Chapter 4. An Inventory Model with price dependent demand under stochastic inflation rate

4.1 Introduction

The determinants of inflation rate in developing economies are enormously vital for policy makers as when the causes of inflation are suitably specified the appropriate policy change can be easily diagnosed and effectively implemented. Inflation in a large economy can be influenced by both internal and external factors. Internal factors include, among others, government deficits, debt financing, monetary policy, institutional economics (shirking, opportunism, economic freedom, risk, etc.) and structural regime changes (revolution, political regime changes, policy constraints, etc.). External factors include terms of trade and foreign interest rate as well as the attitude of the rest of the world (sanctions, risk generating activities, wars, etc.) toward the country. The objective of this chapter is to develop an inventory policy under inflation rate, which takes into account all of these factors.

It is clear that the cost of agricultural commodities, metals, and oil have risen substantially over the last four decade, but many of those increases have not yet reached the consumer. There are two effects that inflation has on an industry. The first is what the higher cost of goods means to their expenses. The other is whether companies in the industry can pass along these costs to consumers – either individuals or other businesses. Rising prices are hurting many industries including retailers. Shoppers may reject higher prices and cut back spending to preserve their household budgets. Retailers will face margin compression if that happens. Earnings in the industry will be damage, and in some cases this will be enough to cause layoffs. The price of the goods and services, such as air transport, food retailing, apparel, appliances, consumer products, enterprise shipping, mining, petrochemicals, consumer electronics etc changes time to time due to random inflation. In this chapter we study the effects of random inflation on the inventory policy.

Misra (1979) developed a discounted-cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Sarker and Pan (1994) surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Some studies were also conducted with variable demand, see, for example, Uthayakumar and Geetha (2009), Maity (2010), Vrat and Padmanabhan (1990), Dutta and Pal (1991), Hariga (1995), Hariga and Ben-Daya (1996) and Chung (2003). In all these studies, it is assumed that the inflation rate remains constant over time and is known. However, in reality, the inflation rate is affected by many factors, which may be economical, political, social or cultural, like world inflation rate, unemployment rate, natural calamities, artificial scarcities, etc. As such, it is more justified to assume the rate of inflation to be random. A study in this direction has been carried out by Mirzazadeh (2009), who considered an inventory model for deteriorating items allowing shortages when demand rate remains constant but inflation conditions are uncertain.

4.2 An Inventory Model for price dependent demand and delay in payments under normally distributed Inflation rate

In this sub-section, we consider a periodic review inventory model for deteriorating items over a finite planning horizon allowing shortages when demand is price dependent. The inflation rate is assumed to be a random and the inventory manager is allowed a permissible delay in payment.

4.2.1 Notations

$[0, H]$ = planning horizon;

A = ordering cost per order;

C = cost price per unit;

I_c = holding cost per unit per unit time;

s = shortage cost per unit per unit time;

P = selling price per unit;

θ = deterioration rate;

I_e = interest earned per annum;

I_r = interest charged per annum;

M = permissible delay in payments;

T = complete inventory cycle length;

T_1 = time taken for stock on hand to become zero in a cycle;

$D(t)$ = demand rate at time t

r = inflation rate;

$f(r)$ = probability density function (pdf) of r;

$M_r(t)$ = moment generating function of r;

d = discount rate representing the value of money;

k = discount rate net of inflation, i.e. $k = d-r$.

4.2.2 Assumptions

1. The length of the planning period is $H = n\ T$, where n is an integer denoting the number of replenishments to be made in the period (0, H) and T is the length of a reorder interval.

2. The demand rate is dependent on the selling price. It remains constant within an inventory cycle and for the i^{th} cycle it is given by $a - bpe^{r(i-1)T}$, $1 \leq i \leq n$, where $a, b \geq 0, a \succ\succ b$. This form of the demand rate arises from the fact that as the price of an item increases, its demand is likely to decrease, and, due to price inflation, the price p increases to $pe^{r(i-1)T}$ in the (i – 1)th reorder interval.

3. The inflation rate r is random but remains unchanged during an inventory cycle.

4. Shortages are allowed and backlogged during the first (n – 1) inventory cycles, but no shortage is allowed during the last cycle.

5. Replenishment is instantaneous on ordering.

4.2.3 Mathematical Model

We consider inventory of a single item which has a constant deterioration rate. A periodic review policy is used, and the inventory manager is allowed a fixed permissible delay in payment. The decision variables of the model are n, the number of replenishments, and Q, the order quantity at each reorder point, which are determined so as to maximize the total expected profit over (0, H). During the i-th reorder interval, denoted by $[\ (i-1)T, iT\ ]$, the inventory level becomes zero at the time point

($(i-1)T+T_1$) and thereafter shortage is allowed to accumulate till the end of the interval before they are backordered. However, in the last, that is the *n*-th reorder interval, no backlogging is allowed. The following figure illustrates the model:

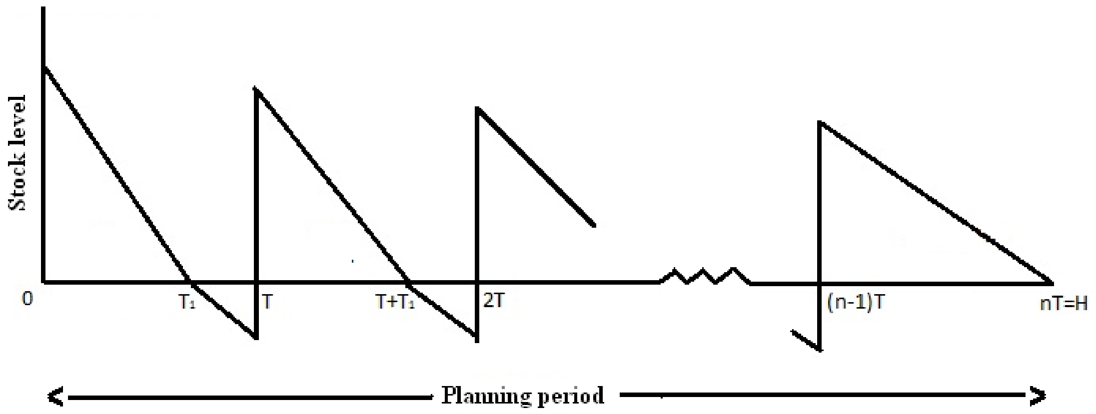

Figure 4.2.1: The inventory policy over a finite planning horizon

Let $I_i(t)$ denote the stock on hand at time (i-1)T +t, 1≤ i≤n, 0 ≤ t ≤ T.

To obtain the differential equations defining transitions in the stock level, we note that in the *s*-th reorder interval, 1 ≤ i ≤ n -1, depletion from stock occurs due to demand and deterioration of items in the interval [0, T_1], and thereafter only due to demand as the stock size becomes zero at T_1. On the other hand, for the *n*-th interval, depletion of stock occurs due to both demand and deterioration in the interval $[(n-1)T, nT]$. Hence, the differential equations are as follows:

(i) for 1 ≤ i ≤ n -1,

$$\frac{dI_i(t)}{dt}+\theta I_i(t)=-a+bpe^{r(i-1)T} \qquad 0\le t\le T_1$$

$$\frac{dI_i(t)}{dt}=-a+bpe^{r(i-1)T} \qquad T_1\le t\le T\ ,$$

(ii) for $i=n$,

$$\frac{dI_n(t)}{dt}+\theta I_n(t)=-a+bpe^{r(n-1)T}, \qquad 0\le t\le T.$$

The boundary condition is $I_i(T_1)=0$ for $1\le i\le n-1$, and $I_n(T)=0$.

Defining $D_i=-a+bpe^{r(i-1)T}$, the above equations give

(i) for $1\le i\le n-1$,

$$I_i(t)=\begin{cases}\dfrac{D_i}{\theta}\left(1-e^{\theta(T_1-t)}\right), & 0\le t\le T_1\\ D_i(T_1-t), & T_1\le t\le T\end{cases}$$

(ii) for $i = n$,

$$I_n(t)=\frac{D_n}{\theta}\left(1-e^{\theta(T-t)}\right),\ 0\le t\le T.$$

Then, the maximum stock height for the i[th] cycle is $\dfrac{D_i}{\theta}\left(1-e^{\theta T_1}\right)$, $1\le i\le n-1$, and that for the last cycle is $\dfrac{D_n}{\theta}\left(1-e^{\theta T}\right)$. Clearly these are functions of T_1 and T, respectively.

