INTRODUCTION
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The Chapter I, on Introduction presents a brief historical development of the work done in the field of "Generalized Polynomial Systems and Their Applications". No attempt has been made to give a comprehensive review of the entire literature on the subject but only those aspects, which have a direct bearing on our work done in the present thesis, have been dealt with in some details. It also presents the scope for under taking researches and their importance in related areas.

1.1. Special Functions. An equation of the form

\[(1.1.1) \ p_0(x)w^n + p_1(x)w^{n-1} + \ldots + p_n(x) = 0,\]

where \( p_0(x), p_1(x), \ldots, p_n(x) \) are polynomials expressions having integral coefficients, is called algebraic equation. The roots of the above equation \[(1.1.2) \ w = f(x)\]

are called algebraic functions. The functions, which are not roots of algebraic equations are called "Transcendental Functions". Logarithmic functions, exponential functions, trigonometrical functions etc. are examples of "Transcendental Functions". Transcendental functions are generally solutions of differential equations or they have integral representations. Transcendental functions such as beta functions, gamma functions, Bessel functions, \( E, G \) and \( H \)-functions, all polynomials etc., which are of complicated nature are known as "Higher Transcendental Functions."

In the study of Higher transcendental functions, if we are not concerned with their general properties, but only with the properties of the functions which occur in the solution of special problems, they are called "Special Functions". Moreover, it is a matter of opinion or
conversion. According to Harry-Bateman (1882-1946) any function which has received individual attention at least in one research paper, may be attributed to *Special Function*.

Here we shall discuss some special functions, particularly, polynomials and their generalizations. We shall also discuss generalized multiple hypergeometric functions of several variables and their applications.

### 1.2 Legendre Function.

Special Functions were first introduced towards the end of eighteenth century in the solutions of the problems of Dynamical Astronomy and Mathematical Physics. In 1782, Laplace introduced the potential theorem. Legendre (1782 or earlier) investigated the expansion of potential function in the form of an infinite series and was thus led to the discovery of functions now known as "Legendre Coefficients" or Legendre polynomials.

Thomson and Tait in their well known "*Natural Philosophy*" (1879) defined spherical harmonics as follows:

Any function $V_n$ of Laplace equation $\nabla^2 \phi = 0$, which is homogeneous of degree $n$ in $x, y, z$ is called a "Solid Spherical Harmonics of Degree $n$". The degree $n$ may be any positive integer and the function need not be rational.

If $x, y, z$ are expressed in terms of polar coordinates $(r, \theta, \phi)$ the solid spherical harmonics of degree $n$ assumes the form $r^nf_n(\theta, \phi)$. The function $f_n(\theta, \phi)$ is called a "*Surface Spherical Harmonics of Degree* $n$".

Laplace equation possesses solutions of the form $r^n e^{i\mu \phi} (\bar{H}(\mu))$ where $\bar{H}(\mu)$ satisfies the ordinary differential equation

$$\left(1-\mu^2\right)\frac{d^2 \bar{H}}{d\mu^2} - 2\mu \frac{d \bar{H}}{d\mu} + \left\{n(n+1) - \frac{m^2}{1-\mu^2}\right\} \bar{H} = 0.$$  

(1.2.1)

The above equation is called *associated Legendre equation*. $\mu$ is
restricted to be a real and to be in the interval (-1,1).

Legendre polynomials were generalized by Gegenbauer, Tchebicheff and Jacobi. Jacobi polynomials are most general polynomials of this family and were first introduced by C.G. Jacobi in 1859.

Jacobi polynomials (See Reinville [125, p254, (1)]) are defined as

\[ P_n^{(\alpha, \beta)}(x) = \frac{(1+x)_n}{n!} F_1 \left[ -n, 1+\alpha + \beta + n; 1-x \mid 1+\alpha; \frac{1}{2} \right] \]

For \( \alpha=\beta=0 \) the above polynomials reduce to Legendre polynomials.

Generating function for Legendre polynomials is given by Rainville ([125,p.157(1)])

\[ (1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n, \]

while their Rodrigues’ formula is given by Rainville ([125, p.162(7)])

\[ P_n(x) = \frac{1}{2^n n!} D^n(x^2 - 1). \]

1.3 Hermite Polynomials. Hermite polynomials, first of all were discussed by Laplace in his two works: “Treatise on Celestial Mechanics” ([109], 1805) and “Theory of Probability” ([110], 1820). The systematic study of these polynomials was made by C.H. Hermite [93]. Hermite polynomials occur in case of the motion of the point mass in a field of force.

