Chapter 2. Inventory Model for linearly price-dependent demand and with constant inflation rate

2.1 Introduction

In most of the profit maximization economic order quantity (EOQ) models, researchers have considered demand as constant. Most of these models consider the demand rate as an independent exogenous variable, which is outside the control of the given organization. This assumption however is quite impracticable for non-essential goods like fashion items, electrical appliances, etc. For such items, it is more realistic to assume the demand to be selling price dependent, as high selling price is likely to dissuade many of the customers from buying the product. In our research, we have considered demand rate to be inversely related to the selling price.

In classical inventory models it is generally assumed that the inventory manager settles his account with the supplier as soon as the ordered quantity arrives. However, in today's business transactions it is frequently observed that the supplier allows his customer a grace period within which he can repay his dues without having to pay any interest, or may delay the payment beyond the permitted time in which case interest is charged. Since, before settling the account with the supplier, the inventory manager can sell the goods, accumulate revenue and earn interest, it makes economic sense for the manager to delay the settlement of his account to the last day of the permissible settlement period.

Now there arises a natural question whether the length of the permissible delay period is influenced by the quantity ordered or not. Intuition leads to the belief that the order quantity should have a direct impact on the length of this period. More precisely we can say that the more we order the longer the delay period is likely to be allotted.

Another situation that seems realistic is that the inventory manager is allowed to pay his dues in two installments within a reorder interval, which is often observed in real life but has not been studied in literature. This basically amounts to giving the manager a loan without interest if he pays within the subsequent grace periods for the installments, beyond which he has to pay interest.

Goyal (1985) first developed an EOQ model under the condition of permissible delay in payments. Shinn et al. (1996) extended the model by considering quantity discount for freight cost. Recently Aggarwal and Jaggi (1995) and Hwang and Shinn (1997) extended Goyal's model to consider deterministic inventory model with constant rate of deterioration. Later Jamal et al. (1997) extended Aggarwal and Jaggi's model to allow for shortages. Pal and Ghosh (2006, 2007) studied deterministic inventory models with quantity dependent permissible delay period. Shah and Shah (1998) developed probabilistic inventory model for deteriorating items when delay in payment is permitted. Ghosh (2008) investigated a stochastic inventory model with stock dependent demand under conditions of permissible delay in payments.

The first study in this direction has been reported by Buzacott (1975), who considered EOQ model with inflation, subject to different types of pricing policies. Misra (1979) developed a discounted-cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Sarker and Pan (1994) surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Some studies were conducted with variable demand, see, for example, Uthayakumar and Geetha (2009), Maity (2010), Vrat and Padmanabhan (1990), Datta and Pal (1991), Hariga (1995), Hariga and Ben-Daya (1996) and Chung (2003)..

In this chapter, we consider a periodic review policy over a finite planning horizon for a deteriorating item with linearly price dependent demand, constant inflation rate and permissible delay in payment. We analyze the model for three different types of permissible delay in payment, namely a) constant, b) order quantity dependent, and c) partial delay in payments.

2.2 Model with constant permissible delay in payment

2.2.1 Assumptions

The assumptions made in this section are as follows:

1. The inventory system involves only one item.

2. Replenishment occurs instantaneously on ordering i.e. lead-time is zero.

3. Demand rate *R(t)* is deterministic and given by

$$R(t) = a - bpe^{rt}, \quad t \le \frac{1}{r}\log\left(\frac{a}{bp}\right)$$

$$= 0, \text{ for } t > \frac{1}{r}\log\left(\frac{a}{bp}\right),$$

where $a, b \geq 0, a \succ\succ b$.

4. Shortages are allowed and completely backlogged.

5. The planning period is of infinite length. The planning horizon is divided into sub-intervals of length T units. Orders are placed at time points $0, T, 2T, 3T,\ldots$. the order quantity at each reorder point being just sufficient to bring the stock height to a certain maximum level S.

6. The length of the permissible delay period is M for repaying the supplier

7. No payment to the supplier is outstanding at the time of placing an order, i.e. $M < T$.

2.2.2 Notations

The following notations have been used in the study:

H : The finite planning horizon

r : The constant inflation rate, $0 < r < 1$

$p(t)$: The selling price at time t, $p(0) = p$

$R(t)$: The price dependent demand rate at time t

θ : The constant deterioration rate

M : The permissible delay in payment

I_r : The interest charged per unit of money per annum by the supplier

I_e : The interest earned per unit of money per annum

T : Length of a replenishment cycle

T_1 : Time to exhaust stock within a replenishment cycle, $0 \leq T_1 < T$

A : The ordering cost per order at time $t = 0$

c : Purchase cost per unit at time $t = 0$

I : Fraction of the purchase cost per unit defining the holding cost per unit per annum

s : shortage cost per unit per annum at time $t = 0$

2.2.3 Mathematical Model

We assume that the planning horizon is divided into *n* reorder intervals of length *T*, so that we have $H = nT$. Further, we assume that the costs and price during a reorder cycle remain the same as that at the beginning of the cycle. Thus, the price in the i^{th} cycle is given by $pe^{r(i-1)T}$, and hence the demand rate is

$$R(t) = a - bpe^{r(i-1)T}, (i-1)T \le t < iT, 1 \le i \le n.$$

Let, $I_i(t)$ denote the inventory level at time point t in the i^{th} cycle, $1 \le i \le n$. Since depletion of stock occurs owing to demand and deterioration, the following differential equations define transitions in inventory:

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = -a + bpe^{r(i-1)T}, \quad 0 \le t \le T_1, \ \ 1 \le i \le n$$

$$\frac{dI_i(t)}{dt} = -a + bpe^{r(i-1)T}, \qquad T_1 \le t \le T \quad 1 \le i \le n$$

where $I_i(T_1) = 0, \ \ \text{for} \, 1 \le i \le n.$

Solving the differential equations, we get

$$I_i(t) = \begin{cases} \dfrac{D_i}{\theta}\left(1 - e^{\theta(T_1 - t)}\right), & 0 \le t \le T_1, 1 \le i \le n \\ D_i(t - T_1), & T_1 \le t \le T, 1 \le i \le n, \end{cases}$$

where $D_i = -a + bpe^{r(i-1)T}$.

To find the optimal values of the decision variables T_1 and *T*, we maximize the value of the total profit over [*0*,*H*] at *t*=*H*. Thus the different cost incurred in the planning horizon will be as follows

The values of the different costs incurred during the planning horizon are as follows:

(i) Total ordering cost:

$$A_H(T, T_1) = A + Ae^{rT} + \cdots + Ae^{r(n-1)T}$$

$$= A\frac{e^{rH} - 1}{e^{rT} - 1}$$

(ii) Since in the reorder interval [$0,T$] , the stock in hand in the interval [$0, T_1$] the total holding cost over [$0,H$]:

$$K_H(T,T_1) = Ic\int_0^{T_1} I_0(t)dt + Ice^{rT}\int_0^{T_1} I_t(t)dt + \cdots + Ice^{r(n-1)T}\int_0^{T_1} I_{n-1}(t)dt$$

$$= \frac{Ic}{\theta^2}\left(e^{\theta T_1} - \theta T_1 - 1\right)\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)$$

(iii) Only a fraction of θ of the stock on hand at any time point in *[$0,T_1$]* deteriorates. The total deterioration cost would be

$$D_H(T,T_1) = \theta c\int_0^{T_1} I_0(t)dt + \theta ce^{rT}\int_0^{T_1} I_t(t)dt + \cdots + \theta ce^{r(n-1)T}\int_0^{T_1} I_{n-1}(t)dt$$

$$= \frac{c}{\theta}\left(e^{\theta T_1} - \theta T_1 - 1\right)\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)$$

(iv) To find the shortage cost, we note that, the shortage at the end of the i^{th} interval before the next order is placed is given by $I_i(T)$. Hence the total shortage cost:

$$S_H(T,T_1) = -s\left(I_0(T) + e^{rT}I_1(T) + \cdots + e^{r(n-1)T}I_{n-1}(T)\right)$$

$$= s(T - T_1)\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)$$

(v) Total purchasing cost:

$$C_H(T,T_1) = cI_0(0) + ce^{rT}\left(I_1(0) + \int_0^{T_1}(a - bp)dt\right) + \cdots + ce^{r(n-1)T}\left(I_{n-2}(0) + \int_0^{T_1}\left(a - bpe^{r(n-1)T}\right)dt\right)$$

$$= \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right) + c(T - T_1)\left(a\frac{e^{rH-rT}-1}{e^{rT}-1} - bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right)$$

(vi) Total selling price:

$$P_H(T,T_1) = p\int_0^{T_1}(a - bp)dt + pe^{rT}\left(\int_0^{T_1}\left(a - bpe^{rT}\right)dt - I_0(T)\right)$$

$$+ \cdots + pe^{r(n-1)T}\left(\int_0^{T_1}\left(a - bpe^{r(n-1)T}\right)dt - I_{n-2}(T)\right)$$

$$= pT_1\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)-pe^{rT}(T-T_1)\left(a\frac{e^{rH-rT}-1}{e^{rT}-1}-bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right)$$

During [0, T_1] the inventory manager earns revenue by selling his goods which he can invest to earn interest. For $T_1 \le M$ the manager sells all his good before the end of the grace period given to him repay his dues. Hence he has no unsold goods and therefore has no to pay interest. However, for $T_1 \ge M$ the manager still has some unsold goods, and has to pay interest on it.

The following figures show the above two cases.

Figure 2.2.1: Case I: Trade credit period is less than time when inventory becomes zero

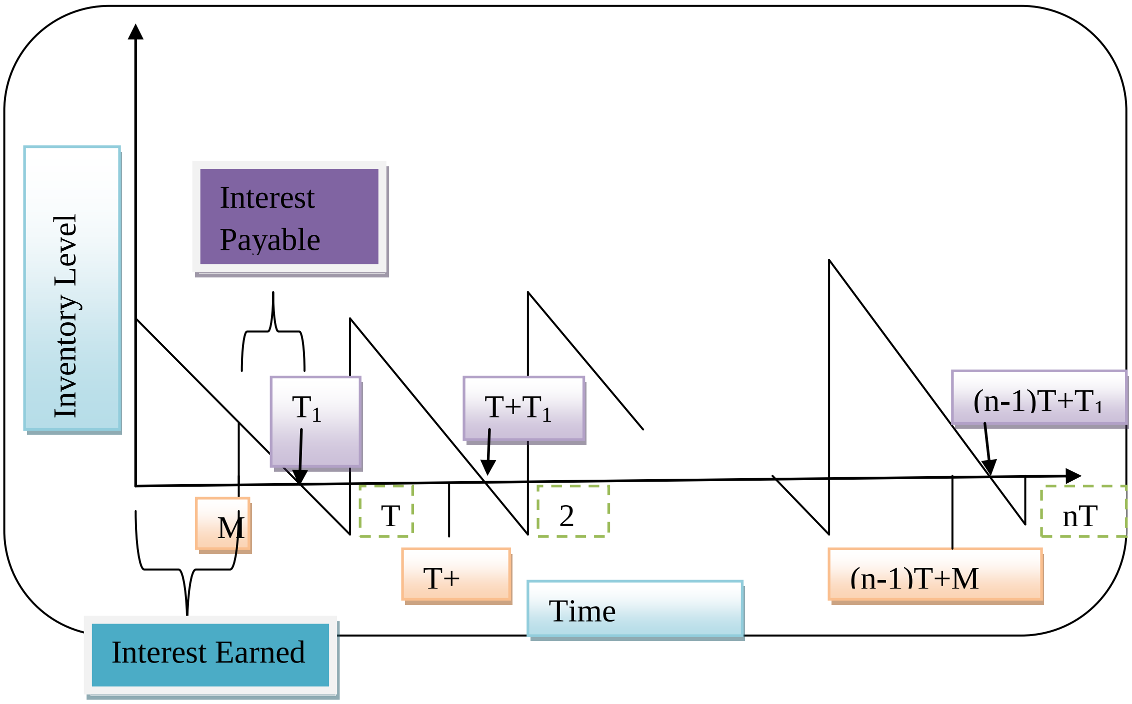
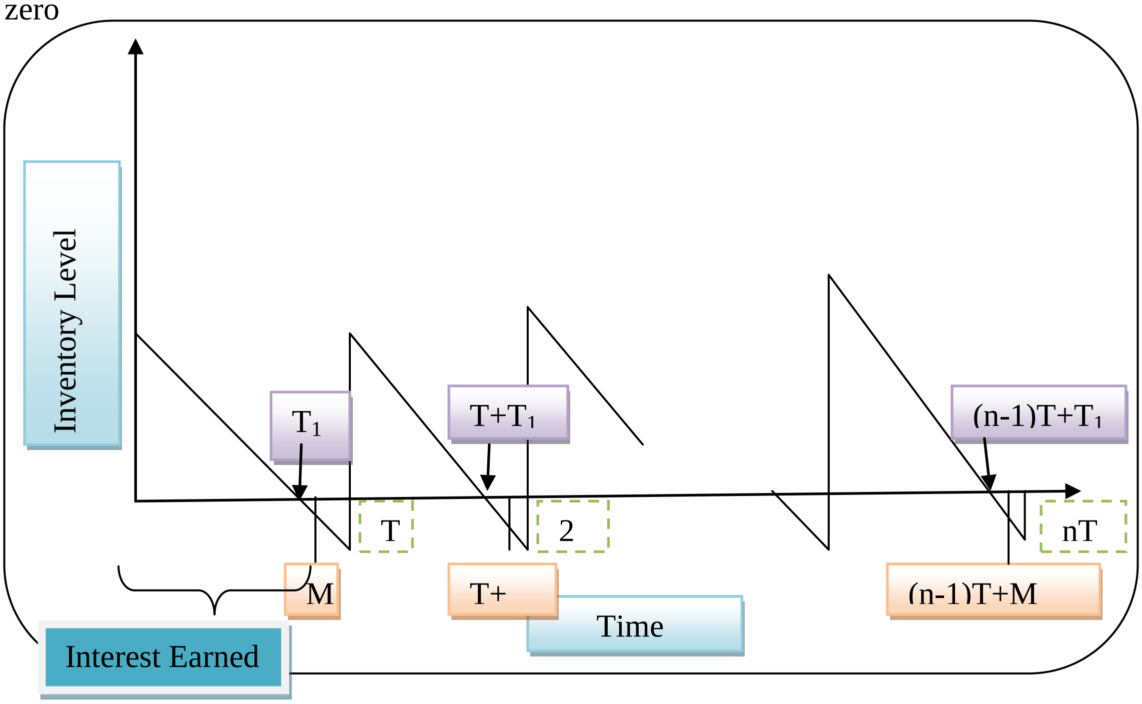