4.2.4 Profit Function

We find the optimal values of T and T_1 that maximize the value of the expected total profit over (0, H) at $t=H$.

In order to get the different components in the profit expression, we note that in the last cycle holding cost is incurred but the shortage cost is zero.

The different components of the present value of the expected profit are as follows:

Expected ordering cost: $C_0(T_1,T)=A\sum_{i=1}^{n}E(e^{-k(i-1)T})=A\sum_{i=1}^{n}e^{-(i-1)dT}M_r((i-1)T)$;

Expected holding cost: $C_h(T_1,T)=$

$$=\sum_{i=1}^{n-1}EIce^{-(i-1)kT}\int_0^{T_1}I_i(t)dt+EIce^{-(n-1)kT}\int_0^{T}I_n(t)dt$$

$$= \frac{Ic}{\theta^2}\left(e^{\theta T_1} - \theta T_1 - 1\right)\sum_{i=1}^{n-1} e^{-(i-1)dT}[aM_r((i-1)T) - bpM_r(2(i-1)T)]$$

$$+ \frac{Ic}{\theta^2}(e^{\theta T} - \theta T - 1)[e^{-(n-1)dT}(aM((n-1)T) - bpM(2(n-1)T))]$$

Expected deterioration cost: $C_d(T_1, T) = \frac{\theta}{I} C_h(T_1, T)$

Expected shortage cost: $C_{sh}(T_1, T) = \sum_{i=1}^{n-1} Ese^{-k(i-1)T} \int_{T_1}^{T} \left(a - bpe^{r(i-1)T}\right) t dt$

$$= s\frac{(T^2 - T_1^2)}{2}\sum_{i=1}^{n-1} e^{-(i-1)dT}[aM_r((i-1)T) - bpM_r(2(i-1)T))]$$

Expected purchase cost: $C_p(T_1, T)$

$$= \sum_{i=1}^{n-1} Ece^{-k(i-1)T}[I_i(0) + \int_{T_1}^{T}\left(a - bpe^{r(i-2)T}\right)dt]$$

$$+ Ece^{-k(n-1)T}[I_n(0) + \int_{T_1}^{T}\left(a - bpe^{r(n-2)T}\right)dt]$$

$$= \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\sum_{i=1}^{n-1} e^{-(i-1)dT}[aM_r((i-1)T) - bpM_r(2(i-1)T)]$$

$$+ \frac{c}{\theta}\left(e^{\theta T} - 1\right)e^{-(n-1)dT}[aM_r((n-1)T) - bpM_r(2(n-1)T)]$$

$$+ c\left(T - T_1\right)\sum_{i=1}^{n} e^{-(i-1)dT}[aM_r((i-1)T) - bpM_r((2i-3)T)]$$

Expected selling price: $C_S(T_1, T)$

$$= \sum_{i=1}^{n-1} E[pe^{-k(i-1)T}(\int_{0}^{T_1}\left(a - bpe^{r(i-1)T}\right)dt - I_{i-1}(T))]$$

$$+ E[pe^{-k(n-1)T}(\int_{0}^{T}\left(a - bpe^{r(n-1)T}\right)dt - I_{n-1}(T))]$$

$$= pT_1\sum_{i=1}^{n-1} e^{-(i-1)dT}[aM_r((i-1)T)-bpM_r(2(i-1)T)]$$
$$+ pTe^{-(n-1)dT}[aM_r((n-1)T)-bpM_r(2(n-1)T)]$$
$$+ p(T-T_1)\sum_{i=1}^{n} e^{-(i-1)dT}[aM_r((i-1)T)\text{-}bpM_r((2i-3)T))].$$

In order to calculate the total interest earned or paid, we note that when $M \le T_1$, the inventory manager earns interest on his sales revenue in the interval [$(i-1)T,(i-1)T+M]$ and pays interest on the unsold stock in the interval $[(i-1)T+M, iT]$ for $1 \le i \le n-1$. On the other hand, when $M \ge T_1$ he does not have to pay any interest. For *i* =*n*, he earns interest in $[(i-1)T,(i-1)T+M]$ and pays interest in the interval $[(n-1)T+M, nT]$.

Hence, we have the following:

Case 1: $M \le T_1$

Total interest earned:

$$C_e^{(1)}(T_1,T) = \sum_{i=1}^{n-1} Ee^{-k(i-1)T}\, pI_e \int_0^{T_1} \left(a-bpe^{r(i-1)T}\right)(T_1-t)dt + \ldots$$
$$+ Ee^{-(n-1)kT}\, pI_e \int_0^{M} \left(a-bpe^{(n-1)rT}\right)(M-t)dt$$

$$= pI_e \frac{T_1^{\ 2}}{2}[\sum_{i=1}^{n-1} e^{-(i-1)dT}\{aM_r((i-1)T)-bpM_r(2(i-1)T)\}]$$
$$+ pI_e \frac{M^2}{2}[e^{-(n-1)dT}\{aM_r((n-1)T)-bpM_r(2(n-1)T)\}]$$

Total interest payable

$$C_r^{(1)}(T_1,T) = \sum_{i=1}^{n-1} Ee^{-k(i-1)T} cI_r \int_M^{T_1} I_i(t)dt + Ee^{-k(n-1)T} cI_r \int_M^{T} I_{n-1}(t)dt$$

$$= \frac{cI_r}{\theta}\left(e^{\theta(T_1-M)} - \theta(T_1-M)-1\right)\sum_{i=1}^{n-1} e^{-(i-1)dT}[aM_r((i-1)T)-bpM_r(2(i-1)T)]$$
$$+\frac{cI_r}{\theta}\left(e^{\theta(T-M)} - \theta(T-M)-1\right)e^{-(n-1)dT}[aM_r((n-1)T)-bpe^{-(n-1)dT}M_r(2(n-1)T)]$$

Hence, for $M \le T_1$, the total profit in the interval $[0, H]$ is

$$C_1^M(T_1,T) = C_S(T_1,T) + C_e^{(1)}(T_1,T) - C_0(T_1,T) - C_h(T_1,T) - C_d(T_1,T) - C_{sh}(T_1,T) - C_r^{(1)}(T_1,T)$$

$$= [pT_1 + pI_e \frac{T_1^2}{2} - \left(e^{\theta T_1} - \theta T_1 - 1\right)\left(\frac{Ic}{\theta^2} + \frac{c}{\theta}\right) - \frac{cI_r}{\theta}\left(e^{\theta(T_1 - M)} - \theta(T_1 - M) - 1\right) \tag{4.2.1}$$

$$- s_1 \frac{T^2 - T_1^2}{2} - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)]K_1(T) + (p-c)(T - T_1)K_2(T) - AK_3(T)$$

$$[pT + pI_e \frac{M^2}{2} - (\frac{Ic}{\theta^2} + \frac{c}{\theta})(e^{\theta T} - \theta T - 1) - \frac{cI_r}{\theta}\left(e^{\theta(T-M)} - \theta(T-M) - 1\right)$$

$$- \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)][e^{-(n-1)dT}(aM((n-1)T) - bpM(2(n-1)T))],$$

where

$$K_1(T) = \sum_{i=1}^{n-1} e^{-(i-1)dT}[aM_r((i-1)T) - bpM_r(2(i-1)T)]$$

$$K_2(T) = \sum_{i=1}^{n} e^{-(i-1)dT}[aM_r((i-1)T) - bpM_r((2i-3)T)]$$

$$K_3(T) = \sum_{i=1}^{n} e^{-(i-1)dT} M_r((i-1)T).$$

Case 2: $M \ge T_1$

Interest earned

$$C_e^{(2)}(T_1,T) = \sum_{i=1}^{n-1} E\left[cI_e e^{-k(i-1)T}\left(\int_0^{T_1}\left(a - bpe^{r(i-1)T}\right)(T_1 - t)dt + \int_0^{T_1}\left(a - bpe^{r(i-1)T}\right)(M - T_1)dt\right)\right]$$

$$+ E\left[cI_e e^{-k(n-1)T}\left(\int_0^{M}\left(a - bpe^{r(n-1)T}\right)(M - t)dt\right)\right]$$

$$= cI_eT_1(M-\frac{T_1}{2})\sum_{i=1}^{n-1} e^{-(i-1)dT}[aM_r((i-1)T)-bpM_r(2(i-1)T)]$$
$$+ pI_e\frac{M^2}{2}[e^{-(n-1)dT}[aM_r((n-1)T)-bpM_r(2(n-1)T)]$$