Generating function for Hermite polynomials is given by Rainville ([125],p.187, (1))

\[ \exp(2xt-t^2) = \sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!} \]

and their Rodrigues’ formula is given by Rainville ([125,p.189, (2)])

\[ H_n(x) = (-1)^n \exp(x^2) D^n \exp(-x^2). \]
1.4 Laguerre Polynomials. E. de Laguerre [108] introduced Laguerre polynomials in 1879. These polynomials occur in case of the motion of two particles (nucleus and electron) that are attached to each other by a force that depends only on the distance between them.

These polynomials satisfy the following differential equation:

\( x \frac{d^2}{dx^2} L_n^{(\alpha)}(x) + (1 + \alpha - x) \frac{d}{dx} L_n^{(\alpha)}(x) + nL_n^{(\alpha)}(x) = 0. \)

The generating function for Laguerre polynomials is given by Rainville ([125 p.209(1)])

\( \sum_{n=0}^{\infty} L_n^{(\alpha)}(x)t^n = (1-t)^{1-\alpha} e^{-xt/(1-t)} , \)

while their Rodrigues’ formula is given by Rainville [125, p.205(5)]

\( L_n^{(\alpha)}(x) = \frac{x^{-\alpha}e^x}{n!} D^n\left[e^{-x}x^{\alpha+n}\right] . \)

1.5 Other Polynomials. There are several hypergeometric polynomials which are non-orthogonal. In 1936 Bateman [9] was interested in constructing inverse Laplace transforms. For this purpose he introduced the polynomials

\( Z_n(x) = {}_2F_1(-n, n+1; 1; x) . \)

Rice [173] made a considerable study of the polynomials defined by

\( H_n(x) = {}_2F_2\left[-n, n+1, \xi, \eta; 1\right] . \)

Bateman [8] studied the polynomials

\( F_n(\zeta) = {}_3F_2\left[-n, n+1, \frac{1}{2}(1+\zeta); 1, 1; \zeta\right] \)

quite extensively, and which were generalized by Pasternak in the following way:
(5)

\[(1.5.4) \quad F_n^{(m)}(z) = F\left[ \begin{array}{c} -n, n+1, \frac{1}{2}(1+z+n), \\ 1, 2, m+1 \end{array} \mid z \right]. \]

Another polynomial, in which the interest is concentrated on a parameter, is Mittag-Leffler polynomial

\[(1.5.5) \quad g_n(z) = 2z \, _2F_1[1-n, 1-z; 2] \]

Bateman (1940) generalized the above polynomials in the form:

\[(1.5.6) \quad g_n(z, r) = \frac{(-r)_n}{n!} \, _2F_1(-n, z; -r; 2). \]

Sister Celine (Fesenmyer [101]) concentrated on the polynomials generated by

\[(1.5.7) \quad (1-t)^{-1} \, _pF_q\left[ \begin{array}{c} a_1, \ldots, a_p; \\ b_1, \ldots, b_q \end{array} \mid -4xt/(1-t)^2 \right] = \sum_{n=0}^{\infty} \, _{p+2}F_{q+2}\left[ \begin{array}{c} -n, n+1, a_1, \ldots, a_p; \\ 1, 1/2, b_1, \ldots, b_q \end{array} \mid x \right] t^n. \]

Her polynomials include Legendre polynomials, some special Jacobi, Rice’s \(H_n(\zeta, p, v)\), Bateman’s \(Z_n(x)\), \(F_n(z)\), and Pasternak’s polynomials etc. as special cases.

**1.6 Hypergeometric Function of One Variable. The Gaussian Hypergeometric Series.** In the study of second order linear differential equations with three regular singular points there arises the function

\[(1.6.1) \quad _2F_1(a, b; c; z) = _2F_1\left[ \begin{array}{c} a, b; \\ c \end{array} \mid z \right] = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \, \frac{z^n}{n!}, c \neq 0, -1, -2, \ldots. \]

The above infinite series obviously reduces to the elementary geometric series
(1.6.2) \( \sum_{n=0}^{\infty} z^n = 1 + z + z^2 + \ldots + z^n + \ldots \)

(1.6.3) (i) \( a = c \) and \( b = 1 \) (ii) \( a = 1 \) and \( b = c \).