Figure 2.2.2: Case II: Trade credit period is greater than time when inventory becomes zero



Hence, we can find the total interest earned and paid during the planning horizon.

(vii) The total interest that he earns during the planning horizon will be

$IEH(T, T_1)$ = total interest earned during $[0, H]$

$$= pI_e\int_0^{T_1}(a-bp)(T_1-t)dt + pI_e e^{rT}\int_0^{T_1}\left(a-bpe^{rT}\right)(T_1-t)dt$$

$$+\ldots+ pI_e e^{r(n-1)T}\int_0^{T_1}\left(a-bpe^{r(n-1)T}\right)(T_1-t)dt \quad , \quad \text{when} \quad M \le T_1$$

$$= pI_e\left(\int_0^{T_1}(a-bp)(T_1-t)dt + \int_{T_1}^{M}(a-bp)(M-t)dt\right)$$

$$+ pI_e e^{rT}\left(\int_0^{T_1}\left(a-bpe^{rT}\right)(T_1-t)dt + \int_{T_1}^{M}\left(a-bpe^{rT}\right)(M-t)dt\right) \quad , \text{ when } \quad M \ge T_1$$

$$+\ldots+ pI_e e^{r(n-1)T}\left(\int_0^{T_1}\left(a-bpe^{r(n-1)T}\right)(T_1-t)dt + \int_{T_1}^{M}\left(a-bpe^{r(n-1)T}\right)(M-t)dt\right)$$

$$= pI_e\frac{T_1^2}{2}\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right), \text{ when } \quad M \le T_1$$

$$= pI_e\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)\left(\frac{T_1^2}{2} + \frac{(M-T_1)^2}{2}\right), \text{ when } M \ge T_1.$$

(viii) Total interest payable: the interest payable by the inventory manger by

$$IP_H(T,T_1) = cI_r \int_M^{T_1} I_0(t)dt + cI_r e^{rT} \int_M^{T_1} I_1(t)dt + \cdots + cI_r e^{(n-1)rT} \int_M^{T_1} I_{n-1}(t)dt$$

$$= \frac{cI_r}{\theta}\left(e^{\theta(T_1-M)} - \theta(T_1-M) - 1\right)\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right), \text{ when } M \le T_1$$

$$= 0, \text{ when } M \ge T_1.$$

Hence, the total profit made in the interval $[0, H]$ is:

$$P^M(T,T_1) = P_1^M(T,T_1), \text{for } M \le T_1$$
$$= P_2^M(T,T_1), \text{for } M \ge T_1,$$

where

$$P_1^M(T,T_1) = [pT_1 + pI_e\frac{T_1^2}{2} - \left(e^{\theta T_1} - \theta T_1 - 1\right)\left(\frac{Ic}{\theta^2} + \frac{c}{\theta}\right) - \frac{cI_r}{\theta}\left(e^{\theta(T_1-M)} - \theta(T_1-M) - 1\right)$$

$$-s(T-T_1) - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)]\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)$$

$$-(T-T_1)\left(c + pe^{rT}\right)\left(a\frac{e^{rH-rT}-1}{e^{rT}-1} - bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right) - A\frac{e^{rH}-1}{e^{rT}-1}$$

$$P_2^M(T,T_1) = [pT_1 + pI_e\left(\frac{T_1^2}{2} + \frac{(M-T_1)^2}{2}\right) - \left(e^{\theta T_1} - \theta T_1 - 1\right)\left(\frac{Ic}{\theta^2} + \frac{c}{\theta}\right)$$

$$-s(T-T_1) - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)]\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)$$

$$-(T-T_1)\left(c + pe^{rT}\right)\left(a\frac{e^{rH-rT}-1}{e^{rT}-1} - bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right) - A\frac{e^{rH}-1}{e^{rT}-1}$$

2.2.4 Optimum Policy

The optimal policy is defined by the optimal values of (T_1, T) that maximize $P^M(T_1, T)$. It is found by solving the so called average profit optimality equations given by

$\frac{\partial P_1^M(T,T_1)}{\partial T_1} = 0$ and $\frac{\partial P_1^M(T,T_1)}{\partial T} = 0$, which reduce to the following equations:

$$c(2+I_r e^{-\theta M}+\frac{I}{\theta})e^{\theta T_1}-pI_eT_1-(cI_r+p+s)$$
$$=\frac{(c+pe^{rT})\left(a\frac{e^{rH-rT}-1}{e^{rT}-1}-bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right)}{\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)} \qquad (2.2.1)$$

$$(T-T_1)\left(c+pe^{rT}\right)\left(a\frac{re^{rH-rT}\left(e^{rT}-1\right)+\left(e^{rH-rT}-1\right)re^{rT}}{\left(e^{rT}-1\right)^2}-2rbp\frac{e^{rH-rT}\left(e^{2rT}-1\right)+\left(e^{2rH-2rT}-1\right)e^{2rT}}{\left(e^{2rT}-1\right)^2}\right)$$
$$-[pT_1+pI_e\frac{T_1^{\,2}}{2}-\left(e^{\theta T_1}-\theta T_1-1\right)\left(\frac{Ic}{\theta^2}+\frac{c}{\theta}\right)-\frac{cI_r}{\theta}\left(e^{\theta(T_1-M)}-\theta(T_1-M)-1\right)-s(T-T_1)$$
$$-\frac{c}{\theta}\left(e^{\theta T_1}-1\right)]\left(a\frac{e^{rH}-1}{\left(e^{rT}-1\right)^2}re^{rT}-bp\frac{e^{2rH}-1}{\left(e^{2rT}-1\right)^2}2re^{2rT}\right)-\left(c+pe^{rT}\right)\times$$
$$\left(a\frac{e^{rH-rT}-1}{e^{rT}-1}-bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right)-pre^{rT}(T-T_1)\left(a\frac{e^{rH-rT}-1}{e^{rT}-1}-bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right)$$
$$-A\frac{e^{rH}-1}{\left(e^{rT}-1\right)^2}re^{rT}]\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)^{-1}=s \qquad (2.2.2)$$

Similarly, the optimal values of (T_1,T) maximizing $P_2^M\left(T_1,T\right)$ satisfy $\frac{\partial P_1^M(T,T_1)}{\partial T_1}=0$ and $\frac{\partial P_1^M(T,T_1)}{\partial T}=0$, which reduce to the following equations:

$$p+s-c[I_r(M-1)-1-\frac{I}{\theta}]+cT_1(I+\theta)-ce^{\theta T_1}\left(2+\frac{I}{\theta}+I_re^{-\theta M}\right)$$
$$=\left(c+pe^{rT}\right)\frac{\left(a\frac{e^{rH-rT}-1}{e^{rT}-1}-bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right)}{\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)} \qquad (2.2.3)$$

$$[\left(cI_r\left(\frac{T_1^{\,2}}{2}+\frac{(M-T_1)^2}{2}\right)+\left(e^{\theta T_1}-\theta T_1-1\right)\left(\frac{Ic}{\theta^2}+\frac{c}{\theta}\right)-pT_1+\frac{cI_r}{\theta}\left(e^{\theta(T_1-M)}-\theta(T_1-M)-1\right)\right.$$

$$\left.+s(T-T_1)+\frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)\left(a\frac{e^{rH}-1}{\left(e^{rT}-1\right)^2}re^{rT}-bp\frac{e^{2rH}-1}{\left(e^{2rT}-1\right)^2}2re^{2rT}\right)-\left(c+pe^{rT}\right)\times$$

$$\left(a\frac{e^{rH-rT}-1}{e^{rT}-1}-bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right)-(T-T_1)\{pre^{rT}\left(a\frac{e^{rH-rT}-1}{e^{rT}-1}-bp\frac{e^{2rH-2rT}-1}{e^{2rT}-1}\right)$$

$$+\left(c+pe^{rT}\right)\left(a\frac{-re^{rH-rT}\left(e^{rT}-1\right)-\left(e^{rH-rT}-1\right)re^{rT}}{\left(e^{rT}-1\right)^2}-bp\frac{-2re^{rH-rT}\left(e^{2rT}-1\right)-\left(e^{2rH-2rT}-1\right)2re^{2rT}}{\left(e^{2rT}-1\right)^2}\right)\}$$

$$-A\frac{e^{rH}-1}{\left(e^{rT}-1\right)^2}re^{rT}]\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)^{-1}=s \tag{2.2.4}$$

2.2.4.1 Some important results

Lemma 2.2.1: Both $P_1^M(T_1,T)$ and $P_2^M(T_1,T)$ are decreasing function of I and s, for fixed T_1 and T.

Proof: We have,

$$\frac{\partial P_1^M(T_1,T)}{\partial I}=-\frac{c}{\theta^2}\left(e^{\theta T_1}-\theta T_1-1\right)\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)\le 0$$

$$\frac{\partial P_2^M(T_1,T)}{\partial I}=-\frac{c}{\theta^2}\left(e^{\theta T_1}-\theta T_1-1\right)\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)\le 0$$

And,

$$\frac{\partial P_1^M(T_1,T)}{\partial s}=-(T-T_1)\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)\le 0$$

$$\frac{\partial P_2^M(T_1,T)}{\partial s}=-(T-T_1)\left(a\frac{e^{rH}-1}{e^{rT}-1}-bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)\le 0$$

Thus, the total profit decreases with increase in the inventory holding cost and the shortage cost. □

Lemma 2.2.2: $P_1^M(T_1,T)$ and $P_2^M(T_1,T)$ are both concave in T_1, for given T.