Interest payable $C_r^{(2)}(T_1,T) = 0$

Total profit in the interval $[0, H]$ is, therefore,

$$C_2^M(T_1,T) = C_S(T_1,T) + C_e^{(2)}(T_1,T) - C_0(T_1,T) - C_h(T_1,T) - C_d(T_1,T)$$
$$- C_{sh}(T_1,T) - C_r^{(2)}(T_1,T).$$

$$\begin{aligned} C_2^M(T_1,T) = {} & [pT_1 + pI_eT_1\left(M-\frac{T_1}{2}\right) - \left(e^{\theta T_1}-\theta T_1-1\right)\left(\frac{Ic}{\theta^2}+\frac{c}{\theta}\right) - s\frac{T^2-T_1^2}{2} \\ & -\frac{c}{\theta}\left(e^{\theta T_1}-1\right)]K_1(T) + (T-T_1)(p-c)K_2(T) - AK_3(T) \\ & +[pT + pI_e\frac{M^2}{2} - (\frac{Ic}{\theta^2}+\frac{c}{\theta})(e^{\theta T}-\theta T-1) - \\ & -\frac{c}{\theta}\left(e^{\theta T_1}-1\right)][e^{-(n-1)dT}(aM((n-1)T)-bpM(2(n-1)T))] \end{aligned} \qquad (4.2.2)$$

4.2.5 Algorithm and some Result

To obtain the optimal value of $\boldsymbol{T} = (T_1, T)$, we first obtain $\boldsymbol{T}_{(1)}$ and $\boldsymbol{T}_{(2)}$ that maximize $C_1^M(T_1,T)$ and $C_2^M(T_1,T)$ respectively. If $C_1^M(\boldsymbol{T}_{(1)}) \le C_2^M(\boldsymbol{T}_{(2)})$, $\boldsymbol{T} = \boldsymbol{T}_{(2)}$ is optimal, else $\boldsymbol{T} = \boldsymbol{T}_{(1)}$ is optimal.

For given T, the optimal value of T_1 minimizing $C_1^M(T_1,T)$ is a solution to

$$\frac{\partial C_1^M(\boldsymbol{\theta})}{\partial T_1} = 0,$$

which gives

$$\left(p + pI_eT_1 - \left(e^{\theta T_1} - 1\right)\left(\frac{Ic}{\theta} + c\right) - cI_r\left(e^{\theta(T_1-M)} - 1\right) + s(T - T_1) - ce^{\theta T_1}\right)K_1(T) + (p-c)K_2(T)$$

$$e^{\theta T_1}\left(\frac{Ic}{\theta} + c + cI_re^{-\theta M} + cT_1\right) + (s - pI_e)T_1 = \frac{Ic}{\theta} + c + cI_r + p + sT + (p-c)\frac{K_2(T)}{K_1(T)} \quad (4.2.3)$$

i.e., $f(T_1 = g(T),$

where

$$f(T_1) = e^{\theta T_1}\left(\frac{Ic}{\theta} + c + cI_re^{-\theta M} + cT_1\right) + (s - pI_e)T_1$$

$f(T_1)$ is an increasing function of T_1 with $0 < f(0) < g(T)$. In order to get a solution to (4.2.3), we must have $f(T) \geq g(T)$, else $T_1 = T$ is the optimal value.

Theorem 4.2.1: For given T, a sufficient condition for $C_1^M(T_1,T)$ to be concave in T_1 is that $pI_e \leq c\theta + s.$

Proof: The proof follows from the fact that

$$\frac{\partial^2 C_1^M(T_1,T)}{\partial T_1^2} = \left(pI_e - e^{\theta T_1}(Ic + c\theta) - \theta cI_re^{\theta(T_1-M)} - c\theta e^{\theta T_1} - s\right)K_1(T) \leq 0,$$

since, $pI_e \leq c\theta + s.$

In case 2, the optimal value of T_1 satisfies $\frac{\partial C_2^M(T_1,T)}{\partial T_1} = 0,$ which gives

$$2cI_rT_1 + e^{\theta T_1}\left(\frac{Ic}{\theta} + c\right) + cI_re^{\theta(T_1-M)} + sT_1 - ce^{\theta T_1} = p + cI_r(M+1) + \frac{Ic}{\theta} + c + sT + (p-c)\frac{K_2(T)}{K_1(T)} \quad (4.2.4)$$

And,

$$\frac{\partial^2 C_2^M(T_1,T)}{\partial T_1^2} = -\left(2cI_r + e^{\theta T_1}(Ic + c\theta) + \theta cI_re^{\theta(T_1-M)} + c\theta e^{\theta T_1} - s\right)K_1(T) < 0,$$

which shows that $C_2^M(T_1,T)$ is concave in T_1.

To obtain the optimal value of T in each case, we first find optimum n that maximizes $C_i^M(T_1, T), i = 1,2,$ where $T = H/n$, and hence find optimum T.

4.2.6 Numerical Examples

In this sub-section we give two examples based on our model, and carry out a sensitivity analysis of the model to changes in its parameters. The optimal values of the decision variables have been obtained using the software MATLAB. We assume that $r \sim N(\mu, \sigma^2)$, which gives $M_r(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$.

Example 4.2.1: Consider the following parameter values: a = 2000;b = 0.1; c=Rs. 20, p= Rs. 25, θ= 0.3, A=Rs. 250, s_1 = Rs. 1.5, I = 0.05, I_e =.12; I_r =.15, d = 0.14, M = 0.1year; H = 5 years, $r \sim N(\mu, \sigma^2)$, where $\mu = 0.08, \sigma^2 = 0.04$.

For $M \le T_1$, optimal n =33, optimal T= 0.15152 year, optimal T_1= 0.15145 year, profit = Rs. 36525.70.

For $M \ge T_1$, optimal n =21, optimal T= 0.15152 year, optimal T_1= 0.09996 year, profit = Rs. 33718.40.

Hence, optimal n = 33, optimal T= 0.15152 year, optimal T_1= 0.15145 year, profit = Rs. 36525.70.

Example 4.2.2: Suppose the values of s_1 and I in example 1 are changed to Re. 0.5 and 0.04, respectively.

For $M \le T_1$, optimal n =12, optimal T= 0. 41667 year, optimal T_1= 0.100055 year, profit = Rs. 40248.90.

For $M \ge T_1$, optimal n =14, optimal T= 0. 35714 year, optimal T_1= 0.039835 year, profit = Rs. 41008.10.

Hence, optimal n =14, optimal T= 0. 35714 year, optimal T_1= 0.039835 year, profit = Rs. 41008.10.

4.2.7 Sensitivity Analysis

To study the sensitivity of the model to changes in its parameters, we start with the following set-up: a=2000; b=.1; c=200; p=230; θ = 0.1; A=250; s=4; I=.05; I_e=.12; I_r=.15; μ=.08; d=.14; σ^2= 0.04; M= 0.1; H=5.

The following tables show how the decision variables, namely n, T and T_1, change with change in the values of the model parameters, and also give the corresponding percentage change in the expected profit as compared to that for the above set of parameter values.

Table 4.2.1: Changes in the values of the decision variables with change in I, and the corresponding % change in the expected profit from that when I = 0.05

I	No of cycles(n)	T	T_1	Optimal profit	% change in profit
0.005	14	0.3571	0.1184	241299	33.65
0.01	18	0.2778	0.1099	219981	21.84
0.03	29	0.1724	0.1025	187121	3.64
0.05	32	0.1563	0.1018	180549	0
0.07	36	0.1389	0.1011	173013	-4.17
0.09	40	0.125	0.1007	166190	-7.95
0.11	41	0.122	0.1006	163775	-9.29
0.13	41	0.122	0.1006	162737	-9.87

Table 4.2.2: Changes in the values of the decision variables with change in *s*, and the corresponding % change in the expected profit from that when *s* = 4

s_1	No of cycles(*n*)	*T*	T_1	Optimal profit	% change in profit
0.5	19	0.2632	0.1086	216699	20.02
2	25	0.2	0.104	196346	8.75
4	32	0.1562	0.1018	180549	0
6	35	0.1428	0.1013	175080	-3.03
8	36	0.1389	0.1011	173170	-4.09
10	40	0.125	0.1007	167437	-7.26
20	47	0.1064	0.1002	159284	-11.78

Table 4.2.3: Changes in the values of the decision variables with change in I_e, and the corresponding % change in the expected profit from that when I_e = 0.12

I_e	No of cycles(n)	*T*	T_1	Optimal profit	% change in profit
0.01	34	0.1471	0.1014	171662	-4.92
0.04	34	0.1471	0.1014	173150	-4.1
0.08	33	0.1515	0.1016	176864	-2.04
0.12	32	0.1563	0.1018	180549	0
0.16	31	0.1613	0.102	184227	2.04
0.18	31	0.1613	0.102	184686	2.29
0.2	31	0.1613	0.102	186062	3.05

Table 4.2.4: Changes in the values of the decision variables with change in I_r, and the corresponding % change in the expected profit from that when I_r = 0.15

I_r	No of cycles(n)	T	T_1	Optimal profit	% change in profit
0.01	13	0.3846	0.1219	246590	36.57
0.05	18	0.2778	0.1099	218604	21.08
0.1	25	0.2	0.104	195488	8.27
0.15	32	0.1563	0.1018	180549	0
0.2	37	0.1351	0.101	172431	-4.5
0.3	47	0.1064	0.1002	159680	-11.56
0.4	48	0.1042	0.1001	158573	-12.17

Table 4.2.5: Changes in the values of the decision variables with change in p, and the corresponding % change in the expected profit from that when p = 230.

p	No of cycles(n)	T	T_1	Optimal profit	% change in profit
210	21	0.2381	0.1066	81069.3	-55.1
220	26	0.1923	0.1036	130527	-27.71
230	32	0.1563	0.1018	180549	0
240	40	0.125	0.1007	229837	27.3
250	42	0.119	0.1005	288655	59.88
260	45	0.1111	0.1003	346202	91.75
270	46	0.1087	0.1002	406266	125

Table 4.2.6: Changes in the values of the decision variables with change in μ, and the corresponding % change in the expected profit from that when $\mu = 0.08$

μ	No of cycles(n)	T	T_1	Optimal profit	% change in profit
0.02	38	0.1316	0.1009	134433	-25.54
0.04	36	0.1389	0.1011	147350	-18.36
0.06	34	0.1471	0.1014	162579	-9.95
0.08	32	0.1563	0.1018	180549	0
0.12	29	0.1724	0.1025	195037	8.02
0.16	26	0.1923	0.1036	212165	17.51
0.2	24	0.2083	0.1046	229307	27.01

Table 4.2.7: Changes in the values of the decision variables with change in the credit period M, and the corresponding % change in the expected profit from that when $M = 0.1$

M	No. of cycles (n)	T	T_1	optimal profit	% change in profit
0.04	52	0.0962	0.0008	198657	10.03
0.07	37	0.1351	0.0718	186778	3.45
0.1	32	0.1562	0.1018	180549	0
0.3	16	0.3125	0.0188	260743	44.42
0.5	8	0.625	0.0625	341895	89.36
0.7	7	0.7143	0.1001	362449	100.75

Table 4.2.8: Changes in the values of the decision variables with change in θ, and the corresponding % change in the expected profit from that when $\theta = 0.1$

θ	No. of cycles (n)	T	T_1	optimal profit	% change in profit
0.01	19	0.2632	0.1086	226885	25.66
0.04	22	0.2273	0.1058	214858	19
0.07	26	0.1923	0.1036	202599	12.21
0.1	32	0.1562	0.1018	180549	0
0.2	41	0.122	0.1006	166890	-7.56
0.4	45	0.1111	0.1003	139254	-22.87
0.6	46	0.1087	0.1002	114205	-36.75

Table 4.2.2 and Table 4.2.3 show that the model is fairly sensitive to changes in the shortage cost and the interest earned, while from the other tables, namely Table 4.2.1, Table 4.2.4, Table 4.2.5, Table 4.2.6, Table 4.2.7, and Table 4.2.8, we may conclude that the model is highly sensitive to changes in the corresponding parameters.

4.2.8 Discussion

The existing literature on inventory policies with inflation generally assumes a constant inflation rate over time. This assumption is, however, violated in many real life situations since the time value of money may be subjected to change owing to change in environmental factors. In this section we have therefore assumed a stochastic inflation rate to capture its change over time. We further consider a permissible delay in payment, which has not been investigated much for policies under inflation. Our model has usefulness when dealing with items like electronic components, fashion items and domestic goods, whose demands are affected by the selling prices, and customers for such goods are often allowed a grace period to repay their dues. The model may be extended to consider other types of demands, such as stock dependent demand, time dependent demand, etc. and also delay period dependent on the order quantity.

4.3 An Inventory Model for Deteriorating Items with price Dependent Demand and Delay in Payments under Uncertain Inflationary Condition

The demand rate can be related to the inflation rates. The theoretical contribution of this sub-section is to consider stochastic inflationary conditions with variable probability density functions (pdfs) over the time horizon, and the demand rate is dependent to the inflation rates (any arbitrary pdfs can be used).

Misra (1979) developed a discounted-cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system.

The present study differs from previous researches on the following situations. In a few models, the inflation rates have been assumed to be stochastic with known pdfs over the time horizon like Mirzazadeh (2008). In the real world, for long-term investment and forecasting, the fluctuations in the inflation rate cannot be ignored. Accordingly, in this model the internal and external inflation rates are assumed to be stochastic. The pdfs of inflation rates may be variable over the time horizon. For example, at the beginning of a time horizon with a length of 20 years, the internal and external inflation rates may be stochastic with the following pdfs

$$r_1 \sim Uniform(a_1, b_1) \text{ and } r_2 \sim Normal\left(\mu_2, \sigma_2^2\right)$$

After ten years, the pdfs of the inflation rates may change to the following

$$r_3 \sim Normal\left(\mu_3, \sigma_3^2\right) \text{ and } r_4 \sim Normal\left(\mu_4, \sigma_4^2\right)$$

These features have been incorporated in the inventory model considered in this sub-section.

4.3.1 Notations

The notations used are the same as in section 4.2.1, with the following additional notations

r_m = the internal (for m=1) and external (for m=2) inflation rates in the first inflationary period and the internal (for m=3) and external (for m=4) inflation rates in the second inflationary period;

$f(r_m)$ = probability density function of $r_m, m=1,2,3,4$;

$M_r(t)$ = moment generating function of r;

k_m = discount rate net of inflation, i.e. $k_m = d-r_m$.

$[0,H_1]$= First inflationary period.

$[H_1,H]$= Second inflationary period.

4.3.2 Assumptions

The assumptions used are the same as in section 4.2.2, with the following additional assumptions

1. The internal and external inflation rates are stochastic with known distribution functions. The pdfs of the inflation rates may change over the time horizon. In this model, even if once change in the pdfs of inflation rates has been considered, the model can be extended to more than one change in the pdfs similar to the explained method.
2. The demand rate is a linear function of the internal and external inflation rates.
3. The demand rate is dependent on the selling price. It remains constant within an inventory cycle and for the i^{th} cycle it is given by $a-bpe^{r(i-1)T}$, $1 \le i \le n$, where $a,b \ge 0, a \succ\succ b$. and r is the average of internal and external inflation rate. This form of the demand rate arises from the fact that as the price of an item increases, its demand is likely to decrease, and, due to price inflation, the price p increases to $pe^{r(i-1)T}$ in the (i – 1)th reorder interval.

According to these assumptions, we can divide the time horizon into two different inflationary periods. The pdfs of inflation rates in these periods may be different to each other. The following notations are used in the model:

The time horizon, [0, H], is divided into n cycles, each of length T so that $T=H/n$. Initial and final inventory levels are both zero. Each inventory cycle except the last cycle can be divided into two parts. The replenishment starts at time zero and the inventory level is Q_0(Initial stock). Then, the level of inventory is decreasing by consumption and deterioration rates. At the moment of T_1, the inventory level leads to zero and shortages occur.

During the time interval [T_1, T], we do not have any deterioration, and therefore the shortages level increases by the demand rate until the moment T. In this moment, the second cycle starts and this behavior continues till the end of the first inflationary period. The pdfs of inflation rates change during the time horizon and the second inflationary period starts at time $H_1=n_1T$. Similar to the first inflationary period, each inventory cycle can be divided into two parts. In the last cycle, shortages are not allowed and the inventory cycle can be divided in one part.

