In this research work we have mainly design new wavelet filter for the localization and feature extraction. We have designed new orthogonal wavelet filter and named it as DR wavelet. All the experiments in this research work are done with this new DR wavelet and with existing HAAR wavelet. We have localized OD and macula with DR wavelet and HAAR wavelet, also we have extracted microaneurysm (MA) and exudates (EX) and retinal blood vessels with DR wavelet and HAAR wavelet. Here in this chapter we are going to discuss about wavelets.

3.1 Introduction to wavelets:

Wavelets are used to denoise two dimensional signals, such as images. Wavelets are the wave like structures, which begins at the zero then increase and again decrease back to zero. Wavelet is mathematical function which is used for the extraction of various data like signals and images. In this research work we are working on the retinal fundus images. Wavelet does fully analysis of data. It helps for the decomposition and compression of an image or signal. Decomposition and compression of image or signal is done without loss of data using wavelet. Wavelet has time domain and frequency domain representation [1]. There are total 15 types of wavelet 1) Haar 2) Doubechies 3) Symlets 4) BiorSplines 5) ReverseBior 6) Meyer 7)DMeyer 8) Gaussian 9) Mexican_hat 10) Morlet 11) Complex Gaussian 12) Shannon 13) Frequency B-Spline 14) Complex Morlet 15) Coiflets

In our research work we have used HAAR wavelet and Designed new wavelet filter (DR).

We can define the wavelet family in various ways like in terms of scaling function, scaling filter and wavelet function.

3.1.1 Scaling filter:

Orthogonal wavelet is defined by the scaling filter. It is low pass finite impulse response (FIR) filter having length of 2N and there sum is 1. These High pass filters of orthogonal wavelet calculated as the quadrature mirror filter of the low pass. Reconstruction filters are time reverse
of decomposition filters. Examples of scaling filter wavelets are Daubechies and Symlet wavelet [1].

3.1.2 Scaling function:

Wavelets are defined by the wavelet function $\psi(t)$ and $\varphi(t)$ in the time domain. Where $\psi(t)$ is mother wavelet and $\varphi(t)$ is scaling function. Example of scaling function is Meyer wavelets, Haar wavelet [1].

3.1.3 Wavelet function:

Wavelet function is a time domain representation function. It represent as $\psi(t)$. Example of wavelet function is Mexican hat wavelets [1].

3.2 HAAR wavelet:

For this work we have used HAAR wavelet filter.

Figure 3.1: HAAR wavelet [2]

HAAR wavelet is simplest wavelet. HAAR wavelet is a square shape function. Invention of HAAR wavelet filter is done by the Hungarian mathematician Alfred Haar. HAAR wavelet is orthogonal wavelet. HAAR wavelet is doing pair up the input values, it store the difference and then passing the sum. This process repeats again and again until get the differences and final sum [2].
The HAAR wavelet's mother wavelet function $\psi(t)$ can be described as

$$\psi(t) = \begin{cases} 
1 & 0 \leq t < 1/2 \\
-1 & 1/2 \leq t < 1 \\
0 & \text{otherwise}
\end{cases} \quad (3.1)$$

The HAAR wavelets scaling function $\phi(t)$ is given as

$$\phi(t) = \begin{cases} 
1 & 0 < t < 1 \\
0 & \text{otherwise}
\end{cases} \quad (3.2)$$

### 3.3 Discrete Wavelet Transform:

DWT is any wavelet transform for which wavelets are discretely sampled. It is very useful in the fields like science, computer science, engineering and mathematics etc. DWT produces non-redundant image representation. DWT provides better spatial and spectral localization of image formation. DWT is used mainly for the image de-noising. The DWT is the signal decomposition in a set of independent, spatially oriented frequency channels. S is the signal which is passed through two complementary filters and emerges in to two signals that are approximation and detail. This process is called decomposition. These output signals can again assembled back to the original signals without any loss of data is called reconstruction. The mathematical manipulation in which decomposition and reconstruction is done called discrete wavelet transform (DWT). In this research work we have done two dimensional DWT. Two dimensional DWT decompose approximation coefficient at level $j$. It has 4 components. The approximation is done at level $j+1$ and horizontal, vertical and diagonal details [3][4]. Following diagram shows the decomposition steps for images:

![Two-Dimensional DWT Diagram](image)

**Figure 3.2: Decomposition steps of Two Dimensional Discrete Wavelet Transform [4]**
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3.4 Orthogonal Wavelet:

Orthogonal wavelet is one type of wavelet. Its associated wavelet transform is orthogonal.

Orthogonal scaling function defined as

$$\phi(x) = \phi \sum_{k=0}^{N-1} a_k \phi(2x - k) \quad \ldots \ldots \quad (3.3)$$

This scaling function is known as the refinement equation, it is fractal function equation. Where $(a_0, a_1, \ldots, a_{(N-1)})$ are real numbers. This is called scaling sequence or scaling mask.

The wavelet is also obtained by the similar linear combination

$$\varphi(x) = \sum_{k=0}^{M-1} b_k \phi(2x - k) \quad \ldots \ldots \quad (3.4)$$

Where $(b_0, b_1, \ldots, b_{(k-1)})$ are the real numbers. This is known as wavelet sequence or wavelet mask.

In orthogonal wavelet scaling sequence should be orthogonal to any shifts of it by an even number of coefficients. The equation is given as follows

$$\sum_{n \in \mathbb{Z}} a_n a_n + 2m = 2\delta_{m,0} \quad \ldots \ldots \quad (3.5)$$
Where in this case there is same number \( M=N \) of coefficients in the scaling sequence as in the wavelet sequence. The wavelet sequence is given as

\[
b_n = (-1)^n a_{N-1-n} \ldots \quad (3.6)
\]

In the some cases opposite sign can be chose [5].

**Summary:**

In this chapter we have discussed about wavelet. Wavelet is mainly used for the de-noising purpose. That means it is useful to remove the noise from the image. Wavelet is better than other present filters. In this chapter we have discussed about wavelet. In that we have explained about scaling function, wavelet function, scaling filter. We have explained about Discrete Wavelet Transform which is mainly used for the decomposition purpose. We have used orthogonal type of wavelet for this work. We have discussed about the HAAR wavelet.
References:


