]  \hspace{1cm} (3.37)

where \( L(D) \) is the likelihood ratio for the data is defined as

\[
L(D) = \frac{p\left(\frac{D_T}{T}\right)}{p\left(\frac{D_F}{F}\right)}
\]  \hspace{1cm} (3.38)

Equation (3.37) can be modified in a convenient form for recursive computation, as \( L_k \) is to be the likelihood ratio for the data received at \( k \)th scan to be correlated with the true track. Likelihood \( L_k \) associated with data set \( D_k \)
must be determined by first defining \( p\left(\frac{T}{D_k}\right) \) and \( p\left(\frac{D_k}{F}\right) \). Probability of true target given data through \( k^{th}\) scan is given by equation (3.39)

\[
P\left(\frac{T}{D_k}\right) = \frac{L_k P\left(\frac{T}{D_{k-1}}\right)}{L_k P\left(\frac{T}{D_{k-1}}\right) + 1 - P\left(\frac{T}{D_{k-1}}\right)}
\]  

(3.39)

Dropping subscript \( k \), for a true target \( p\left(\frac{D_k}{T}\right) \) is taken to be the product of the probability of detection and the Gaussian likelihood function defined as

\[
g_{ij} = \frac{\exp\left(-d_{ij}^2 / 2\right)}{(2\pi)^{m/2} \sqrt{|S|}}
\]  

(3.40)

It is likelihood function for scan \( k \), associated with the assignment of observation \( j \) to track \( i \) by assuming Gaussian distribution for the residual. Similarly, \( p\left(\frac{D_k}{F}\right) \) is taken to be the probability of a false target return times the likelihood function \( (1/V_G) \) associated with the assumed uniform distribution of false returns within the volume \( V_G \) of the gated region. Thus, likelihood function is,

\[
g = \frac{P_D e^{-d^2 / 2} V_G}{P_F(2\pi)^{m/2} \sqrt{|S|}}
\]  

(3.41)

Where \( d^2 \) is the normalized distance functions and \( |S| \) is determinant of the residual covariance matrix. Equation (3.40) can be simplified by noting
that $P_F = \beta_{FT} V_G$, where $\beta_{FT}$ is the false target density. Thus equation (3.41) becomes

$$g = \frac{P_F e^{-d^2/2}}{\beta_{FT} (2\pi)^{m/2} \sqrt{|S|}}$$  \hfill (3.42)

The log likelihood score of $i^{th}$ hypotheses is given by equation (3.34).

The new target probability can be defined in terms of new target density and false target density as

$$p_0(T) = \frac{\beta_{NT}}{\beta_{NT} + \beta_{FT}}$$  \hfill (3.43)

For missed detection, likelihood score of hypotheses for scan $k$ is given by equation (3.44)

$$L_k = \frac{1 - P_D}{1 - P_F}$$  \hfill (3.44)

Equation (3.34) to (3.44) provides a convenient sequential scoring for ZBMHT scheme that can be adjusted to the environment.

### 3.4.6 Hypothesis Management

From the score of the hypotheses, the most likely observation to track are picked out and correlated. The remaining hypotheses are eliminated for that particular observation which gets associated with the related track. The false target score and the new target score are always calculated for every observation. If any one of that score is high, then a new potential track is constructed for the new target and the observation eliminated for the false target case. The tracks get updated by the observation and are not used for
other observations until next scan starts. The not updated track is pruned and the numbers of tracks to predict also get reduced.

3.5 RESULTS AND DISCUSSION

The performance of proposed ZBMHT has been validated using MATLAB simulation and its performance is compared with existing JPDA and n BMHT schemes. For multiple target simulation study, five numbers of linear target scenario and four number of crossing target scenario is considered. For both cases the values $f_{VT} = 0.5$, $f_{NT} = 0.5$, $P_D = 0.8$ and $m = 2$ are assumed.

The surveillance region consists of a 10 km by 10 km square region (the origin is located at the southwest corner) in which the targets are placed.

3.5.1 Performance Analysis of Existing Correlation Logics

The performances of existing correlation logics are simulated for single, five linear and four crossing target scenarios. The single target tracking scenario using PDA is shown in Figure 3.5.

![Figure 3.5 Single target tracking scenario using PDA](image_url)
A single target is initially located near the southwest corner of the region moving towards northeast direction. The following assumptions are made to obtain the recursive state estimator (Bar-Shalom et al 2005), i.e. only one target of interest, track has been initialized and past information about the target is summarized approximately by assuming the PDF of the current state. Figure 3.5 shows tracking results of single target using PDA. From the results, it is observed that PDA works well for a single target environment. The Table 3.1 shows the position error between the original and estimated values using PDA correlation logic.

<table>
<thead>
<tr>
<th>Scan No.</th>
<th>Original position (m)</th>
<th>Estimated (Filtered) position (m)</th>
<th>Position error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.9774</td>
<td>12.05</td>
<td>0.0726</td>
</tr>
<tr>
<td>2</td>
<td>14.3513</td>
<td>14.0274</td>
<td>0.3238</td>
</tr>
<tr>
<td>3</td>
<td>16.3746</td>
<td>16.4013</td>
<td>0.0267</td>
</tr>
<tr>
<td>4</td>
<td>18.4891</td>
<td>18.4246</td>
<td>0.0645</td>
</tr>
<tr>
<td>5</td>
<td>20.5304</td>
<td>20.5391</td>
<td>0.0087</td>
</tr>
<tr>
<td>6</td>
<td>22.6641</td>
<td>22.5804</td>
<td>0.0837</td>
</tr>
<tr>
<td>7</td>
<td>26.6488</td>
<td>26.7086</td>
<td>0.0599</td>
</tr>
<tr>
<td>8</td>
<td>24.6586</td>
<td>24.7141</td>
<td>0.0554</td>
</tr>
<tr>
<td>9</td>
<td>28.6668</td>
<td>28.6987</td>
<td>0.0319</td>
</tr>
<tr>
<td>10</td>
<td>30.6917</td>
<td>30.7168</td>
<td>0.0252</td>
</tr>
</tbody>
</table>

From the Table 3.1, it is observed that, the error will be reduced in successive scans for single target.
Next, consider a two linear target scenario using PDA as shown in Figure 3.6. Initially both the targets were located near the southwest corner of the region and moving towards northeast direction.

![Graph showing tracking of targets](image)

**Figure 3.6 Multiple targets tracking scenario using PDA**

Figure 3.6 reveals that, when more than one target falls within the gate, the association performed by PDA is very poor. When two closely spaced target scenarios is shown in Figure 3.6, it is observed that, during the second scan the PDA wrongly associates the data with the other target also, it exhibits poor performance for multiple target environments. Therefore PDA is not suitable for multiple target environments.

Another association method is JPDA, which probabilistically associate the sensor information with the tracks. Tracking of five linear target scenarios using JPDA is shown in Figure 3.7.
Figure 3.7 Tracking of multiple linear target scenario using JPDA

Figure 3.8 Tracking of multiple crossing target scenario using JPDA
From Figure 3.7, it is observed that for multiple target environments the estimation error is high and also error is not converging. Tracking of multiple crossing target scenarios using JPDA is shown in Figure 3.8.

From the Figure 3.8, it is observed that JPDA exhibits poor performance for crossing targets and it associates the data with other tracks during the initial scans.

The tracking performance for multiple linear targets using n-BMHT is shown in Figure 3.9. It is found that the performance of n-BMHT for multiple linear targets is good.

![n-BMHT results for multiple linear target scenario](image)

**Figure 3.9** n-BMHT results for multiple linear target scenario

The results of n-BMHT multiple crossing target scenario is shown in Figure 3.10.
From Figure 3.10, it can be found that the performance of n-BMHT for crossing targets is poor. During the second scan, the algorithm wrongly associates the data with other track.

### 3.5.2 Performance analysis of proposed ZBMHT

The input observations are considered for inclusion in existing tracks and for initiation of new tracks. First, a gate, based upon the maximum acceptable measurement plus tracking prediction error magnitudes, is placed around the predicted track.

The simulation of ZBMHT is done by the tracking of a single target among multiple targets. Assume that the observations are received at regular intervals T=1 sec. Only those observations that are within the track gate are considered for further processes. The EKF is initialized by using the first three measurements which determine the position, velocity and acceleration estimates. The position measured by radar is converted into Cartesian
coordinate system, because of the transformation of measurements from spherical to Cartesian coordinates, the measurement noise covariance components in Cartesian coordinates becomes correlated and are updated for every iteration. The measurement noise is assumed to be zero mean Gaussian with variance of $[10 \ 0.001 \ 0.001 \ 10^2]$ in range, azimuth, elevation and range rate respectively. The score of each hypothesis is calculated using equation (3.27). From the score the most likely observation-to-track pairing gets correlated.

The filter is initialized in each of the orthogonal axis and the $F$ matrix given by equation (3.5) is used to update the states.

![Figure 3.11](image.png)

**Figure 3.11 Multiple linear target scenario for ZBMHT**

The scenario with multiple linear targets for ZBMHT is shown in Figure 3.11. From the Figure 3.11, it is found that the performance of ZBMHT for multiple linear targets is good. The trajectories of the two
crossing target scenario with ten samples was developed to test the performance of the ZBMHT is shown in Figure 3.12. Target 1 is initially located near the southwest corner of the region moving northeast and target 2 is initially located in the northwest corner of the region moving southeast.