4.3.3 Mathematical Model

We consider inventory of a single item which has a constant deterioration rate. A periodic review policy is used, and the inventory manager is allowed a fixed permissible delay in payment. The decision variables of the model are n, the number of replenishments, and Q, the order quantity at each reorder point, which are determined so as to maximize the total expected profit over [0, H]. During the i-th reorder interval, denoted by [$(i-1)T, iT$] , the inventory level becomes zero at the time point ($(i-1)T+T_1$) and thereafter shortage is allowed to accumulate till the end of the interval before they are backordered. However, in the last, that is the n-th reorder interval, no backlogging is allowed. The following figure illustrates the model:

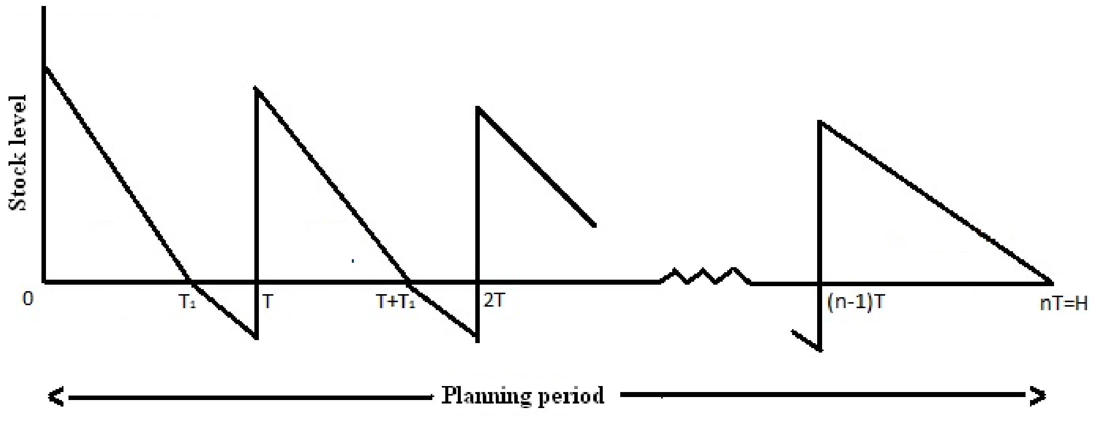

Figure 4.3.1: The inventory policy over a finite planning horizon

Let $I_i(t)$ denote the stock on hand at time $(i\text{-}1)T + t$, $1 \le i \le n$, $0 \le t \le T$.

To obtain the differential equations defining transitions in the stock level, we note that in the i-th reorder interval, $1 \le i \le n-1$, depletion from stock occurs due to demand and deterioration of items in the interval $[0, T_1]$, and thereafter only due to demand as the stock size becomes zero at T_1. On the other hand, for the n-th interval, depletion of stock occurs due to both demand and deterioration in the interval $[(n-1)T, nT]$. Hence, the differential equations are as follows:

(i) for $1 \le i \le n_1$,

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = -a + bpe^{r(i-1)T} \qquad 0 \le t \le T_1$$

$$\frac{dI_i(t)}{dt} = -a + bpe^{r(i-1)T} \qquad T_1 \le t \le T,$$

Where $r = w_1 r_1 + (1 - w_1) r_2$, $0 \le w_1 \le 1$.

(ii) for $n_1 + 1 \le i \le n-1$,

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = -a + bpe^{r(i-1)T} \qquad 0 \le t \le T_1$$

$$\frac{dI_i(t)}{dt} = -a + bpe^{r(i-1)T} \qquad T_1 \le t \le T,$$

Where $r = w_2 r_3 + (1 - w_2) r_4$, $0 \le w_2 \le 1$.

(iii) for $i = n$,

$$\frac{dI_n(t)}{dt} + \theta I_n(t) = -a + bpe^{r(n-1)T}, \qquad 0 \le t \le T.$$

Where $r = w_2 r_3 + (1 - w_2) r_4$, $0 \le w_2 \le 1$.

The boundary condition is $I_i(T_1) = 0$ for $1 \le i \le n-1$, and $I_n(T) = 0$.

Defining $D_i = -a + bpe^{r(i-1)T}$, the above equations give

(i) for $1 \le i \le n_1$,

$$I_i(t) = \begin{cases} \dfrac{D_i}{\theta}\left(1 - e^{\theta(T_1 - t)}\right), & 0 \le t \le T_1 \\ D_i(T_1 - t), & T_1 \le t \le T \end{cases}$$

where $D_i = -a + bpe^{r(i-1)T}$

(ii) for $n_1 < i \le n-1$,

$$I_{s-1}(t) = \begin{cases} \dfrac{D_i}{\theta}\left(1 - e^{\theta(T_1 - t)}\right), & 0 \le t \le T_1 \\ D_i(T_1 - t), & T_1 \le t \le T \end{cases}$$

(ii) for $i = n$,

$$I_n(t) = \frac{D_n}{\theta}\left(1 - e^{\theta(T - t)}\right), \; 0 \le t \le T.$$

Then, the maximum stock height for the i^{th} cycle is $\frac{D_i}{\theta}\left(1 - e^{\theta T_1}\right)$, $1 \le i \le n-1$, and that for the last cycle is $\frac{D_n}{\theta}\left(1 - e^{\theta T}\right)$. Clearly these are functions of T_1 and T, respectively.

4.3.4 Profit Function

We find the optimal values of T and T_1 that maximize the value of the expected total profit over $[0, H]$ at $t = H$. In order to get the different components in the profit expression, we note that in the last cycle holding cost is incurred but the shortage cost is zero.

The different components of the present value of the expected profit are as follows:

Expected ordering cost: $C_0(T_1, T) = A\sum_{i=1}^{n} E(e^{-k(i-1)T})$

$$= A\sum_{i=1}^{n_1} e^{-(i-1)dT} M_{r_1}\left(w_1(i-1)T\right) M_{r_2}\left((1 - w_1)(i-1)T\right)$$

$$+ A\sum_{i=n_1+1}^{n} e^{-(i-1)dT} M_{r_3}\left(w_2(-1)T\right) M_{r_4}\left((1 - w_2)(i-1)T\right)$$

Expected holding cost : $C_h(T_1, T)$

$$= \sum_{i=1}^{n-1} E\left[\left(Ice^{-(i-1)kT}\int_0^{T_1} I_i(t)dt\right)\right] + E\left[\left(Ice^{-(n-1)kT}\int_0^{T} I_n(t)dt\right)\right]$$

$$= \frac{Ic}{\theta^2}\left(e^{\theta T_1} - \theta T_1 - 1\right)\left[\sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}\left(w_1(i-1)T\right)M_{r_2}\left((1-w_1)(i-1)T\right)\right.$$
$$- bpM_{r_1}\left(2w_1(i-1)T\right)M_{r_2}\left(2(1-w_1)(i-1)T\right)\}$$
$$+ \sum_{i=n_1+1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}\left(w_2(i-1)T\right)M_{r_4}\left((1-w_2)(i-1)T\right)$$
$$- bpM_{r_3}\left(2w_2(i-1)T\right)M_{r_4}\left(2(1-w_2)(i-1)T\right)\}$$
$$+ e^{-(n-1)dT}\{aM_{r_3}\left(w_2(i-1)T\right)M_{r_4}\left((1-w_2)(i-1)T\right)$$
$$\left. - bpM_{r_3}\left(2w_2(i-1)T\right)M_{r_4}\left(2(1-w_2)(i-1)T\right)\}\right]$$

Expected deterioration cost $C_d(T_1, T) = \frac{\theta}{I} C_h(T_1, T)$

Expected shortage cost $C_{sh}(T_1, T) = \sum_{i=1}^{n-1} E\left[se^{-k(i-1)T}\int_{T_1}^{T}\left(a - bpe^{r(i-1)T}\right)tdt\right]$

$$= s\frac{(T^2 - T_1^2)}{2}\left[\sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}\left(w_1(i-1)T\right)M_{r_2}\left((1-w_1)(i-1)T\right) - bpM_{r_1}\left(2w_1(i-1)T\right)\right.$$
$$\times M_{r_2}\left(2(1-w_1)(i-1)T\right)\} + \sum_{s=n_1+1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}\left(w_2(i-1)T\right)$$
$$\left.\times M_{r_4}\left((1-w_2)(i-1)T\right) - bpM_{r_3}\left(2w_2(i-1)T\right)M_{r_4}\left(2(1-w_2)(i-1)T\right)\}\right]$$