Hence it is called hypergeometric series or more precisely, Gauss's hypergeometric series after the famous German mathematician Carl Friedrich Gauss (1777-1855), who in the year 1812 introduced this series into analysis and gave the \( F \)-notation for it.

By D'Alembert's ratio test, it is easily seen that the hypergeometric series in (1.6.1) converges absolutely within the unit circle, that is, when \( |z| < 1 \), provided that the denominator parameter \( c \) is neither zero nor negative integer. However, we notice if either or both of the numerator parameters \( a \) and \( b \) in (1.6.1) is zero or negative integer, the hypergeometric series terminates and the series is automatically convergent. Further tests readily show that the hypergeometric series in (1.6.1) when \( |z| = 1 \) (that is, on the unit circle), is

i) absolutely convergent if \( \Re(c-a-b) > 0 \),

ii) conditionally convergent if \( -1 < \Re(c-a-b) \leq 0 \), \( z \neq 1 \).

iii) divergent if \( \Re(c-a-b) \leq -1 \).

In case (i), for a number of summation theorems for the hypergeometric series (1.6.1) when \( z \) takes on other special values, see Bailey ([7], 1935, pp. 9-11), Erdélyi et al. ([81], 1953, pp. 104-105), Slater ([138], 1966, p.243), Luke ([111], 1975, pp. 271-273) and Srivastava-Manocha ([154], 1984, pp.29-31).

**Generalized Hypergeometric Series.** A natural generalization of above Gaussian Hypergeometric series \( _2F_1(a,b;c;z) \) is accomplished by introducing any arbitrary number of numerator and denominator parameters. The resulting series

(1.6.4) \( \sum_{n=0}^{\infty} \frac{(a_1)_n\ldots(a_p)_n}{(b_1)_n\ldots(b_q)_n} \frac{z^n}{n!} = _rF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z) \)
(7) is known as the generalized Gauss series, or simply, the generalized hypergeometric series. Here \( p \) and \( q \) are positive integers or zero (interpreting an empty product as 1), we assume that the variable \( z \), the numerator parameters \( a_1, \ldots, a_p \) and denominator parameters \( b_1, \ldots, b_q \) take on complex values, provided that

\[(1.6.5) \ b_j \neq 0, -1, -2, \ldots; \ j = 1, \ldots, q.\]

Supposing that none of the numerator parameters is zero or negative integer (otherwise question of convergence will not arise, and with usual restriction (1.6.5) the \( _p F_q \) series in (1.6.4)

(i) converges for \( |z| < \infty \) if \( p \leq q \)
(ii) converges for \( |z| < 1 \) if \( p = q + 1 \) and
(iii) diverges for all \( z, z \neq 0 \), if \( p > q + 1 \)

Further more, if we set

\[(1.6.5) \ \omega = \sum_{j=1}^{q} b_j - \sum_{j=1}^{p} a_j,\]

then the series \( _p F_q \) with \( p = q + 1 \), is

(i) absolutely convergent for \( |z| = 1 \) if \( Re(\omega) > 0 \),
(ii) conditionally convergent for \( |z| = 1, \ z \neq 1 \) if \( -1 \leq Re(\omega) \leq 0 \).
(iii) divergent for \( |z| = 1 \), if \( Re(\omega) \leq -1 \).

1.7 A Further Generalization of \( _p F_q \). An interesting further generalization of the series \( _p F_q \) is due to Fox [87] and Wright ([158], [159]), who studied asymptotic expansion of the generalized hypergeometric function defined by

\[(1.7.1) \ _p \Psi_q \left[ \begin{array}{c} (a_1, A_1), \ldots, (a_p, A_p) \\ (b_1, B_1), \ldots, (b_q, B_q) \end{array} \right] z = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p} (a_j + A_j n)}{\prod_{j=1}^{q} (b_j + B_j n)} \frac{z^n}{n!} \]

where the coefficients \( A_1, \ldots, A_p \) and \( B_1, \ldots, B_q \) are positive real numbers such
that

\[(1.7.2) \quad 1 + \sum_{j=1}^{p} B_j - \sum_{j=1}^{q} A_j \geq 0.\]

By comparing (1.6.4) and (1.7.1), we have

\[(1.7.3) \quad \psi_q^p \left[ (a_1,1), \ldots, (a_p,1); \frac{\prod_{j=1}^{p} (a_j)}{q \prod_{j=1}^{q} (b_j)} \right] = F_q^p \left[ a_1, \ldots, a_p; \frac{b_1, \ldots, b_q}{z} \right].\]

### 1.8 Hypergeometric Series in Two Variables.

The great success of the hypergeometric series in one variable has stimulated the development of a corresponding theory in two or more variables. Appell [4] has defined four double hypergeometric series $F_1$, $F_2$, $F_3$, $F_4$ (known as Appell series), analogous to Gauss's $\,_{2}F_{1}(a,b;c;z)$. The standard work on the theory of Appell series is the monograph by Appell and Kampé de Fériet [6], which contains an extensive bibliography of all relevant papers up to 1926 (by for example, L. Pochhammer, J. Horn, E. Picard, E. Goursat). See Erdélyi et al. [81, pp. 222-245] for a review of a subsequent work on the subject; see also Bieley ([7], Chapter 9), Slater ([138], Chapter 8) and Exton ([99] pp. 23-28). Horn puts

\[ f(m,n) = \frac{F(m,n)}{F'(m,n)}, \quad g(m,n) = \frac{G(m,n)}{G'(m,n)}, \]

where $F, F', G, G'$ are polynomials in $m, n$ of respective degrees $p, p', q, q'$. $F'$ is assumed to have factor $m+1$, and $G'$ a factor $n+1$; $F$ and $F'$ have no common factor except possibly, $m+1$; and $G$ and $G'$ have no common factor except possibly $n+1$. The greatest of the four numbers $p, p', q, q'$ is the order of the hypergeometric series. Horn investigated, in particular, the hypergeometric series order two and found that, apart from certain series which are either expressible in terms of
one variable or are products of two hypergometric series, in one variable, there are essentially thirty four distinct convergent series of order two (Horn [95], correction in Borngässer [14]).

**Horn Series.** Horn [95] defined the ten hypergeometric series in two variables and denoted them by \( G_1, G_2, G_3, H_1, \ldots, H_7 \); he thus completed the set of all fourteen possible second order (complete) hypergeometric series Appell and Kampé de Fériet ([6], p.143 et seq.), see also Erdélyi et al. ([81], pp. 224-228).

**Cofluent Hypergeometric Series in Two Variables.**

Seven confluent forms of the four Appell series were defined by Humbert [96] and he denoted these confluent hypergeometric series in two variables by \( \phi_1, \phi_2, \phi_3, \psi_1, \psi_2, \psi_3, E_1, E_2 \).

In addition, there exist thirteen confluent forms of the Horn series which are denoted by Horn [95] and Borngässer [14] \( \Gamma_1, \Gamma_2, H_1, \ldots, H_{11} \). Thus there are *twenty* possible confluent hypergeometric series in two variables.

The work of Humbert has been described reasonably fully by Appell and Kampé de Fériet ([6], pp. 124-135), and the definitions and convergence conditions of all these twenty confluent hypergeometric series in two variables are given also in Erdélyi et al. ([81], pp. 225-228).

For more details see Srivastava and Karlsson [155].

**Kampé de Fériet Series and its Generalization.** Just as the Gaussian series \( _2F_1 \) was generalized to \( _pF_q \) by increasing the number of numerator and denominator parameters, the four Appell series were unified and generalized by Kampé de Fériet [103], who defined a general hypergeometric series in two variables (see Appell and Kampé de Fériet [6, p.150 (29)]). The notation introduced by Kampé de Fériet [loc. cit]
for his double hypergeometric series of superior order was subsequently abbreviated by Burchnall and Chaundy ([15], p.112).

A further generalization of the Kampé de Fériet series is due to Srivastava and Daoust ([147], 1969), who indeed defined the extension of the \( p \psi_q \) series (1.8.3) in two variables.

Later on in 1976, a generalization of Kampé de Fériet series is also seen in the literature due to Srivastava and Panda ([153], p.423,(26)) but it is special case of Srivastava and Daoust ([147], 1969).

1.9 Triple Hypergeometric Series.

Lauricella [107, p. 114] introduced fourteen complete hypergeometric series in three variables of the second order. He denoted his triple hypergeometric series by the symbols \( F_1, F_2, F_3, \ldots, F_{14} \) of which four series \( F_1, F_2, F_3 \) and \( F_5 \) correspond respectively to the three variable Lauricella series \( F_A^{(3)}, F_B^{(3)}, F_C^{(3)} \) and \( F_D^{(3)} \).