Proof: We have

$$\frac{\partial^2 P_1^M(T_1,T)}{\partial T_1^2} = -\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)\frac{T^2}{H}\left(ce^{\theta T}(I+2) + cI_r e^{\theta(T_1-M)} - pI_e\right) \le 0, \text{ since}$$

$$\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right) \ge 0 \text{ and } \left(ce^{\theta T}(I+2) + cI_r e^{\theta(T_1-M)} - pI_e\right) \ge 0.$$

Hence $P_1^M(T_1,T)$ is a concave function of T_1, for given T.

Similarly it can be shown that $P_2^M(T_1,T)$ is concave in T_1, for given T. □

From Lemma 2 it follows that for given T, the optimal value of T_1 is given by the following:

(i) For $M \le T_1$, T_1 is the unique solution to (2.2.1) if the solution is $\ge M$, else $T_1 = M$;

(ii) For $T_1 \le M$, T_1 is the unique solution to (2.2.3) if the solution is $< M$, else $T_1 = M$;

2.2.5 Numerical Examples

Example 2.2.1: Suppose A = Rs. 250, c=Rs. 20, p=Rs. 24, s =Rs 0.1, I=Rs. 0.1/unit/yr, I_e=12%, I_r=15%, r=2%, θ=0.02, M =0.1 year, H =4 years, a=2000, b= 0.1.

Using the software MATLAB, we have the following result:

For $M \le T_1$,

T_{1opt}= 0.226, T_{opt}= 0.433 and $\max P_1^M(T_1,T) = 39227.11$

For $M \ge T_1$,

T_{1opt} = 0.098, T_{opt}=0.443 and $\max P_2^M(T_1,T) = 38060.3$.

Hence, the optimal solution is T_1= 0.226 Yrs, T= 0.433 Yrs, and $P^M(T_1,T) = 39227.11$.

Example 2.2.2: Suppose A=250, c=Rs. 20, p=Rs. 25, s =Rs 0.1, I=0.1/unit/yr, I_e=12%, I_r=15%, r = 15%, θ = 0.4, M =0.1 year, H =2 years, a=2000, b= 0.1.

MATLAB result is:

For $M \le T_1$, T_{1opt}= 0.01, T_{opt}= 0.395 and max $P_1^M(T_1,T) = 21067.82$.

For $M \ge T_1$, T_{1opt}= 0.002, T_{opt}= 0.393 and max $P_2^M(T_1,T) = 21073.55$.

Hence, the optimal solution is T_1= 0.002 Yrs, T= 0.393 Yrs, and $P^M(T_1,T) = 21073.55$.

2.2.6 Sensitivity Analysis

The following tables show the change in the optimal values of T_1 and T and the maximum attainable profit with change in the model parameters in examples 1 and 2.

Table 2.2.1: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in profit with change in the length of the planning horizon and the permissible delay period in Example 2.2.1

H		0.8	1	2	4	6
M=0.01	T_1	0.294	0.294	0.271	0.165	0.124
	T	0.4	0.5	0.286	0.4	0.154
	Profit	8413	8430	16522	39002	59460
	% Change	78.55	78.51	57.88	0.57	51.58
M=0.05	T_1	0.298	0.298	0.298	0.206	0.165
	T	0.4	0.5	0.333	0.4	0.187
	Profit	8630	9647	16735	39100	62517
	% Change	78.00	75.41	57.34	0.32	59.37
M=0.1	T_1	0.311	0.311	0.311	0.256	0.217
	T	0.4	0.5	0.311	0.363	0.461
	Profit	8857	9875	18965	39227	67590
	% Change	77.42	74.83	51.65	0.00	72.30
M=0.3	T_1	0.423	0.423	0.423	0.423	0.42
	T	0.8	0.5	0.5	0.44	0.5
	Profit	9405	10424	19519	39714	69914
	% Change	76.02	73.43	50.24	1.24	78.23
M=0.6	T_1	0.675	0.675	0.675	0.675	0.675
	T	0.8	1	1	0.8	0.75
	Profit	9730	11750	21848	41005	70256
	% Change	75.20	70.05	44.30	4.53	79.10

Table 2.2.2: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in profit with change in inflation rate and deterioration rate in Example 2.2.1 (H=1)

r		0.02	0.05	0.08	0.1	0.12	0.15	0.2
θ=0.02	T_1	0.271	0.217	0.163	0.125	0.085	0.019	0.01
	T	0.286	0.309	0.327	0.344	0.364	0.396	0.463
	Profit	8522.5	8816.1	9201.4	9516	9882.4	10539	11850
	% Change	0.00	3.44	7.97	11.66	15.96	23.66	39.04
θ=0.08	T_1	0.132	0.109	0.084	0.065	0.045	0.01	0
	T	0.265	0.286	0.312	0.334	0.357	0.395	0.46
	Profit	8210.5	8615.9	9091.2	9453	9854.1	10538	11866
	% Change	3.66	1.10	6.67	10.91	15.62	23.65	39.23
θ=0.15	T_1	0.083	0.069	0.054	0.042	0.029	0.01	0
	T	0.255	0.278	0.307	0.33	0.355	0.395	0.46
	Profit	8090	8537.3	9047.4	9427	9842.7	10537	11866
	% Change	5.07	0.17	6.16	10.62	15.49	23.64	39.23
θ=0.2	T_1	0.065	0.055	0.043	0.033	0.023	0.003	0
	T	0.251	0.275	0.305	0.329	0.354	0.394	0.46
	Profit	8046	8508.5	9031.3	9418	9838.6	10537	11866
	% Change	5.59	0.16	5.97	10.51	15.44	23.64	39.23
θ=0.4	T_1	0.036	0.03	0.024	0.018	0.013	0.002	0
	T	0.246	0.271	0.302	0.326	0.353	0.393	0.46
	Profit	7968.8	8457.7	9002.8	9402	9831.1	10537	11866
	% Change	6.50	0.76	5.64	10.32	15.35	23.64	39.23

Table 2.2.3: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in profit with selling price and interest earned in Example 2.2.1

I_e		0.01	0.05	0.1	0.14	0.16
p=22	T_1	0.077	0.114	0.161	0.35	0.539
	T	0.721	0.717	0.713	0.706	0.716
	Profit	3359	3381.5	3408.8	3506	3583.4
	% Change	0.00	0.67	1.48	4.38	6.68
p=24	T_1	0.137	0.177	0.226	0.416	0.576
	T	0.446	0.441	0.433	0.416	0.576
	Profit	7058.1	7129.5	7219.8	7591.6	7831.7
	% Change	110.13	112.25	114.94	126.01	133.16
p=26	T_1	0.194	0.236	0.287	0.43	0.596
	T	0.361	0.355	0.345	0.43	0.596
	Profit	10985	11114	11280	11837	12111
	% Change	227.03	230.87	235.81	252.40	260.55
p=28	T_1	0.26	0.305	0.328	0.446	0.618
	T	0.326	0.319	0.328	0.446	0.618
	Profit	15051	15246	15483	16085	16393
	% Change	348.08	353.89	360.94	378.86	388.03
p=30	T_1	0.322	0.327	0.341	0.464	0.642
	T	0.322	0.327	0.341	0.464	0.642
	Profit	19241	19468	19709	20336	20680
	% Change	472.82	479.58	486.75	505.42	515.66

Table 2.2.5: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in profit with selling price and interest earned in Example 2.2.1

I_r		0.01	0.05	0.1	0.15	0.2
c=15	T_1	0.371	0.371	0.347	0.347	0.347
	T	0.4	0.4	0.363	0.363	0.363
	Profit	91035	87145	84723	80766	76810
	% Change	132.07	122.16	115.98	105.89	95.81
c=18	T_1	0.406	0.39	0.371	0.371	0.347
	T	0.5	0.433	0.4	0.4	0.363
	Profit	59113	57069	54961	51230	48778
	% Change	50.69	45.48	40.11	30.60	24.35
c=20	T_1	0.375	0.329	0.226	0.226	0.226
	T	0.571	0.5	0.433	0.433	0.4
	Profit	47354	44914	42086	39227	36781
	% Change	20.72	14.50	7.29	0.00	6.24
c=22	T_1	0.337	0.337	0.329	0.329	0.226
	T	0.571	0.571	0.5	0.433	0.4
	Profit	29971	28743	26501	24364	22170
	% Change	23.60	26.73	32.44	37.89	43.48
c=23	T_1	0.362	0.267	0.267	0.204	0.187
	T	0.667	0.571	0.571	0.5	0.433
	Profit	20346	18706	17699	15347	13675
	% Change	48.13	52.31	54.88	60.88	65.14

Table 2.2.6: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in profit with shortage cost and holding cost in Example 2.2.1

I		0.05	0.1	0.15	0.2	0.25
s=.1	T_1	0.322	0.32	0.347	0.35	0.36
	T	0.333	0.333	0.364	0.364	0.4
	Profit	42187	39210	37401	35679	33605
	% Change	0.00	7.06	11.34	15.43	20.34
s=.5	T_1	0.277	0.297	0.321	0.319	0.34
	T	0.286	0.307	0.333	0.333	0.364
	Profit	41003	39000	37365	35147	31971
	% Change	2.81	7.55	11.43	16.69	24.22
s=1	T_1	0.259	0.248	0.266	0.265	0.288
	T	0.267	0.267	0.286	0.286	0.307
	Profit	39152	38456	36202	34777	30109
	% Change	7.19	8.84	14.19	17.56	28.63
s=2	T_1	0.238	0.236	0.252	0.266	0.263
	T	0.25	0.25	0.267	0.286	0.286
	Profit	37963	36710	34129	32304	28187
	% Change	10.01	12.98	19.10	23.43	33.19
s=5	T_1	0.231	0.241	0.25	0.261	0.28
	T	0.236	0.25	0.267	0.286	0.307
	Profit	33870	31725	309631	28715	26043
	% Change	19.71	24.80	26.60	31.93	38.27

From the above tables we have the following observations:

(i) As the permissible delay period (M) increases, the time taken for stock to be exhausted (T_1) in an inventory cycle increases.

(ii) As the length of the planning period (H) increases T_1 decreases.

(iii) As inflation rate (r) increases T_1 decreases, but T increases.

(iv) Both T_1 and T decrease as deterioration rate (θ) increases.

(v) As selling price (p) increases, both T_1 and T increase.

(vi) Further, the percentage change in profit with change in the model parameters shows that the model is highly sensitive to change in H, M and p , it is moderately sensitive to change in θ, r, I and s, and quite insensitive to change in I_e.

2.2.7 Discussion

The subsection studies a dynamic inventory model for deteriorating items. The demand for the item is dependent on the selling price and unmet demand is backlogged. The replenishment source allows the inventory manager a certain fixed period of time to settle his accounts. No interest is charged during this period, but beyond it the manager has to pay an interest. The effect of inflation on various costs is also taken into consideration. The optimum ordering policy is determined by maximizing the total profit over the planning horizon.

2.3 Model with order quantity dependent permissible delay in payments

In this section, we assume that the permissible delay period *M* depends on the quantity ordered.