Expected purchase cost: $C_p(T_1, T)$

$$= \sum_{i=1}^{n-1} E\left[\left(ce^{-k(i-1)T}\left(I_i(0) + \int_{T_1}^{T}\left(a - bpe^{r(i-2)T}\right)dt\right)\right)\right]$$
$$+ E\left[\left(ce^{-k(n-1)T}\left(I_n(0) + \int_{T_1}^{T}\left(a - bpe^{r(n-2)T}\right)dt\right)\right)\right]$$

$$= \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}\left(w_1(i-1)T\right)M_{r_2}\left((1-w_1)(i-1)T\right)-bpM_{r_1}\left(2w_1(i-1)T\right)$$

$$\times M_{r_2}\left(2(1-w_1)(i-1)T\right)\}+\sum_{i=n_1+1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}\left(w_2(i-1)T\right)M_{r_4}\left((1-w_2)(i-1)T\right)$$

$$-bpM_{r_3}\left(2w_2(i-1)T\right)M_{r_4}\left(2(1-w_2)(i-1)T\right)\}]+\frac{c}{\theta}\left(e^{\theta T}-1\right)e^{-(n-1)dT}$$

$$\times\{aM_{r_3}\left(w_2(i-1)T\right)M_{r_4}\left((1-w_2)(i-1)T\right)-bpM_{r_3}\left(2w_2(i-1)T\right)$$

$$\times M_{r_4}\left(2(1-w_2)(i-1)T\right)\}+c\left(T-T_1\right)\sum_{i=1}^{n} e^{-(i-1)dT}\{aM_{r_3}\left(w_2(i-1)T\right)$$

$$\times M_{r_4}\left((1-w_2)(i-1)T\right)-bpM_{r_3}\left(w_2(2i-3)T\right)M_{r_4}\left((1-w_2)(2i-3)T\right)\}$$

Expected selling price:

$$C_S(T_1,T)=\sum_{i=1}^{n-1} E\left[\left(pe^{-k(i-1)T}\left(\int_0^{T_1}\left(a-bpe^{r(i-1)T}\right)dt-I_{i-2}(T)\right)\right)\right]$$

$$+E\left[\left(pe^{-k(n-1)T}\left(\int_0^{T}\left(a-bpe^{r(n-1)T}\right)dt-I_{n-2}(T)\right)\right)\right]$$

$$= pT_1[\sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}\left(w_1(i-1)T\right)M_{r_2}\left((1-w_1)(i-1)T\right)$$

$$-bpM_{r_1}\left(2w_1(i-1)T\right)M_{r_2}\left(2(1-w_1)(i-1)T\right)\}+\sum_{i=n_1+1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}\left(w_2(i-1)T\right)$$

$$\times M_{r_4}\left((1-w_2)(i-1)T\right)-bpM_{r_3}\left(2w_2(i-1)T\right)M_{r_4}\left(2(1-w_2)(i-1)T\right)\}]$$

$$+pTe^{-(n-1)dT}\{aM_{r_3}\left(w_2(i-1)T\right)M_{r_4}\left((1-w_2)(i-1)T\right)$$

$$-bpM_{r_3}\left(2w_2(i-1)T\right)M_{r_4}\left(2(1-w_2)(i-1)T\right)\}+p\left(T-T_1\right)e^{-(n-1)dT}$$

$$\times\{aM_{r_3}\left(w_2(i-1)T\right)M_{r_4}\left((1-w_2)(i-1)T\right)-bpM_{r_3}\left(w_2(2i-3)T\right)M_{r_4}\left((1-w_2)(2i-3)T\right)\}$$

In order to calculate the total interest earned or paid, we note that when $M \le T_1$, the inventory manager earns interest on his sales revenue in the interval [$(i-1)T,(i-1)T+M$] and pays interest on the unsold stock in the interval $[(i-1)T+M, iT]$ for $1 \le i \le n-1$. On the other hand, when $M \ge T_1$ he does not have to pay any interest. For $i = n$, he earns interest in $[(i-1)T,(i-1)T+M]$ and pays interest in the interval $[(n-1)T+M, nT]$.

Hence, we have the following:

Case 1: $M \le T_1$

Total interest earned:

$$C_e^{(1)}(T_1,T)=\sum_{i=1}^{n-1} E\left[\left(pI_e\int_0^{T_1}\left(a-bpe^{r(i-1)T}\right)(T_1-t)dt\right)e^{-k(i-1)T}\right]+\ldots$$
$$+E\left[\left(pI_e\int_0^{M}\left(a-bpe^{(n-1)rT}\right)(M-t)dt\right)e^{-(n-1)kT}\right]$$
$$=pI_e\frac{T_1^2}{2}[\sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}(w_1(i-1)T)M_{r_2}((1-w_1)(i-1)T)-bpM_{r_1}(2w_1(i-1)T)$$
$$\times M_{r_2}(2(1-w_1)(i-1)T)\}+\sum_{i=n_1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}(w_2(i-1)T)M_{r_4}((1-w_2)(i-1)T)$$
$$-bpM_{r_3}(2w_2(i-1)T)M_{r_4}(2(1-w_2)(i-1)T)\}]+pI_e\frac{M^2}{2}e^{-(n-1)dT}[\{aM_{r_1}(w_1(i-1)T)$$
$$\times M_{r_2}((1-w_1)(i-1)T)-bpM_{r_1}(2w_1(i-1)T)M_{r_2}(2(1-w_1)(i-1)T)\}$$
$$+\{aM_{r_3}(w_2(i-1)T)M_{r_4}((1-w_2)(i-1)T)-bpM_{r_3}(2w_2(i-1)T)M_{r_4}(2(1-w_2)(i-1)T)\}]$$

Total interest payable:

$$C_r^{(1)}(T_1,T)=\sum_{i=1}^{n-1} E\left[\left(cI_r\int_M^{T_1}I_i(t)dt\right)e^{-k(i-1)T}\right]+E\left[\left(cI_r\int_M^{T}I_n(t)dt\right)e^{-k(n-1)T}\right]$$
$$=\frac{cI_r}{\theta}\left(e^{\theta(T_1-M)}-\theta(T_1-M)-1\right)[\sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}(w_1(i-1)T)M_{r_2}((1-w_1)(i-1)T)$$
$$-bpM_{r_1}(2w_1(i-1)T)M_{r_2}(2(1-w_1)(i-1)T)\}$$
$$+\sum_{i=n_1+1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}(w_2(i-1)T)M_{r_4}((1-w_2)(i-1)T)-bpM_{r_3}(2w_2(i-1)T)$$
$$\times M_{r_4}(2(1-w_2)(i-1)T)\}]$$
$$+\frac{cI_r}{\theta}\left(e^{\theta(T-M)}-\theta(T-M)-1\right)e^{-(n-1)dT}[\{aM_{r_1}(w_1(i-1)T)M_{r_2}((1-w_1)(i-1)T)$$
$$-bpM_{r_1}(2w_1(i-1)T)M_{r_2}(2(1-w_1)(i-1)T)\}+\{aM_{r_3}(w_2(i-1)T)M_{r_4}((1-w_2)(i-1)T)$$
$$-bpM_{r_3}(2w_2(i-1)T)M_{r_4}(2(1-w_2)(i-1)T)\}]$$