The remaining ten series \( F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, \ldots, F_{14} \) of Lauricella’s set apparently fell into oblivion except that there is an isolated appearance of the triple hypergeometric series \( F_8 \) in a paper by Mayr [115, p.265] who came across this series while evaluating certain infinite integrals. Saran [128] initiated a systematic study of these ten triple hypergeometric series of Lauricella’s set. Saran’s notations are \( F_E, F_F, F_G, F_K, F_M, F_N, F_P, F_R, F_S \) and \( F_T \) for the series \( F_4, F_{14}, F_8, F_3, F_{11}, F_0, F_{12}, F_{10}, F_7, F_{13} \) respectively (see also Chandel [22]).

Srivastava Triple Hypergeometric Series \( H_A, H_B \) and \( H_C \):

In the course of further investigation of Lauricella’s fourteen hypergeometric series in three additional complete triple hypergeometric series of the second order. These three series \( H_A, H_B, H_C \) had been neither included in the Lauricella’s set, nor were they previously mentioned in the literature. \( H_C \) is new and interesting generalization of Appell’s series.
(11)

$F_1$, $H_B$ generalizes the Appell series $F_2$, while $H_A$ provides a generalization of both $F_1$ and $F_2$.

A unification of Lauricella’s fourteen hypergeometric series $F_1, \ldots, F_{14}$ and the additional series $H_A$, $H_B$, $H_C$ was introduced by Srivastava [140, p.428], who defined general triple hypergeometric series.

While transforming Pochhammer’s double-loop contour integrals associated with the series $F_8$ and $F_{14}$ (i.e. $F_G$ and $F_F$ respectively) belonging to Lauricella’s set of hypergeometric series in three variables, the two interesting triple hypergeometric series $G_A$, $G_B$ of Horn’s type were encountered by Pandey ([122], pp.115-116). An investigation of the system of partial differential equation associated with the triple hypergeometric series $H_C$ of Srivastava ([139], [141]) led Srivastava [145, p.105 (3.5)] to new series $G_C$. Other triple hypergeometric series studied in the literature are introduced by Dhawan [79], Samar [127] and Exton ([85]).

1.10 The Quadruple Hypergeometric Functions.

Until the Exton [83] defined and examined a few of their properties, no specific study had been made of any hypergeometric function of four variables apart from the four Lauricella’s function $F_A^{(4)}, F_B^{(4)}, F_C^{(4)}$ and $F_D^{(4)}$ certain of their limiting cases. On account of the large number of such functions which arise from a systematic study of all the possibilities he restricted himself to those functions which are complete and of the second order and which involve at least one product of the type $(a, k+m+n+p)$, in series representation; $k, m, n$ are indices of quadruple summation. Exton ([83], [84]) defined twenty one quadruple hypergeometric series. (see also Chandel and Dwivedi [47]).

Recently Sharma and Parihar [132] introduced eighty three hypergeometric functions of four variables. It is worthy to note that out
of these eighty three functions, nineteen functions had already been included in the set of 21 functions introduced by Exton ([83], [84]) in different notations (see, Remark due to Chandel and Kumar [58]). Further very recently Chandel, Agrawal and Kumar [36] have also introduced seven more hypergeometric functions of four variables:

1.11 Multiple Hypergeometric Series of Several Variables.

While several authors, for example, Green [88], Hermite [94] and Dedon [80] have discussed what amount to certain specified hypergeometric functions. It was left to Lauricella [107] to approach this topic systematically. Beginning with the Appell functions Lauricella proceeded to define and study the four important functions \( F_A^{(n)} \), \( F_B^{(n)} \), \( F_C^{(n)} \) and \( F_D^{(n)} \) which bear his name.

A number of confluent forms of the above Lauricella’s functions denoted by \( \phi_2^{(n)} \) and \( \psi_2^{(n)} \) exist in the literature (for instance see Erdélyi [82,p.446 (7.2)]; Humbert [98,p.429], see also Appell and Kampé de Fériet [6,p.134 (34)].