2.3.1 Notations

The notations used are the same as in section 2.2.2

2.3.2 Assumptions

All the assumption remain the same as in section 2.2.1 except that regarding the permissible delay in payments The length of the permissible delay period *M* for repaying the supplier is assumed to be given by

$$M = \begin{cases} M_1 & \text{if } q \le q_0 \\ \mathrm{M}_2 & \text{if } q \ge q_0 \end{cases} \tag{2.3.1}$$

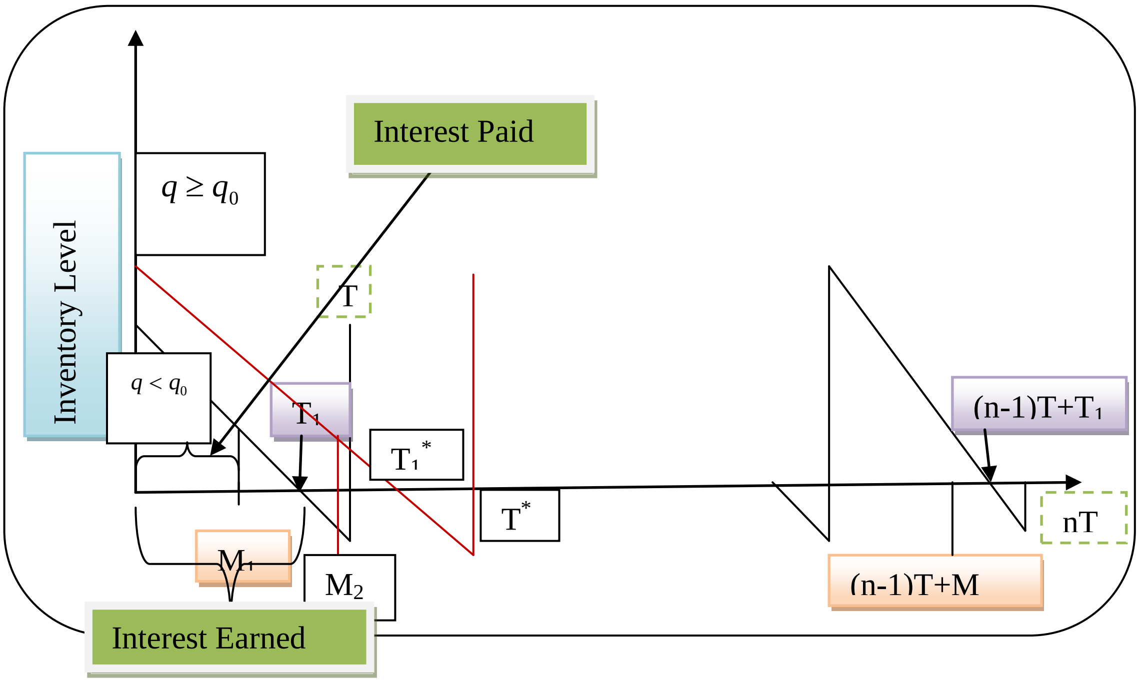
where *q* is the ordered quantity and q_0 a specified value of *q*, and $M_2 > M_1$.

2.3.3 Mathematical Model

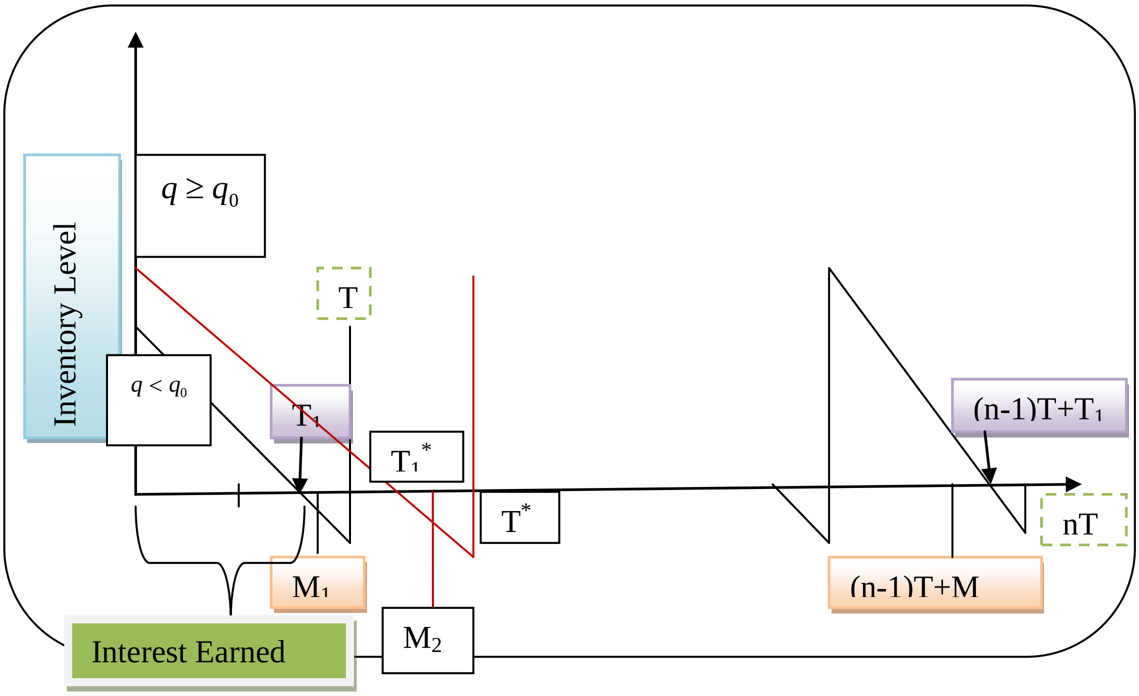
As in section 2.2, we assume that the planning horizon is divided into n reorder intervals of length *T*, so that we have $H = nT$. Further, we assume that the costs and price during a reorder cycle remain the same as that at the beginning of the cycle, and no shortage or excess stock is allowed at the end of the last reorder cycle.

We consider the two possible situations that may occur, namely (i) both interest is earned and paid, and (ii) only interest earned. The following figures show the two situations:

Case 1: $T_1 \geq M$ **Figure 2.3.1:** Interest earned and Paid



Case 2: $T_1 \leq M$ **Figure 2.3.2:** Case of only Interest earned



For any *M*, we have, as before, two profit expressions, same as those obtained in Section 2.2, for the two situations as indicated below:

$$P^M(T,T_1) = P_1^M(T,T_1), \text{ for } M \le T_1$$
$$= p_2^M(T,T_1), \text{ for } M \ge T_1,$$

where

$$P_1^M(T,T_1) = [\,pT_1 + pI_e \frac{T_1^{\,2}}{2} - \left(e^{\theta T_1} - \theta T_1 - 1\right)\left(\frac{Ic}{\theta^2} + \frac{c}{\theta}\right) - \frac{cI_r}{\theta}\left(e^{\theta(T_1 - M)} - \theta(T_1 - M) - 1\right)$$

$$- s(T - T_1) - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)]\left(a\frac{e^{rH} - 1}{e^{rT} - 1} - bp\frac{e^{2rH} - 1}{e^{2rT} - 1}\right)$$
$$- (T - T_1)(c + pe^{rT})\left(a\frac{e^{rH - rT} - 1}{e^{rT} - 1} - bp\frac{e^{2rH - 2rT} - 1}{e^{2rT} - 1}\right) - A\frac{e^{rH} - 1}{e^{rT} - 1} \qquad (2.3.2)$$

$$P_2^M(T,T_1) = [\,pT_1 + pI_e\left(\frac{T_1^{\,2}}{2} + \frac{(M - T_1)^2}{2}\right) - \left(e^{\theta T_1} - \theta T_1 - 1\right)\left(\frac{Ic}{\theta^2} + \frac{c}{\theta}\right)$$

$$- s(T - T_1) - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)]\left(a\frac{e^{rH} - 1}{e^{rT} - 1} - bp\frac{e^{2rH} - 1}{e^{2rT} - 1}\right)$$
$$- (T - T_1)(c + pe^{rT})\left(a\frac{e^{rH - rT} - 1}{e^{rT} - 1} - bp\frac{e^{2rH - 2rT} - 1}{e^{2rT} - 1}\right) - A\frac{e^{rH} - 1}{e^{rT} - 1} \qquad (2.3.3)$$

2.3.4 Some Results

As the profit expressions for given *M* remain the same as in Section 2.2, Lemma 2.2.1 of sub-section 2.2.4.1, given by

Lemma 2.3.1: $P_1^M(T_1,T)$ and $P_2^M(T_1,T)$ are both concave in T_1, for given *T*,

also holds good for the present profit function.

Further, we have the following lemma:

Lemma 2.3.2: $P^M(T,T_1)$ is an increasing function of *M*.

Proof: We have

$$P^M(T,T_1) = P_1^M(T,T_1), \text{ for } T_1 \ge M$$
$$= P_2^M(T,T_1), \text{ for } T_1. \le M$$

For $T_1 \geq M$,

$$\frac{dP^M(T,T_1)}{dM} = \frac{dP_1^M(T,T_1)}{dM}$$

$$= [-\frac{cI_r}{\theta}\left(-\theta e^{\theta(T_1-M)} + \theta\right)]\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right)$$

$$= cI_r\left(e^{\theta(T_1-M)} - 1\right)\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right) \geq 0, \quad \text{as } T_1 \geq M.$$

For $T_1 \leq M$,

$$\frac{dP^M(T,T_1)}{dM} = \frac{dP_2^M(T,T_1)}{dM} = [pI_e(M-T_1)]\left(a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}\right) \geq 0 \quad \text{as } T_1 \leq M.$$

□

Based on Lemmas 2.3.1 and 2.3.2, we develop an algorithm to find the optimal values of T_1 and T.

2.3.5 Algorithm

Step1: Find (T_1^*, T^*) maximizing $P^{M_2}(T_1, T)$.

(i) If $\sum_{i=1}^{n} R_i(t) T^* \geq nQ_0$, (T_1^*, T^*) is optimal.

(ii) Else, go to step 2.

Step2: Find (T_1^{**}, T^{**}) maximizing $P^{M_1}(T_1, T)$. Compute $P^{M_1}(T_1^{**}, T^{**})$ and $P^{M_2}(T_1^0, T^0)$,

where $\sum_{i=1}^{n} R_i(t) T^0 = nQ_0$ and T_1^0 is the optimal value of T_1 for given $T = T_0$.

(i) If $P^{M_1}(T_1^{**}, T^{**}) > P^{M_2}(T_1^0, T^0)$, then (T_1^{**}, T^{**}) is optimal.

(ii) Else, (T_1^0, T^0) is optimal.

2.3.6 Numerical Example

Example 2.3.1: Suppose A = Rs. 250, c=Rs. 20, p=Rs. 25, s =Rs 0.1, I=0.1/unit/yr, I_e=12%, I_r=15%, r=2%, θ=.02, H =4 years, a=20000, b= 0.1.

$$M = \begin{cases} \frac{1}{12} \text{ years} & \text{if } q < 8000 \\ \frac{2}{12} \text{ years} & \text{if } q \geq 8000 \end{cases}$$

Using the above algorithm in MATLAB, we have the following result:

Hence, the optimal solution is T_1= 0.186 Yrs, T= 0.235 Yrs, and $P^M(T_1, T) = 378721$.

2.3.7 Sensitivity Analysis

The following tables show the change in the optimal values of T_1 and T and the maximum attainable profit with change in the model parameters.

Table 2.3.1: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in the length of the time horizon and the permissible delay period in Example 2.3.1

H		2	4	5	6	8
$M = \begin{cases} 0.01 & q < 2000 \\ 0.02 & q \geq 2000 \end{cases}$	T_1	0.043	0.044	0.044	0.045	0.048
	T	0.048	0.048	0.049	0.05	0.053
	Profit	231471	458784	644441	754897	914602
	% Change	0.00	538.82	978.90	1240.73	1619.29
$M = \begin{cases} 0.05 & q < 4000 \\ 0.08 & q \geq 4000 \end{cases}$	T_1	0.048	0.048	0.051	0.053	0.054
	T	0.053	0.054	0.057	0.06	0.062
	Profit	219732	450111	620037	715877	905172
	% Change	27.83	518.26	921.06	1148.24	1596.94
$M = \begin{cases} 0.1 & q < 7000 \\ 0.2 & q \geq 7000 \end{cases}$	T_1	0.103	0.107	0.125	0.129	0.129
	T	0.143	0.148	0.152	0.154	0.154
	Profit	201774	429871	617413	709412	872169
	% Change	70.39	470.29	914.84	1132.91	1518.71
$M = \begin{cases} 0.3 & q < 12000 \\ 0.5 & q \geq 12000 \end{cases}$	T_1	0.42	0.423	0.423	0.45	0.461
	T	0.4	0.444	0.455	0.462	0.5
	Profit	173698	397410	574132	658741	778141
	% Change	136.95	393.34	812.24	1012.80	1295.83

$M = \begin{cases} 0.6 & q < 25000 \\ 1 & q \geq 25000 \end{cases}$	T_1	0.675	0.675	0.675	0.675	0.675
	T	0.67	0.8	0.83	1	1
	Profit	113478	301411	487135	517412	689713
	% Change	279.69	165.79	606.03	677.79	1086.22