Hence, for $M \le T_1$, the total profit in the interval $[0, H]$ is

$$C_1^M(T_1,T) = C_S(T_1,T) + C_e^{(1)}(T_1,T) - C_0(T_1,T) - C_h(T_1,T) - C_d(T_1,T) - C_{sh}(T_1,T) - C_r^{(1)}(T_1,T)$$

$$= [pT_1 + pI_e\frac{T_1^2}{2} - \left(e^{\theta T_1} - \theta T_1 - 1\right)\left(\frac{Ic}{\theta^2} + \frac{c}{\theta}\right) - \frac{cI_r}{\theta}\left(e^{\theta(T_1-M)} - \theta(T_1-M) - 1\right)$$
$$- s_1\frac{T^2 - T_1^2}{2} - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)]K_1(T) + (p-c)(T-T_1)K_2(T) - AK_3(T)$$
$$+ [pT + pI_e\frac{M^2}{2} - (\frac{Ic}{\theta^2} + \frac{c}{\theta})(e^{\theta T} - \theta T - 1) - \frac{cI_r}{\theta}\left(e^{\theta(T-M)} - \theta(T-M) - 1\right)$$
$$- \frac{c}{\theta}\left(e^{\theta T} - 1\right)][e^{-(n-1)dT}[\{aM_{r_1}(w_1(n-1)T)M_{r_2}((1-w_1)(n-1)T)$$
$$- bpM_{r_1}(2w_1(n-1)T)M_{r_2}(2(1-w_1)(n-1)T)\} +$$
$$\{aM_{r_3}(w_2(n-1)T)M_{r_4}((1-w_2)(n-1)T) - bpM_{r_3}(2w_2(n-1)T)M_{r_4}(2(1-w_2)(n-1)T)\}],$$

where

$$K_1(T) = [\sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}(w_1(i-1)T)M_{r_2}((1-w_1)(i-1)T) - bpM_{r_1}(2w_1(i-1)T)M_{r_2}(2(1-w_1)(i-1)T)\}$$
$$+ \sum_{i=n_1+1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}(w_2(i-1)T)M_{r_4}((1-w_2)(i-1)T) - bpM_{r_3}(2w_2(i-1)T)M_{r_4}(2(1-w_2)(i-1)T)\}]$$

$$K_2(T) = \sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}(w_1(i-1)T)M_{r_2}((1-w_1)(i-1)T) - bpM_{r_1}(w_1(2i-3)T)M_{r_2}((1-w_1)(2i-3)T)\}$$
$$+ \sum_{i=n_1+1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}(w_2(i-1)T)M_{r_4}((1-w_2)(i-1)T) - bpM_{r_3}(w_2(2i-3)T)M_{r_4}((1-w_2)(2i-3)T)\}$$

$$K_3(T) = \sum_{i=1}^{n_1} e^{-(i-1)dT} aM_{r_3}(w_2(i-1)T)M_{r_4}((1-w_2)(i-1)T) + \sum_{i=n_1+1}^{n-1} e^{-(i-1)dT} aM_{r_3}(w_2(i-1)T)M_{r_4}((1-w_2)(i-1)T)$$

Case 2: $M \geq T_1$

Interest earned:

$$C_e^{(2)}(T_1,T) = \sum_{i=1}^{n-1} E\left[cI_e e^{-k(i-1)T}\left(\int_0^{T_1}\left(a-bpe^{r(i-1)T}\right)(T_1-t)dt + \int_0^{T_1}\left(a-bpe^{r(i-1)T}\right)(M-T_1)dt\right)\right]$$
$$+E\left[cI_e e^{-k(n-1)T}\left(\int_0^{M}\left(a-bpe^{r(n-1)T}\right)(M-t)dt\right)\right]$$

$$= cI_e T_1(M-\frac{T_1}{2})[\sum_{i=1}^{n_1} e^{-(i-1)dT}\{aM_{r_1}\left(w_1(i-1)T\right)M_{r_2}\left((1-w_1)(i-1)T\right)$$
$$-bpM_{r_1}\left(2w_1(i-1)T\right)M_{r_2}\left(2(1-w_1)(i-1)T\right)\}$$
$$+\sum_{i=n_1}^{n-1} e^{-(i-1)dT}\{aM_{r_3}\left(w_2(i-1)T\right)M_{r_4}\left((1-w_2)(i-1)T\right)$$
$$-bpM_{r_3}\left(2w_2(i-1)T\right)M_{r_4}\left(2(1-w_2)(i-1)T\right)\}]$$
$$+pI_e\frac{M^2}{2}e^{-(n-1)dT}[aM_{r_1}\left(w_1(n-1)T\right)M_{r_2}\left((1-w_1)(n-1)T\right)-bpM_{r_1}\left(2w_1(n-1)T\right)$$
$$\times M_{r_2}\left(2(1-w_1)(n-1)T\right)+aM_{r_3}\left(w_2(n-1)T\right)M_{r_4}\left((1-w_2)(n-1)T\right)$$
$$-bpM_{r_3}\left(2w_2(n-1)T\right)M_{r_4}\left(2(1-w_2)(n-1)T\right)]$$

Interest payable $C_r^{(2)}(T_1,T) = 0$

Total profit in the interval $[0,H]$ is, therefore,

$$C_2^M(T_1,T) = C_S(T_1,T) + C_e^{(2)}(T_1,T) - C_0(T_1,T) - C_h(T_1,T) - C_d(T_1,T)$$
$$-C_{sh}(T_1,T) - C_r^{(2)}(T_1,T).$$

$$C_2^M(T_1,T) = [pT_1 + pI_eT_1\left(M-\frac{T_1}{2}\right) - \left(e^{\theta T_1}-\theta T_1-1\right)\left(\frac{Ic}{\theta^2}+\frac{c}{\theta}\right) - s\frac{T^2-T_1^2}{2}$$
$$-\frac{c}{\theta}\left(e^{\theta T_1}-1\right)]K_1(T) + (T-T_1)(p-c)K_2(T) - AK_3(T)$$
$$+[pT + pI_e\frac{M^2}{2} - (\frac{Ic}{\theta^2}+\frac{c}{\theta})(e^{\theta T}-\theta T-1)$$
$$-\frac{c}{\theta}\left(e^{\theta T}-1\right)][e^{-(n-1)dT}\{aM_{r_1}\left(w_1(n-1)T\right)M_{r_2}\left((1-w_1)(n-1)T\right)$$
$$-bpM_{r_1}\left(2w_1(n-1)T\right)M_{r_2}\left(2(1-w_1)(n-1)T\right)\} + \{aM_{r_3}\left(w_2(n-1)T\right)$$
$$\times M_{r_4}\left((1-w_2)(n-1)T\right) - bpM_{r_3}\left(2w_2(n-1)T\right)M_{r_4}\left(2(1-w_2)(n-1)T\right)\}]$$

4.3.5 Solution Procedure

The problem is to determine the optimal values of n, T and T_1 so as to maximize the total expected inventory system profits. For this, the algorithm begins by setting discrete variable n=1, and takes the partial derivatives of $C^M(n,T,T_1)$ with respect to T and T_1. Equating the partial derivatives to zero derives the following necessary conditions of optimality

$$\frac{dC^M(n,T,T_1)}{dT_1}=0 \quad \text{and} \quad \frac{dC^M(n,T,T_1)}{dT}=0$$

For a given value of n, derive T^* and T_1^* from the above equation. $C^M(n,T^*,T_1^*)$) is derived by substituting (n,T^*,T_1^*) into expected profit function. Then, n increases by increment of one continually and $C^M(n,T^*,T_1^*)$ drive again. The above stages repeat until the maximum

$C^M(n,T^*,T_1^*)$ can be found. The (n^*,T^*,T_1^*) and $C^M(n^*,T^*,T_1^*)$ values constitute the optimal solution and satisfy the following conditions;

$$\Delta C^M(n^*-1,T^*,T_1^*)>0>\Delta C^M(n^*,T^*,T_1^*)$$

Where $\Delta C^M(n^*,T^*,T_1^*)=C^M(n^*+1,T^*,T_1^*)-C^M(n^*,T^*,T_1^*)$

To ensure concavity of the objective function, the derived values of (n^*,T^*,T_1^*) must satisfy the following sufficient conditions;

$$\frac{d^2C^M(n,T,T_1)}{dT_1^2}\le 0 \quad \text{and} \quad \frac{d^2C^M(n,T,T_1)}{dT^2}\le 0$$

4.3.6 Numerical Example

The following numerical example is provided to clarify how the proposed model is applied. The internal and external inflation rates are stochastic with the following pdfs at the beginning of time horizon

Example 4.3.1: Consider the following parameter values: a = 20000; b = 0.1; *c*=Rs. 200, *p*= Rs. 220, θ= 0.02, *A*=Rs. 5000, s_1 = Rs. 15, *I* = 0.05, I_e =.12; I_r =.15, *d* = 0.14, *M* = 0.1year; w_1=0.4;w_2=0.5;

The time horizon, H, is 20 years and the internal & external inflation rates for the first inflationary period are $r_1 \sim N(0.08, 0.004)$ and $r_2 \sim N(0.06, 0.002)$ after 10 (H_1) years, the internal & external inflation rates will change as follows $r_3 \sim N(0.10, 0.008)$ and $r_4 \sim N(0.07, 0.004)$.