Exton ([84]) introduced two multiple hypergeometric series \( (k)_l E_D^{(n)} \) and \( (k)_l E_D^{(n)} \) related to Lauricella’s \( F_D^{(n)} \)

Prompted by this work, Chandel [25] defined and studied the multiple hypergeometric function \( (k)_l E_C^{(n)} \) closely related to Lauricella’s \( F_C^{(n)} \):

**Intermediate Lauricella’s Functions.**

By taking commendable idea of interpolation between Lauricella’s function, Chandel and Gupta [52] introduced three multiple hypergeometric functions \( (k)_l F_{AC}^{(n)} \), \( (k)_l F_{AD}^{(n)} \) and \( (k)_l F_{BD}^{(n)} \) related to Lauricella’s functions.

Chandel and Gupta [52] also introduced five confluent forms of
their above series: 

\[
\phi_{AC}^{(a)}(\theta_1, \ldots, \theta_{AC}) \phi_{AD}^{(a)}(\theta_1, \ldots, \theta_{AD}) \phi_{BD}^{(a)}(\theta_1, \ldots, \theta_{BD})
\]

Prompted by this work Karlsson [100] also introduced the fourth possible intermediate Lauricella function \( (k)F_{CD}^{(n)} \).

Recently, Chandel and Vishwakarma ([76],[77],[78]) introduced and studied many confluent forms of the above series.

### 1.12 Generalization of Lauricella's Series

An interesting unification and generalization of Lauricella's multiple series \( F_A^{(n)} \) and \( F_B^{(n)} \) and Horn's double series \( H_2 \) was considered by Erdélyi (1939). He denoted his series by \( H_{n,p} \).

Srivastava and Daoust [148, p. 454] (also see Srivastava and Manocha [154, p.64, (18),(19),(20)]) considered a multivariable extension of the series \( _p\psi_q \) defined by (1.8.3). Their multiple hypergeometric series, known as the generalized Lauricella series in several variables is defined as

\[
S_{(A';\ldots;B')_{(a)}}^{(A:B';\ldots;B')_{(a)}} \left[ \begin{array}{c} x_1 \\ \vdots \\ x_n \end{array} \right] = S_{(A';\ldots;B')_{(a)}}^{(A:B';\ldots;B')_{(a)}} \left[ \begin{array}{c} (a): \theta_1', \ldots, \theta_{(a)}' \\ (b'): \phi_1', \ldots, \phi_{(b')}, \delta_1', \ldots, \delta_{(b')}, \alpha_1', \ldots, \alpha_{(a')}, \gamma_1', \ldots, \gamma_{(a')}, \beta_1', \ldots, \beta_{(a')} \end{array} \right] x_1, \ldots, x_n
\]

\[
= \sum_{m_1,\ldots,m_n=0}^{\infty} \prod_{j=1}^{A} (a_j + \sum_{i=1}^{n} m_i \theta_j^{(i)}) \prod_{j=1}^{B'} (b_j' + m_i \phi_j') \prod_{j=1}^{B} (b_j^{(a)} + m_i \phi_j^{(a)}) \\
\prod_{j=1}^{C} (c_j + \sum_{i=1}^{n} m_i \psi_j^{(i)}) \prod_{j=1}^{D'} (d_j' + m_i \delta_j') \prod_{j=1}^{D} (d_j^{(a)} + m_i \delta_j^{(a)}) \\
\frac{x_1^{m_1}}{m_1!} \ldots \frac{x_n^{m_n}}{m_n!}
\]
or alternatively by

\[
F_{A;B;\ldots;E}^{C;D;\ldots;F} \left[ \begin{array}{c}
(a) : \theta_1, \ldots, \theta_n \\
(b') : \phi_1', \ldots, (b^{(n)})' : \phi_n'
\end{array} \right] = \left[ \begin{array}{c}
(c) : \psi_1', \ldots, \psi_n' \\
(d') : \delta_1', \ldots, (d^{(n)})' : \delta_n'
\end{array} \right] : x_1, \ldots, x_n
\]

\[
= \sum_{m_1, \ldots, m_n=0}^{\infty} \frac{A}{m_1! \ldots m_n!} \prod_{j=1}^{\infty} \frac{(a_j + m_1 \theta_j + \ldots + m_n \theta_j^{(n)}) \prod_{j=1}^{b'} (b'_j + m_1 \phi'_j + \ldots + m_n \phi'_j) \ldots \prod_{j=1}^{b^{(n)}'} (b^{(n)}_j + m_1 \phi^{(n)}_j + \ldots + m_n \phi^{(n)}_j)}{c_j + m_1 \psi_j(i) \ldots (d'_j + m_1 \delta_j(i) + \ldots + m_n \delta_j^{(n)}(i))}
\]