In this simulation, the crossing of two targets occurs at the fourth sample (scan number). The proposed ZBMHT accurately associates the data during crossing scenarios.

![Figure 3.12 ZBMHT results for multiple (two) crossing target scenario with 10 samples](image)

From Figure 3.12, it is observed that ZBMHT works well in multiple crossing target environments. The same two target crossing scenario is considered with 500 samples as shown in Figure 3.13.
Figure 3.13  ZBMHT results for multiple (two) crossing target scenario with 500 samples

A multiple target and multiple crossing target scenario is shown in Figure 3.14. From the Figure 3.14, it is observed that the proposed ZBMHT accurately associates the data with the respective track.

Figure 3.14 ZBMHT results for multiple crossing target scenario
To show the effectiveness of the ZBMHT algorithm, consider a scenario, in which three targets are moving and only one is being tracked. To track a particular target in multiple target scenario, the initial state of the target is given as input to the ZBMHT algorithm. The algorithm sets the gate around the target and also tracks other targets which fall in this gate.

Figure 3.15 shows the simulation of ZBMHT under three target scenario. The targets 1, 2, 3 are assumed to move from location 750 450 300, 300 250 150, 250 350 145 in Cartesian coordinates respectively. For three targets five hypotheses are generated and applied as input to the EKF for further estimation. To track target 1, the initial state of that target is given as input. The EKF shown in Figure 3.2 predict the location of the target for next instant. Based on the prediction the likelihood estimation is performed for each hypothesis and tracks are updated. The not updated tracks are pruned and the remaining observations are used to form new track.

![Figure 3.15 Simulation of multiple (three) target tracking with ZBMHT](image)
Figure 3.16 Simulation of multiple (four) target tracking with ZBMHT

Figure 3.16 shows the simulation of ZBMHT under four target scenario. One more target move from the location 175, 225, 150 is considered.

In the multiple target environments, the proposed ZBMHT accurately tracks the desired target with less error. The position error between the measurement and filtered trajectories of spherical coordinate is shown in Figure 3.17.
Figure 3.17 Error between the original and estimated value of spherical coordinates

From the Figure 3.17, it is observed that the error is converging and it is close to zero at 2 msec.

3.5.3 Complexity Analysis

The computation time is the time taken for executing tracking algorithms in a system with Pentium processor (2.66GHz) and 1GB RAM.

The execution time of the ZBMHT is compared with JPDA and n-BMHT and the results are shown in Figure 3.18.
From Figure 3.18, it inferred that the execution time of proposed ZBMHT is linear also less compared to other correlation logics. For less number of samples the time required to execute the correlation logic n-BMHT is lesser than the proposed scheme, but when number of sample increases, execution time of n-BMHT increases exponentially. The execution time of JPDA correlation logic is high compared to ZBMHT and n-BMHT.

The execution time of the proposed ZBMHT is compared with n-BMHT and JPDA and the results are shown in Figure 3.19.
From the execution time comparison, it is found that as the number of targets increases, the execution time complexity also increases exponentially in n-BMHT and JPDA where as it is almost constant for ZBMHT.

Consider the single (linear) target simulation scenario. The probability values considered for single linear target scenario are, $\beta_{\text{Tr}}=0.5$; $\beta_{\text{NT}}=0.5$; where $\beta_{\text{NT}}$ is probability occurrence of new target, $P_D$ is the probability of detecting a given target at a given range. Figure 3.20 shows the position estimation error for a single linear target obtained for different correlation logics.
Estimation (position) Error for single linear target when \(PD = 0.5\)

From Figure 3.20, it is observed that the estimation error of the proposed ZBMHT is lesser than n-BMHT and JPDA. Figure 3.21 shows the estimation error for single linear target with probability of detection 0.8.
From the Figure 3.21, it is observed that when $P_D$ is high the error in estimation is less and tracking accuracy is improved.

### 3.6 SUMMARY

Multitarget tracking deals with data association and tracking. In this research, a new correlation algorithm named as ZBMHT is proposed to reduce the time taken for data association and to increase the accuracy. The results are compared with the existing data association algorithms namely JPDA and n-BMHT. The JPDA uses batch processing approach that processes all observations together. The n-BMHT data association logic maintains previous n backscan hypothesis for data association which requires more time for processing. But the proposed ZBMHT constructs hypotheses based on the present observations and associate the data with existing tracks. Therefore the time taken for ZBMHT is lesser than the other data association logics especially when the number of targets is more than two. Moreover ZBMHT does not require any memory to store the previous scan data like n-BMHT. The computational complexity of ZBMHT is lesser than n-BMHT and JPDA due to less number of combinations of observations in multiple target environments. In summary this chapter provided a solution to the data association problem using ZBMHT for radar sensor network, with enhanced tracking accuracy and reduced computational time and memory utilization.