Optimum values are n=148, T_1=0.0896, T=0.141, Profit= Rs. 4855440.732

4.3.7 Sensitivity Analysis

To study the sensitivity of the model to changes in its parameters, we start with the following set-up: a=20000; b=.1; c=200; p=220; θ = 0.02; A=5000; s=15; I=.05; I_e=.12; I_r=.15; M=.1 years;

The time horizon, H, is 20 years and the internal & external inflation rates for the first inflationary period are $r_1 \sim N(0.08, 0.004)$ and $r_2 \sim N(0.06, 0.002)$ after 10 (H_1) years, the internal & external inflation rates will change as follows $r_3 \sim N(0.10, 0.008)$ and $r_4 \sim N(0.07, 0.004)$;

The following tables show how the decision variables, namely n, T and T_1, change with change in the values of the model parameters, and also give the corresponding percentage change in the expected profit as compared to that for the above set of parameter values.

Table 4.3.1: Changes in the values of the decision variables with change in I, and the corresponding % change in the expected profit from that when I = 0.05

I	n	T_1	T	Profit	% Change
0.005	172	0.1	0.116279	4946756	1.880679
0.01	166	0.1	0.120482	4935325	1.645238
0.025	156	0.1	0.128205	4902644	0.972169
0.05	142	0.089635	0.140845	4855441	0
0.075	134	0.076603	0.149254	4822919	-0.6698
0.1	130	0.066882	0.153846	4799368	-1.15484
0.15	124	0.053344	0.16129	4767014	-1.8212
0.2	120	0.044365	0.166667	4745252	-2.2694

Table 4.3.2: Changes in the values of the decision variables with change in *M*, and the corresponding % change in the expected profit from that when *M* = 0.1

M	n	T_1	T	Profit	% Change
0.01	104	0.008836	0.192308	4666227	-3.9
0.05	110	0.044745	0.181818	4707083	-3.1
0.1	142	0.089635	0.140845	4855441	0
0.15	134	0.134471	0.149254	5122154	5.49
0.2	100	0.179242	0.2	5304320	9.24
0.5	40	0.447417	0.5	5938767	22.3
0.7	29	0.625743	0.689655	6298587	29.7
1	20	0.892521	1	6744541	38.9

Table 4.3.3: Changes in the values of the decision variables with change in *s*, and the corresponding % change in the expected profit from that when *s* = 15.

s_1	*n*	T_1	T	Profit	% Change
1	30	0.060536	0.666667	5296568	9.09
5	68	0.066825	0.294118	5074353	4.51
10	106	0.076588	0.188679	4935281	1.64
15	142	0.089635	0.140845	4855441	-0
20	182	0.1	0.10989	4822938	-0.7
25	200	0.1	0.1	4820683	-0.7

Table 4.3.4: Changes in the values of the decision variables with change in θ, and the corresponding % change in the expected profit from that when θ = .02.

θ	n	T_1	T	Profit	% Change
0.001	166	0.118083	0.120482	4941332	1.77
0.005	160	0.1	0.125	4913352	1.19
0.01	152	0.1	0.131579	4892269	0.76
0.02	142	0.089635	0.140845	4855441	0
0.05	128	0.063609	0.15625	4791564	-1.3
0.1	120	0.04286	0.166667	4741554	-2.3
0.2	118	0.025937	0.169492	4695613	-3.3
0.25	118	0.021661	0.169492	4681346	-3.6

Table 4.3.5: Changes in the values of the decision variables with change in I_e, and the corresponding % change in the expected profit from that when I_e = 0.12.

Ie	n	T_1	T	Profit	% Change
0.02	106	0.058801	0.188679	4682929	-3.6
0.05	114	0.078188	0.175439	4727481	-2.6
0.08	124	0.085186	0.16129	4778421	-1.6
0.1	132	0.087806	0.151515	4815500	-0.8
0.12	142	0.089635	0.140845	4855441	0
0.15	162	0.091544	0.123457	4922216	1.38
0.2	200	0.093521	0.1	5060053	4.21

Table 4.3.6: Changes in the values of the decision variables with change in I_r, and the corresponding % change in the expected profit from that when I_r = 0.15.

I_r	n	T_1	T	Profit	% Change
0.01	112	0.178571	0.178571	5220290	7.51
0.05	124	0.16129	0.16129	5157430	6.22
0.1	141	0.141844	0.141844	5070816	4.44
0.15	142	0.089635	0.140845	4855441	0
0.2	188	0.089662	0.106383	4634925	-4.5
0.25	198	0.089665	0.10101	4126653	-15

Table 4.3.7: Changes in the values of the decision variables with change in d, and the corresponding % change in the expected profit from that when d = 0.14.

d	n	T_1	T	Profit	% Change
0.01	146	0.089528	0.136986	21532924	343
0.05	146	0.089564	0.136986	12829147	164
0.08	144	0.089592	0.138889	8993563	85.2
0.1	144	0.089594	0.138889	7219639	48.7
0.14	142	0.089635	0.140845	4855441	-0
0.18	141	0.089667	0.141844	3453374	-29
0.2	138	0.089674	0.144928	2975287	-39
0.25	134	0.089696	0.149254	2157472	-56

Table 4.3.8: Changes in the values of the decision variables with change in w_1, and the corresponding % change in the expected profit from that when w_1 = 0.4.

w_1	n	T_1	T	Profit	% Change
0.05	142	0.089641	0.140845	4791553	-1.3
0.1	142	0.089635	0.140845	4797153	-1.2
0.2	142	0.089635	0.140845	4811818	-0.9
0.4	142	0.089635	0.140845	4855441	-0
0.6	142	0.089636	0.140845	4919613	1.32
0.8	140	0.089634	0.142857	5007313	3.13
0.9	140	0.089634	0.142857	5061326	4.24
0.95	140	0.089633	0.142857	5091152	4.85

Table 4.3.9: Changes in the values of the decision variables with change in w_2, and the corresponding % change in the expected profit from that when w_2 = 0.5.

w_2	n	T_1	T	Profit	% Change
0.05	142	0.089636	0.140845	4587022	-5.5
0.1	142	0.089649	0.140845	4564920	-6
0.25	143	0.089672	0.13986	4570490	-5.9
0.5	142	0.089635	0.140845	4855441	-0
0.75	142	0.089294	0.142857	5611643	15.6
0.8	138	0.089048	0.144928	5837993	20.2
0.9	130	0.087822	0.153846	6339839	30.6
0.95	118	0.086105	0.169492	6566926	35.2

Table 4.3.10: Changes in the values of the decision variables with change in *b*, and the corresponding % change in the expected profit from that when $b = 0.1$

b	n	T_1	T	Profit	% Change
0.01	144	0.089735	0.138889	4888105	0.67
0.05	142	0.089689	0.140845	4873584	0.37
0.08	142	0.089657	0.140845	4862698	0.15
0.1	142	0.089635	0.140845	4855441	-0
0.15	140	0.089576	0.142857	4837303	-0.4
0.2	140	0.089521	0.142857	4819201	-0.7
0.25	140	0.089465	0.142857	4801098	-1.1
0.3	139	0.089408	0.143885	4779276	-1.6

On the basis of the results of Table 4.3.1-4.3.10, the following observations can be made.

i) The increase in the parameters w_1, w_2, I_e and M leads a positive change in the total profit and the other parameters leads to a negative change in the optimal profit.

ii) The parameters M, d and w_2 are highly sensitive to optimal profit and the other parameters are moderately sensitive to optimal profit.

4.3.8 Discussion

In this subsection, we developed an inventory model for deteriorating items and stock dependent demand with inflationary environment. Shortages are permitted which are completely backlogged. The effect of trade credits has been developed in this study. Here the inflation and the time value of money are considered and the resultant effect of this is taken into account. Here the demand rate is linear and price dependent. The inflation rate a mixture distribution of two normal random variables with suitable parameters under two inflationary periods. The model is illustrated with some numerical data. A numerical example is given to illustrate the theoretical results and a conclusion is made that the model is quite stable and suitable to realistic situations. The performance of different parameters has been exemplified with the help of sensitivity analysis. From sensitivity analysis, it is observed that model is enough stable with respect to the change in system parameters.