Table 2.3.2: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in profit with change in the inflation rate and the deterioration rate in Example 2.3.1

r		0.02	0.05	0.08	0.1	0.12	0.15	0.2
θ=0.02	T_1	0.186	0.184	0.182	0.182	0.181	0.179	0.17
	T	0.235	0.222	0.21	0.2	0.19	0.182	0.174
	Profit	378721	397711	431231	463188	484794	512365	566711
	% Change	0.00	5.01	13.87	22.30	28.01	35.29	49.64
θ=0.06	T_1	0.18	0.178	0.173	0.168	0.166	0.162	0.16
	T	0.267	0.25	0.25	0.235	0.2	0.19	0.19
	Profit	347521	367489	388987	409212	419770	471002	493112
	% Change	8.24	2.97	2.71	8.05	10.84	24.37	30.20
θ=0.15	T_1	0.2	0.19	0.181	0.175	0.16	0.153	0.14
	T	0.285	0.267	0.25	0.235	0.2	0.19	0.19
	Profit	314703	330117	357981	380147	400501	413291	433646
	% Change	16.90	12.83	5.48	0.38	5.75	9.13	14.50

Table 2.3.3: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in profit with change in shortage cost and the fraction of the holding cost in Example 2.3.1

s		0.05	0.1	0.5	1	2
I=0.08	T_1	0.186	0.186	0.204	0.205	0.221
	T	0.235	0.235	0.25	0.25	0.267
	Profit	376871	372569	365900	352114	343358
	% Change	0.00	1.14	2.91	6.57	8.89
I=0.1	T_1	0.186	0.186	0.204	0.205	0.221
	T	0.235	0.235	0.25	0.25	0.267
	Profit	375009	371721	363495	350783	341647
	% Change	0.49	1.37	3.55	6.92	9.35
I=0.15	T_1	0.189	0.197	0.207	0.224	0.241
	T	0.235	0.25	0.25	0.285	0.285
	Profit	370364	368731	360225	347931	341647
	% Change	1.73	2.16	4.42	7.68	9.35

Table 2.3.4: Showing the change in the optimal values of T_1 and T and percentage (absolute) change in profit with change in interest earned and interest payable in Example 2.3.1

I_e		0.05	0.1	0.12	0.15	0.2
I_r=0.10	T_1	0.227	0.207	0.198	0.191	0.185
	T	0.267	0.25	0.235	0.222	0.21
	Profit	367349	376840	385764	396730	425600
	% Change	0.00	0.26	0.50	0.80	1.59
I_r=0.15	T_1	0.225	0.202	0.186	0.182	0.171
	T	0.267	0.25	0.235	0.222	0.21
	Profit	362784	370963	378721	386712	413945
	% Change	0.12	0.10	0.31	0.53	1.27
I_r=0.20	T_1	0.201	0.2	0.182	0.179	0.164
	T	0.267	0.25	0.235	0.222	0.21
	Profit	351601	355103	362541	366681	387814
	% Change	0.43	0.33	0.13	0.02	0.56

From the above tables we have the following observations:

(i) As the permissible delay period (M) increases, the time taken for stock to be exhausted (T_1) in an inventory cycle increases.

(ii) As the length of the planning period (H) increases T_1 decreases.

(iii) As the deterioration rate (θ) increases T_1 decreases, but T increases.

(iv) Both T_1 and T decrease if inflation rate (r) increases.

(v) As interest earned (I_e) increases, both T_1 and T decreases.

(vi) The percentage changes in profit are significantly high for changes in the values of permissible delay period (M), planning period (H), deterioration rate (θ) and inflation rate (r).

2.3.8 Discussion

The subsection studies an inventory problem, where the shortage is completely backlogged and the permissible delay in payment depends on the order quantity. An algorithm is suggested to find the optimal ordering policy, which helps the inventory manager to decide whether it would be worthwhile to take advantage of a longer credit period for repaying the supplier by ordering a larger amount of the commodity.

2.4 Model with partial permissible delay in payment

In this section, we consider a similar model as in Sections 2.2 and 2.3, namely a periodic review inventory model for deteriorating items allowing shortages and under inflation, when demand is price dependent. However, here the inventory manager is allowed to pay his dues in two installments within a reorder interval, which is often observed in real life but has hardly been studied in literature. This basically amounts to giving the manager a loan without interest during two subsequent time periods, beyond which he has to pay an interest.

2.4.1 Notations

The notations used are the same as in section 2.2.2 . Some additional notations are as follows:

T_α= Time by which the inventory manager has to pay a fraction α of total due;

$T_{1-\alpha}$ = Time by which the inventory manager has to pay the remaining fraction (1-α) of total due.; where $T_\alpha < T_{1-\alpha}$.

2.4.2 Assumption

All the assumption remain the same as in section 2.2.1 except that regarding the permissible delay in payments The permissible delay period M for repaying the supplier is divided into two parts according to the partial payment policy considered, namely

$$T_\alpha \text{ and } T_{1-\alpha} = M(> T_\alpha). \tag{2.4.1}$$

2.4.3 Mathematical Model

The inventory policy is to place an order at the beginning of each reorder interval and the order quantity is just sufficient to bring up the stock height to a certain level S. The decision variables of the policy are S and T.

As before, $H/T = n$ is assumed to be an integer, i.e. the planning horizon is divided into n reorder intervals.

Using the boundary condition that $I_i(T_1)=0$, we obtain, as before, the inventory level at any arbitrary time point *t* on the i-th reorder interval, $1 \le i \le n$, as

$$I_i(t)=\frac{D_i}{\theta}\left(e^{\theta(T_1-t)}-1\right) \quad 0 \le t \le T_1 \;\; 1 \le i \le n,$$
$$I_i(t)=D_i(T_1-t) \quad T_1 \le t \le T \;\; 1 \le i \le n,$$

where $D_i = a - bpe^{r(i-1)T}$.

Hence, the order quantity at the i-th reorder point is $S = I_i(0)=\frac{D_i}{\theta}\left(e^{\theta T_1}-1\right)$. We may, therefore, take the independent decision variables to be T_1 and *T*.

The different terms in the expression of the expected profit in [*0, H*] are as follows:

(i) Ordering cost in [*0,H*] = $A + Ae^{rT} + ... + Ae^{r(n-1)T} = A\left(\frac{e^{rH}-1}{e^{rT}-1}\right)$

(ii) Carrying cost in [*0,H*] $= Ic\int_0^{T_1} I_1(t)dt + Ice^{rT}\int_0^{T_1} I_2(t)dt + ... + Ice^{r(n-1)T}\int_0^{T_1} I_n(t)dt$

$$= \frac{Ic}{\theta^2}\left(e^{\theta T_1}-\theta T_1-1\right)\left\{D_1 + D_2 e^{rT} + D_n e^{r(n-1)T}\right\}$$
$$= \frac{Ic}{\theta^2}\left(e^{\theta T_1}-\theta T_1-1\right)K(T),$$

where $K(T)=\left\{D_1 + D_2 e^{rT} + D_n e^{r(n-1)T}\right\} = a\frac{e^{rH}-1}{e^{rT}-1} - bp\frac{e^{2rH}-1}{e^{2rT}-1}$;

(iii) Deteriorating cost in [*0,H*] = $\theta c\int_0^{T_1} I_1(t)dt + \theta ce^{rT}\int_0^{T_1} I_2(t)dt + ... + \theta ce^{r(n-1)T}\int_0^{T_1} I_n(t)dt$

$$= \frac{\theta c}{\theta^2}\left(e^{\theta T_1}-\theta T_1-1\right)\left\{D_1 + D_2 e^{rT} + D_n e^{r(n-1)T}\right\}$$
$$= \frac{\theta c}{\theta^2}\left(e^{\theta T_1}-\theta T_1-1\right)K(T);$$

(iv) Shortage cost in [*0,H*] = $-s\left(I_1(T) + e^{rT}I_2(T) + ... + e^{r(n-1)T}I_n(T)\right) = s(T-T_1)K(T)$;

(v) Selling price in [*0,H*] $= pT_1\left\{D_1 + D_2 e^{rT} + D_n e^{r(n-1)T}\right\} + \left\{-pI_0(T) - pe^{rT}I_0(T) - pe^{r(n-1)T}I_n(T)\right\}$

$$= pT_1K(T) + p(T-T_1)K(T) = pTK(T);$$

(vi) Purchasing cost in $[0,H]$ $= cI_1(0)+ce^{rT}I_2(0)+...+ce^{r(n-1)T}I_n(0)+\{-ce^{rT}I_0(T)-ce^{2rT}I_0(T)-ce^{rnT}I_n(T)\}$

$$=\left\{\frac{c}{\theta}\left(e^{\theta T_1}-1\right)+ce^{rT}\left(T-T_1\right)\right\}K(T);$$

(vii) Interest earned and interest charged in $[0,H]$:

To find the interest earned and interest charged, we look at the different cases that can arise, namely $T_1 \le T_\alpha \le T_{1-\alpha} \le T$, $T_\alpha \le T_1 \le T_{1-\alpha} \le T$ and $T_\alpha \le T_{1-\alpha} \le T_1 \le T$.

Case1: $T_1 \le T_\alpha \le T_{1-\alpha} \le T$

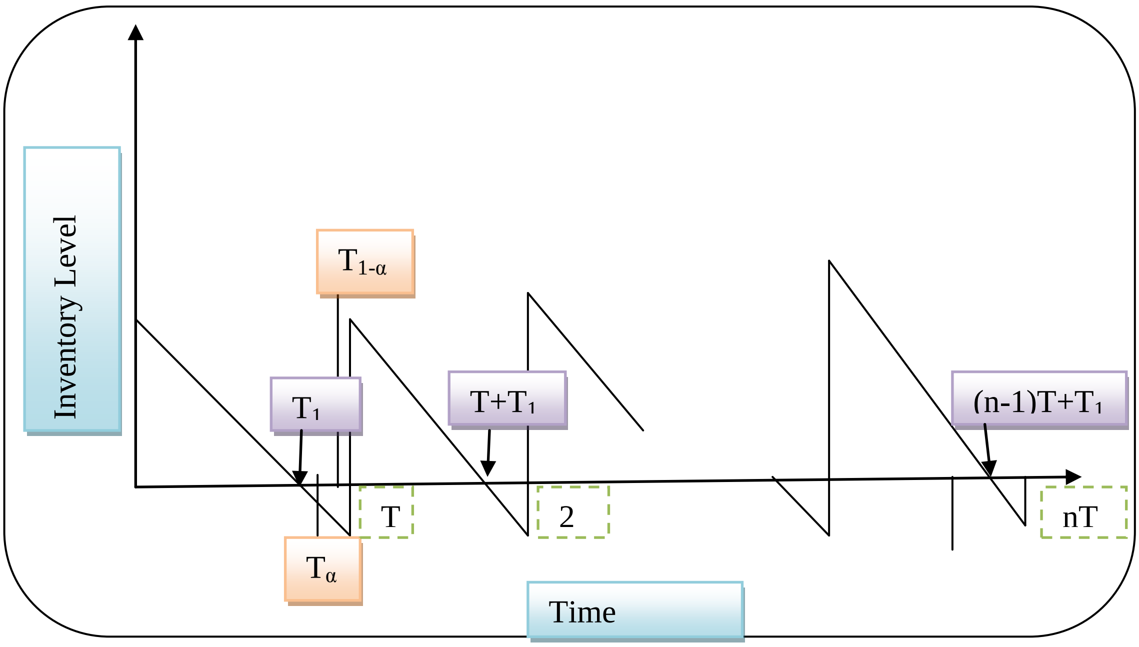


Figure 2.4.1: Showing the the positions of T_α and $T_{1-\alpha}$. with respect to T_1 and T in Case 1

In this case, the inventory manager earns interest on the goods he sells, and the interest earned is given by

$$I_e\left(PT_1(T_\alpha-T_1)+\left(PT_1-\alpha*\frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T_{1-\alpha}-T_\alpha)+\left(PT_1-\frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T-T_{1-\alpha})\right)$$

$$=I_eK(T)\left\{pT_1(T-T_1)-\frac{c}{\theta}\left(e^{\theta T_1}-1\right)\left(T-T_{1-\alpha}+\alpha(T_{1-\alpha}-T_\alpha)\right)\right\}$$

However, the manager does not have to pay any interest to the supplier.

Case 2: $T_\alpha \le T_1 \le T_{1-\alpha} \le T$

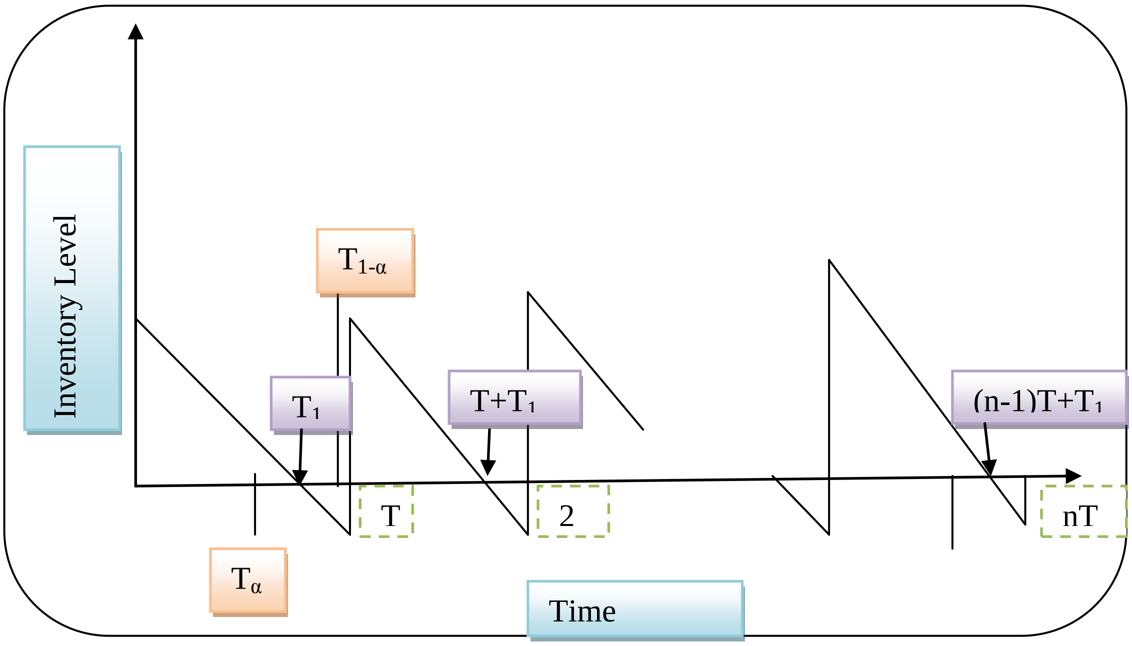


Figure 2.4.2: Showing the the positions of T_α and $T_{1-\alpha}$. with respect to T_1 and T in case 2

(a) If the total selling price in the interval $(0,T_\alpha)$ is greater than α-fraction of the total cost price, the inventory manager will be able to pay the first installment for settling the account in the i^{th} cycle $, 1 \le i \le n$, i.e., the manager pays the first installment at T_α if

$$pe^{r(i-1)T} D_i T_\alpha \ge \alpha c e^{r(i-1)T} \frac{D_i}{\theta}\left(e^{\theta T_1} - 1\right)$$

or, $$T_1 \le \frac{1}{\theta}\log\left(1 + \frac{\theta p T_\alpha}{\alpha c}\right) = T_{20}, \text{ say.}$$

Hence, the manager earns interest, but does not have to pay any interest. His earned interest is given by

$$I_e K(T)\left\{\left(pT_1 - \frac{\alpha c}{\theta}\left(e^{\theta T_1} - 1\right)\right)\left(T_{1-\alpha} - T_1\right) + \left(pT_1 - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\right)\left(T - T_{1-\alpha}\right)\right\}$$

(b) If the total selling price in the interval $(0,T_\alpha)$ is less than α-fraction of the total cost price, the manager will not be able to pay the first installment at T_α. He can pay it only at

$T_{1-\alpha}$, and during the intermittent period high interest will be charged on that amount. He can, however, continue to collect revenue on the sold items.

His interest earned is, therefore, given by

$$I_e K(T)\left\{pT_1(T_{1-\alpha}-T_1)+\left(pT_1-\frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right)(T-T_{1-\alpha})\right\}$$

$$\text{i.e} = pe^{r(i-1)T} D_i T_\alpha \le \alpha c e^{r(i-1)T}\frac{D_i}{\theta}\left(e^{\theta T_1}-1\right)$$

and the interest charged is $= I_r K(T)\dfrac{\alpha c}{\theta}\left(e^{\theta T_1}-1\right)(T_{1-\alpha}-T_\alpha)$.

In this case $T_1 \ge T_{20}$.

Case 3: $T_\alpha \le T_{1-\alpha} \le T_1 \le T$

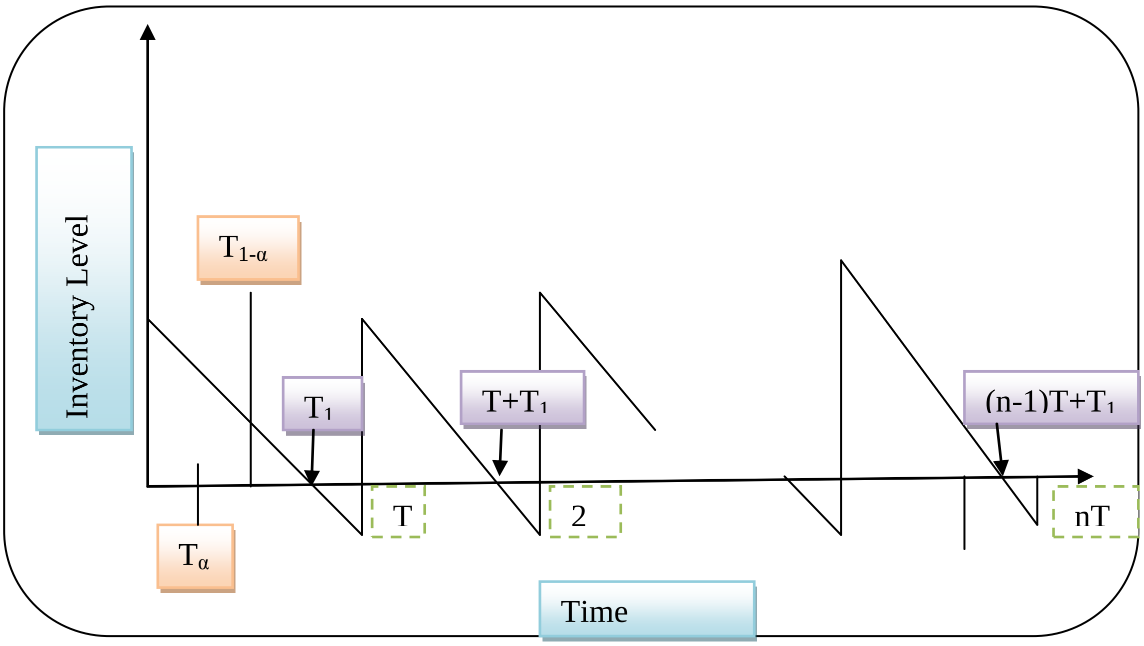


Figure 2.4.3: Showing the positions of T_α and $T_{1-\alpha}$ with respect to T_1 and T in Case 3

(a) If in the i^{th} cycle the total selling price in the interval $(0,T_\alpha)$ is greater than α-fraction of the total cost price, i.e. $pe^{r(i-1)T}D_i T_{1-\alpha} \ge ce^{r(i-1)T}\dfrac{D_i}{\theta}\left(e^{\theta T_1}-1\right)$, so that the manager is able to pay the first installment to the supplier at T_α, and the total selling price in $\left(0, T_{1-\alpha}\right)$ is greater than total cost price i.e. $pe^{r(i-1)T} D_i T_\alpha \geq \alpha c e^{r(i-1)T} \frac{D_i}{\theta}\left(e^{\theta T_1}-1\right)$, then T_1 satisfies

$$T_1 \leq \frac{1}{\theta}\log\left(1+\frac{\theta p T_\alpha}{\alpha c}\right) = T_{30}, \text{ say, and } T_1 \leq \frac{1}{\theta}\log\left(1+\frac{\theta p T_{1-\alpha}}{c}\right) = T_{31}, \text{ say,}$$

i.e $T_1 \leq \min(T_{30}, T_{31})$.

In this case, the interest earned in [*0, H*] is

$$I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1) \ ,$$

while the interest charged is 0.

(b) In the i^{th} cycle, if the total selling price in the interval $\left(0, T_\alpha\right)$ greater than α-fraction of cost price, but the total selling price in $\left(0, T_{1-\alpha}\right)$ is less than the total cost price, i.e. $T_{30} \leq T_1 \leq T_{31}$, the manager can pay the first installment in time, but not the second installment. Hence, he will earn interest as well as pay interest to the supplier. The interest earned is given by

$$I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1)$$

And the interest paid is

$$I_r K(T)\frac{(1-\alpha)c}{\theta}\left(e^{\theta T_1}-1\right)(T-T_1).$$

(c) In the i^{th} cycle, if total selling price in the interval $\left(0, T_\alpha\right)$ is less than α-fraction of cost price, i.e. the customer is not able to pay the first installment, and the total selling price in $\left(0, T_{1-\alpha}\right)$ is less than the total cost price, i.e $pe^{r(i-1)T} D_i T_\alpha \leq \alpha c e^{r(i-1)T} \frac{D_i}{\theta}\left(e^{\theta T_1}-1\right)$

and $pe^{r(i-1)T} D_i T_{1-\alpha} \leq c e^{r(i-1)T} \frac{D_i}{\theta}\left(e^{\theta T_1}-1\right)$, or, $T_1 \geq \max(T_{30}, T_{31})$, then interest earned in [*0, H*] $= I_e K(T)\left\{ pT_1 - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\right\}(T - T_1),$

interest charged in [*0,H*] = $I_r K(T)\frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\{\alpha(T_1 - T_\alpha) + (1-\alpha)(T_1 - T_{1-\alpha})\}.$

(d) In the i^{th} cycle, if the total selling price in the interval $(0, T_\alpha)$ is less than *α*-fraction of the total cost price and the total selling price in $(0, T_{1-\alpha})$ is greater than the total cost price, i.e.

$pe^{r(i-1)T} D_i T_\alpha \le \alpha c e^{r(i-1)T}\frac{D_i}{\theta}\left(e^{\theta T_1} - 1\right)$ and $pe^{r(i-1)T} D_i T_{1-\alpha} \ge c e^{r(i-1)T}\frac{D_i}{\theta}\left(e^{\theta T_1} - 1\right)$, or

$T_{31} \le T_1 \le T_{30},$

Interest earned in [*0, H*] = $I_e K(T)\left\{ pT_1 - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\right\}(T - T_1)$

Interest Charged in [*0, H*]*:* $I_r K(T)\frac{\alpha c}{\theta}\left(e^{\theta T_1} - 1\right)(T_{1-\alpha} - T_\alpha).$

Thus, the total profit in [*0, H*] is given by

$$P(T_1,T) = P_1(T_1,T) = I_e K(T)\left\{ pT_1(T - T_1) - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\left(T - T_{1-\alpha} + \alpha\left(T_{1-\alpha} - T_\alpha\right)\right)\right\} + G(T_1,T), \text{ in case 1}$$

$$= P_2^1(T_1,T) = I_e K(T)\left\{\left(pT_1 - \frac{\alpha c}{\theta}\left(e^{\theta T_1} - 1\right)\right)(T_{1-\alpha} - T_1) + \left(pT_1 - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\right)(T - T_{1-\alpha})\right\}$$
$$+ G(T_1,T), \text{ in case 2(a)}$$

$$= P_2^2(T_1,T) = I_e K(T)\left\{ pT_1(T_{1-\alpha} - T_1) + \left(pT_1 - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\right)(T - T_{1-\alpha})\right\}$$
$$- I_r K(T)\frac{\alpha c}{\theta}\left(e^{\theta T_1} - 1\right)(T_{1-\alpha} - T_\alpha) + G(T_1,T), \text{ in case 2(b)}$$

$$= P_3^1(T_1,T) = I_e K(T)\left\{ pT_1 - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\right\}(T - T_1) + G(T_1,T), \text{ in case 3(a)}$$

$$= P_3^2(T_1,T) = I_e K(T)\left\{ pT_1 - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\right\}(T - T_1) - I_r K(T)\frac{(1-\alpha)c}{\theta}\left(e^{\theta T_1} - 1\right)(T - T_1)$$
$$+ G(T_1,T), \text{ in case 3(b)}$$

,

$$= P_3^3(T_1,T) = I_e K(T)\left\{ pT_1 - \frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\right\}(T - T_1) - I_r K(T)\frac{c}{\theta}\left(e^{\theta T_1} - 1\right)\times$$
$$\{\alpha(T_1 - T_\alpha) + (1-\alpha)(T_1 - T_{1-\alpha})\} + G(T_1,T), \text{ in case 3(c)}$$

$$= P_3^4(T_1,T) = I_e K(T)\left\{pT_1 - \frac{c}{\theta}\left(e^{\theta T_1}-1\right)\right\}(T-T_1) - I_r K(T)\frac{\alpha c}{\theta}\left(e^{\theta T_1}-1\right)(T_{1-\alpha}-T_\alpha) + G(T_1,T),$$

in case 3(d)

where

$$G(T_1,T) = K(T)\left\{pT - \frac{c}{\theta}\left(e^{\theta T_1}-1\right) - ce^{rT}(T-T_1) - s(T-T_1) - \frac{Ic+\theta c}{\theta^2}\left(e^{\theta T_1}-\theta T_1-1\right)\right\}$$
$$- A\frac{e^{rH}-1}{e^{rT}-1}.$$

In each case, the optimum values of T_1 and T are obtained by solving the equations

$$\frac{\partial P(T_1,T)}{\partial T_1} = 0, \qquad \frac{\partial P(T_1,T)}{\partial T} = 0.$$

Then, by comparing the five profit expressions at the corresponding optimal values of T_1 and T we obtain the optimum T_1 and T that maximizes the profit.

2.4.4 Some Results

Theorem 2.4.1: $P(T_1,T)$, is a concave function of T_1, for given T.

Proof: Since $K(T)\ge 0, T-T_{1-\alpha}\ge 0, T_{1-\alpha}-T_\alpha \ge 0,$ we have the following:

Case 1: $T_1 \le T_\alpha \le T_{1-\alpha} \le T$

$$\frac{\partial^2 P_1(T_1,T)}{\partial T_1^{\,2}} = \left[I_e K(T)\left\{-2p - c\theta e^{\theta T_1}\left(T-T_{1-\alpha}+\alpha(T_{1-\alpha}-T_\alpha)\right)\right\} + K(T)\left\{-c\theta e^{\theta T_1} - (Ic+c\theta)e^{\theta T_1}\right\}\right]$$
$$= -K(T)\left[I_e\left\{c\theta e^{\theta T_1}\left(T-T_{1-\alpha}+\alpha(T_{1-\alpha}-T_\alpha)\right)+2p\right\} + \left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\}\right] \le 0,$$

Case2: $T_\alpha \le T_1 \le T_{1-\alpha} \le T$

$$\frac{\partial^2 P_2^1(T_1,T)}{\partial T_1^2} = -K(T)[I_e\left\{\alpha c\theta e^{\theta T_1}(T_{1-\alpha}-T_1) + \left(p-\alpha c e^{\theta T_1}\right) + \left(p-\alpha c e^{\theta T_1}\right) + \theta c e^{\theta T_1}(T-T_{1-\alpha})\right\} +$$
$$\left\{c\theta e^{\theta T_1} + (Ic+c\theta)e^{\theta T_1}\right\}] \le 0$$

$$\frac{\partial^2 P_2^2(T_1,T)}{\partial T_1^2} = -K(T)\left[I_e\left\{2p + c\theta e^{\theta T_1}(T - T_{1-\alpha})\right\} + I_c\alpha\theta c e^{\theta T_1}(T_{1-\alpha} - T_\alpha) - \left\{c\theta e^{\theta T_1} + (Ic + c\theta)e^{\theta T_1}\right\}\right] \le 0.$$

<u>Case3:</u> $T_\alpha \le T_{1-\alpha} \le T_1 \le T$

$$\frac{\partial^2 P_3^1(T_1,T)}{\partial T_1^2} = -K(T)\left[I_e\left\{\theta c e^{\theta T_1}(T - T_1) + 2\left(p - c e^{\theta T_1}\right)\right\} + \left\{c\theta e^{\theta T_1} + (Ic + c\theta)e^{\theta T_1}\right\}\right] \le 0$$

$$\frac{\partial^2 P_3^2(T_1,T)}{\partial T_1^2} = -K(T)[I_e\left\{\theta c e^{\theta T_1}(T - T_1) + 2\left(p - c e^{\theta T_1}\right)\right\}$$
$$+ I_c\frac{(1-\alpha)c}{\theta}\left\{\theta^2 e^{\theta T_1}(T - T_1) - \theta e^{\theta T_1} + \theta e^{\theta T_1}\right\} + \left\{c\theta e^{\theta T_1} + (Ic + c\theta)e^{\theta T_1}\right\}] \le 0$$

$$\frac{\partial^2 P_3^3(T_1,T)}{\partial T_1^2} = -K(T)I_e\left\{\theta c e^{\theta T_1}(T - T_1) + 2\left(p - c e^{\theta T_1}\right)\right\}$$
$$+ I_c\left[c\theta e^{\theta T_1}\left\{\alpha(T_1 - T_\alpha) + (1-\alpha)(T_1 - T_{1-\alpha})\right\} + 2c e^{\theta T_1}\right] + \left\{c\theta e^{\theta T_1} + (Ic + c\theta)e^{\theta T_1}\right\} \le 0$$

$$\frac{\partial^2 P_3^4(T_1,T)}{\partial T_1^2} = -K(T)[I_e\left\{\theta c e^{\theta T_1}(T - T_1) + 2\left(p - c e^{\theta T_1}\right)\right\}$$
$$+ I_c\alpha\theta c e^{\theta T_1}(T_{1-\alpha} - T_\alpha) + \left\{c\theta e^{\theta T_1} + (Ic + c\theta)e^{\theta T_1}\right\}] \le 0.$$

Hence, $P(T_1, T)$ is concave in T_1 for a given T. □

Theorem 2.4.2*:* For $T \le \frac{1}{\theta}\log_e(p/c)$, optimal T_1 is an increasing function in T.

Proof: If $T \le \frac{1}{\theta}\log_e(p/c)$, then $T_1 \le \frac{1}{\theta}\log_e(p/c)$. Hence we have the following -

<u>Case1</u>: $T_1 \le T_\alpha \le T_{1-\alpha} \le T$

Optimal T_1 satisfies $\frac{\partial P_1(T_1,T)}{\partial T_1} = 0$, which gives

$$I_e\left\{pT - 2pT_1 - c e^{\theta T_1}\left(T - T_{1-\alpha} + \alpha(T_{1-\alpha} - T_\alpha)\right)\right\} + \left\{-c e^{\theta T_1} + c e^{rT} + s - \frac{Ic + \theta c}{\theta}\left(e^{\theta T_1} - 1\right)\right\} = 0$$

Differentiating the above expression with respect to T, we get

$$I_e\left\{p - 2p\frac{\partial T_1}{\partial T} - c\theta e^{\theta T_1}\frac{\partial T_1}{\partial T}\left(T - T_{1-\alpha} + \alpha(T_{1-\alpha} - T_\alpha)\right) - c e^{\theta T_1}\right\} +$$
$$\left\{-c\theta e^{\theta T_1}\frac{\partial T_1}{\partial T} + cre^{rT} - (Ic + \theta c)e^{\theta T_1}\frac{\partial T_1}{\partial T}\right\} = 0$$

or, $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT} + I_e\{p - ce^{\theta T_1}\}}{I_e c\theta e^{\theta T_1}\left(T - T_{1-\alpha} + \alpha(T_{1-\alpha} - T_\alpha)\right) + c\theta e^{\theta T_1} + (Ic + c\theta)e^{\theta T_1} + 2p},$$

which is ≥ 0 if $p - ce^{\theta T_1} \geq 0$.

Similarly, we get

<u>Case2</u>: $T_\alpha \leq T_1 \leq T_{1-\alpha} \leq T$

(a) $$\frac{\partial T_1}{\partial T} = \frac{I_e(p - ce^{\theta T_1}) + cre^{rT}}{I_e\{(p - \alpha ce^{\theta T_1}) + (p - \alpha ce^{\theta T_1}) + \theta ce^{\theta T_1}(T - T_{1-\alpha})\} + c\theta e^{\theta T_1} + (Ic + \theta c)e^{\theta T_1} - I_e\alpha\theta ce^{\theta T_1}(T_{1-\alpha} - T_1)} \geq 0$$

(b) $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT} + I_e(p - ce^{\theta T_1})}{I_e(2p + c\theta e^{\theta T_1}(T - T_{1-\alpha})) + I_c\alpha ce^{\theta T_1}(T_{1-\alpha} - T_\alpha) + c\theta e^{\theta T_1} + (Ic + \theta c)e^{\theta T_1}} \geq 0$$

<u>Case3:</u> $T_\alpha \leq T_{1-\alpha} \leq T_1 \leq T$

(a) $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT}}{I_e\{c\theta e^{\theta T_1}(T - T_1) + 2(p - ce^{\theta T_1})\} + ce^{\theta T_1} + (Ic + \theta c)e^{\theta T_1}} \geq 0$$

(b) $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT}}{I_e\{c\theta e^{\theta T_1}(T - T_1) + 2(p - ce^{\theta T_1})\} + ce^{\theta T_1} + (Ic + \theta c)e^{\theta T_1} + I_r(1-\alpha)c\theta e^{\theta T_1}(T - T_1)} \geq 0$$

(c) $$\frac{\partial T_1}{\partial T} = cre^{rT} /[I_e\{c\theta e^{\theta T_1}(T - T_1) + 2(p - ce^{\theta T_1})\} + ce^{\theta T_1} + (Ic + \theta c)e^{\theta T_1} + I_c \times$$
$$(c\theta e^{\theta T_1}\{\alpha(T_1 - T_\alpha) + (1-\alpha)(T_1 - T_{1-\alpha})\} + 2ce^{\theta T_1})] \geq 0$$

(d) $$\frac{\partial T_1}{\partial T} = \frac{cre^{rT}}{I_e\{c\theta e^{\theta T_1}(T - T_1) + 2(p - ce^{\theta T_1})\} + ce^{\theta T_1} + (Ic + \theta c)e^{\theta T_1} + I_r\alpha\theta^2 ce^{\theta T_1}(T_{1-\alpha} - T_\alpha)} \geq 0.$$

Hence the theorem. □

Theorem 2.4.3: $P(T_1, T)$ is a decreasing function of θ and *s*.

Proof: From the profit function, we get
$$\frac{dP_1(T_1,T)}{d\theta} = -\frac{K(T)}{H}\left(Ic\left(\sum_{i=3}^{\infty}\frac{\theta^{i-3}T_1^i}{i!}\right) + c\left(\sum_{i=2}^{\infty}\frac{\theta^{i-2}T_1^i}{i!}\right) + c\left(\sum_{i=2}^{\infty}\frac{\theta^{i-2}T_1^i}{i!}\right)\{I_e(T - T_{1-\alpha} + \alpha(T_{1-\alpha} - T_\alpha)) + 1\}\right) \leq 0$$

and
$$\frac{dP_1(T_1,T)}{ds} = -\frac{K(T)}{H}(T - T_1) \leq 0.$$

Similarly, it can be shown that

$$\frac{dP_i^{(j)}(T_1,T)}{d\theta} \le 0, \quad \frac{dP_i^{(j)}(T_1,T)}{ds} \le 0, \; i, j = 2,3.$$

Hence the theorem. □

2.4.5 Sensitivity Analysis

Since it is difficult to find optimum values of the decision variables in closed form, we numerically find solutions to the equations $\frac{dP(T_1,T)}{dT_1} = 0$ and $\frac{dP(T_1,T)}{dT} = 0$, for given sets of model parameters, using the statistical software MATLAB. The following tables show the change in optimal inventory policy with change in some important parameters of the model. We assume that A = ₹200, c = ₹20, a = 2000, b = 0.1, θ = 0.04, H = 2 years.

Table 2.4.1: Showing change in optimum (T_1, T)-values and corresponding percentage (absolute) profit with change in r for some combinations of (T_α , $T_{1-\alpha}$) when p = 25, s = 0.4, , I = 0.01, α = 0.7.

($T_{1-\alpha}$, T_a)	r	T_1	T	Profit	% Change
(0.07, 0.02)	0.01	0.1279	0.6879	8867.1	0
	0.03	0.1176	0.3873	8693.7	1.96
	0.05	0.1202	0.3018	8655.4	2.39
	0.07	0.1246	0.2578	8674.7	2.17
	0.1	0.132	0.2199	8768.7	1.11
(0.7, 0.04)	0.01	0.1326	0.6767	8884.3	0.19
	0.03	0.1224	0.381	8722.4	1.63
	0.05	0.1249	0.297	8693.8	1.95
	0.07	0.1292	0.2537	8722.1	1.64
	0.1	0.1365	0.2163	8829.2	0.43
(0.7, 0.06)	0.01	0.107	0.5868	8924.5	0.65
	0.03	0.1069	0.3388	8800.6	0.75
	0.05	0.1069	0.2624	8794.6	0.82
	0.07	0.1068	0.2218	8838.8	0.32
	0.1	0.1067	0.1856	8960.5	1.05

($T_{1\text{-}\alpha}$, T_a)	r	T_1	T	Profit	% Change
(0.1, 0.04)	0.01	0.133	0.675	8886.25	0.22
	0.03	0.123	0.38	8725.59	1.6
	0.05	0.1247	0.229	8758.82	1.22
	0.07	0.1247	0.194	8846.68	0.23
	0.1	0.1247	0.162	9029.97	1.84
(0.1, 0.07)	0.01	0.1247	0.574	8964.39	1.1
	0.03	0.1247	0.331	8864.52	0.03
	0.05	0.1246	0.257	8880.8	0.15
	0.07	0.1246	0.217	8947.02	0.9
	0.1	0.1246	0.182	9102.14	2.65
(0.1, 0.09)	0.01	0.1603	0.591	9005.09	1.56
	0.03	0.1602	0.341	8923.66	0.64
	0.05	0.1602	0.264	8962.85	1.08
	0.07	0.1601	0.2234.	9054.49	2.11
	0.1	0.16	0.187	9251.35	4.33

Table 2.4.2: Showing change in optimum (T_1, T)-values and corresponding percentage (absolute) profit with change in I for some values of T_α , $T_{1\text{-}\alpha}$ when p = 25, s = 0.4, r = 0.06, α = 0.7

($T_{1\text{-}\alpha}$, T_a)	I	T_1	T	Profit	% Change
	0.01	0.1424	0.2537	8448.6	0
	0.03	0.1358	0.2549	8416.6	0.38
	0.05	0.1297	0.256	8387.5	0.72
	0.07	0.1241	0.257	8361	1.04
(0.07, 0.02)	0.1	0.1166	0.2583	8325.4	1.46
	0.01	0.1466	0.2467	8503.3	0.65
	0.03	0.1398	0.2484	8468.4	0.23
	0.05	0.1336	0.2499	8436.7	0.14
	0.07	0.128	0.2512	8407.9	0.48
(0.7, 0.04)	0.1	0.1203	0.2529	8369.3	0.94

$(T_{1-\alpha}, T_a)$	I	T_1	T	Profit	% Change
(0.1, 0.04)	0.01	0.125	0.159	8662.12	2.53
	0.03	0.1248	0.166	8621.66	2.05
	0.05	0.1248	0.173	8582.89	1.59
	0.07	0.1247	0.18	8545.61	1.15
	0.1	0.1246	0.189	8492.16	0.52
(0.1, 0.07)	0.01	0.1248	0.191	8734.86	3.39
	0.03	0.1247	0.197	8700.96	2.99
	0.05	0.1247	0.203	8668.08	2.6
	0.07	0.1247	0.208	8636.12	2.22
	0.1	0.1246	0.217	8589.76	1.67
(0.1, 0.09)	0.01	0.1603	0.186	8898.49	5.33
	0.03	0.1603	0.196	8841.56	4.65
	0.05	0.1602	0.205	8787.45	4.01
	0.07	0.1602	0.215	8735.78	3.4
	0.1	0.1601	0.228	8662.2	2.53
(0.7, 0.06)	0.01	0.107	0.204	8593.6	1.72
	0.03	0.107	0.2082	8570.2	1.44
	0.05	0.1069	0.2122	8547.2	1.17
	0.07	0.1068	0.2182	8524.7	0.9
	0.1	0.1068	0.2221	8491.7	0.51

Table 2.4.3: Showing change in optimum (T_1, T)-values and corresponding percentage (absolute) profit with change in p for some values of T_α , $T_{1-\alpha}$ when s = 0.4, r = 0.06, α = 0.7.

($T_{1-\alpha}$, T_a)	p	T_1	T	Profit	% Change
(0.07, 0.02)	22	0.14	0.2531	2066.9	0
	24	0.1393	0.254	6310.7	205.32
	26	0.1388	0.2547	10554	410.6
	28	0.0978	0.1894	14810	616.52
	30	0.1048	0.1853	19100	824.08
(0.7, 0.04)	22	0.1446	0.2455	2321	12.29
	24	0.1436	0.247	6464.1	212.74
	26	0.1427	0.2482	10607	413.16
	28	0.1419	0.2493	14848	618.38
	30	0.1047	0.1852	18989	818.7
(0.7, 0.06)	22	0.1491	0.2371	2178.6	5.4
	24	0.1026	0.208	6433.4	211.26
	26	0.1112	0.2047	10730	419.11
	28	0.1197	0.2036	15021	626.76
	30	0.128	0.2047	19308	834.16
(0.1, 0.04)	22	0.1098	0.1741	2167.98	4.89
	24	0.1197	0.1656	6484.89	213.75
	26	0.1297	0.1604	10796.7	422.36
	28	0.1396	0.1586	15099.9	630.56
	30	0.1496	0.1601	19392	838.22
(0.1, 0.07)	22	0.1098	0.1989	2259.69	9.33
	24	0.1197	0.1948	6566.54	217.7
	26	0.1297	0.1937	10867.2	425.77
	28	0.1396	0.1956	15159.9	633.46
	30	0.1496	0.2001	19443.6	840.71
(0.1, 0.09)	22	0.141	0.1865	2419.22	17.05
	24	0.1538	0.1882	6722.95	225.27
	26	0.1666	0.1943	11012.7	432.81
	28	0.1794	0.2041	15288.3	639.67
	30	0.1921	0.2172	19550.6	845.89

Table 2.4.4: Showing change in optimum (T_1, T)-values and corresponding percentage (absolute) profit with change in I_e for some combinations of (T_α , $T_{1-\alpha}$) when p = 25, s = 0.4, , r = 0.06, α = 0.7, I =0.2, I_c =0.14

($T_{1-\alpha}$, T_a)	I_e	T_1	T	Profit	% Change
(0.07, 0.02)	0.03	0.2179	0.2783	8810.2	0
	0.05	0.1798	0.2772	8745.3	0.74
	0.07	0.1562	0.2765	8709.3	1.15
	0.1	0.1339	0.2758	8681.6	1.46
	0.12	0.1238	0.2756	8672.8	1.56
(0.7, 0.04)	0.03	0.2267	0.2709	8877.3	0.76
	0.05	0.187	0.2712	8800.7	0.11
	0.07	0.1622	0.2714	8757.4	0.6
	0.1	0.1386	0.2715	8722.8	0.99
	0.12	0.1279	0.2716	8710.8	1.13
(0.7, 0.06)	0.03	0.2351	0.2624	8949.1	1.58
	0.05	0.1938	0.2645	8859.6	0.56
	0.07	0.1679	0.2657	8808.3	0.02
	0.1	0.1109	0.2373	8802.3	0.09
	0.12	0.107	0.2391	8814.7	0.05
(0.1, 0.04)	0.01	0.2175	0.2787	8807.6	0.03
	0.03	0.1247	0.203	8774.9	0.4
	0.05	0.1247	0.2058	8785.1	0.28
	0.07	0.1246	0.2099	8800.7	0.11
	0.1	0.1246	0.2126	8811.4	0.01
(0.1, 0.07)	0.03	0.2306	0.2672	8909.7	1.13
	0.05	0.1247	0.2256	8867.5	0.65
	0.07	0.1247	0.2281	8879.3	0.78
	0.1	0.1247	0.2318	8897.1	0.99
	0.12	0.1246	0.2343	8909.2	1.12
(0.1, 0.09)	0.01	0.2388	0.2582	8983.8	1.97
	0.03	0.2165	0.2261	8973.3	1.85
	0.05	0.172	0.2306	8981	1.94
	0.07	0.1605	0.2366	8991.4	2.06
	0.1	0.1602	0.2406	9007.7	2.24

Table 2.4.5: Showing change in optimum (T_1, T)-values and corresponding percentage (absolute) profit with change in I_r for some combinations of ($T_{1-\alpha}$, T_α) when p = 25, s = 0.4, r = 0.06, α = 0.7, I =0.2, I_e =0.12

($T_{1-\alpha}$, T_a)	I_r	T_1	T	Profit	% Change
(0.07, 0.02)	0.13	0.1245	0.2749	8679.5	0
	0.15	0.123	0.2762	8666.2	0.15
	0.17	0.1215	0.2776	8653.2	0.3
	0.2	0.1193	0.2794	8634	0.52
	0.22	0.1177	0.2806	8621.5	0.67
(0.7, 0.04)	0.13	0.1283	0.2711	8715	0.41
	0.15	0.1274	0.272	8706.7	0.31
	0.17	0.1266	0.2729	8698.4	0.22
	0.2	0.1252	0.2742	8686.2	0.08
	0.22	0.1243	0.275	8678.1	0.02
(0.7, 0.06)	0.13	0.107	0.2389	8816	1.57
	0.15	0.107	0.2394	8813.3	1.54
	0.17	0.1069	0.2398	8810.7	1.51
	0.2	0.1068	0.2403	8808.1	1.48
	0.22	0.1067	0.2403	8808.1	1.48
(0.1, 0.04)	0.13	0.1248	0.2142	8817.97	1.6
	0.15	0.1247	0.211	8804.89	1.44
	0.17	0.1247	0.2077	8792.1	1.3
	0.2	0.1247	0.2026	8773.49	1.08
	0.22	0.1246	0.1992	8761.48	0.94
(0.1, 0.07)	0.13	0.1247	0.2343	8909.66	2.65
	0.15	0.1247	0.2343	8867.54	2.17
	0.17	0.1247	0.2344	8879.26	2.3
	0.2	0.1246	0.2345	8897.13	2.51
	0.22	0.1246	0.2344	8909.22	2.65
(0.1, 0.09)	0.13	0.1602	0.2402	9009.62	3.8
	0.15	0.1602	0.2409	9005.68	3.76
	0.17	0.1602	0.2416	9001.75	3.71
	0.2	0.1601	0.2426	8995.88	3.65
	0.22	0.16	0.2433	8991.98	3.6

From the above tables, we make the following observations:

(i) Optimal T_1 is a non-increasing function of holding cost per unit per unit time (I), Interest earned (I_e) and interest charged (I_r).

(ii) Optimal cycle length T is non-decreasing in holding cost per unit per unit time(I), and interest charged (I_r)., but is non-increasing in inflation rate (r).

(iii) Maximum profit is a non-increasing function of I and I_c, but a non-decreasing function of selling price (p).

2.4.6 Discussion

The subsection studies a periodic review inventory model for deteriorating items when demand is dependent on the selling price and the deterioration rate is constant. The inventory manager has the provision to pay his dues to the supplier in two installments – a proportion α of his dues in the first installment and the remaining in the second installment. Failure to make payment in time imposes an interest on the unpaid amount. Value inflation of money is also taken into account, which is essential when the planning period is sufficiently long.