where

\[
\theta_j(i), j = 1, \ldots, A; \phi_j(i), j = 1, \ldots, B; \psi_j(i), j = 1, \ldots, C;
\]

\[
\delta_j(i), j = 1, \ldots, D; 1 \leq i \leq n;
\]

are real and positive and (a) is taken to abbreviate the sequence of \( A \) parameters \( a_1, \ldots, a_A \); \( b(i) \) abbreviates the sequence of \( B(i) \) parameters \( b_1(i), \ldots, b_{B(i)}(i) \), \( i = 1, \ldots, n \); with similar interpretations for \( (c) \) and \( (d(i)) \), \( i = 1, \ldots, n \);

For \( n=2 \), the above series reduces to the series defined by Srivastava and Daoust [147].

This multiple hypergeometric function will be used in the chapter 8 of this thesis.

**Extension of Most Generalized Hypergeometric Function of Srivastava and Daoust.** As natural further generalization of the (Srivastava-Daoust) generalized Lauricella function of several complex variables [148], \( H \)-function of two variables of Mittal-Gupta [118] and \( G \)-function of two variables of Agarwal [2] (also see Chandel-Agrawal [32]), is given by Srivastava and Panda ([151], p.271, (4.1); [146], p.
by means of the multiple contour integral
\begin{align*}
(1.12.4) \quad & H^{0,\lambda}_{\lambda, \mu} [a, v, \ldots, b, \ldots, c, d, \ldots, e, \ldots, f, \ldots] \left[\left\{ (a)_j : \theta_j, \ldots, \theta_{j'} \right\}, \left\{ (b)_j : \theta_j, \ldots, \theta_{j'} \right\}, \left\{ (c)_j : \psi_j, \ldots, \psi_{j'} \right\}, \left\{ (d)_j : \delta_j, \ldots, \delta_{j'} \right\}, \left\{ (e)_j : \delta_j, \ldots, \delta_{j'} \right\} \right], \quad z_1, \ldots, z_r \\
= & \frac{1}{(2\pi\omega)^r} \prod_{i=1}^r \int_{z_i} \phi_i(\xi_i) \phi_r(\xi_r) \psi(\xi_1, \ldots, \xi_r) z_1^{\xi_1} \ldots z_r^{\xi_r} d\xi_1 \ldots d\xi_r, \quad \omega = \sqrt{-1}
\end{align*}
where
\begin{align*}
(1.12.5) \quad & \phi_i(\xi_i) = \frac{\prod_{j=1}^{\lambda} \Gamma(a_j - \delta_j \xi_i) \prod_{j=1}^{\theta_j} \Gamma(1-b_j + \phi_j \xi_i)}{\prod_{j=1}^{\mu} \Gamma(1-a_j + \sum_{i=1}^{r} \theta_j \xi_i) \prod_{j=1}^{\gamma} \Gamma(b_j - \phi_j \xi_i)} \quad \forall i \in \{1, \ldots, r\}; \\
(1.12.6) \quad & \psi(\xi_1, \ldots, \xi_r) = \frac{\prod_{j=1}^{A} (1-a_j + \sum_{i=1}^{r} \theta_j \xi_i)}{\prod_{j=1}^{\lambda} (a_j - \sum_{i=1}^{r} \theta_j \xi_i) \prod_{j=1}^{C} (1-c_j + \sum_{i=1}^{r} \psi_j \xi_i)}
\end{align*}
an empty product is interpreted as 1, the coefficients $\theta_j, j=1, \ldots, A; \phi_j, j=1, \ldots, B; \psi_j, j=1, \ldots, C; \delta_j, j=1, \ldots, D$ $\forall i \in \{1, \ldots, r\}$ are positive numbers, and $\lambda, \mu, v, A, B, C, D$ are integers such that $0 \leq \lambda \leq A, 0 \leq \mu \leq D, C \geq 0, \text{ and } 0 \leq v \leq B, \forall i \in \{1, \ldots, r\}$. The contour $L_j$ in the complex $\xi_i$-plane is of the Mellin-Barnes type which runs from $-\omega \infty$ to $+\omega \infty$ with indentations, if necessary, in such a manner that all the poles of $\Gamma(a_j - \delta_j)$, $j=1, \ldots, \mu$, are to the right, and those of $\Gamma(1-b_j + \phi_j \xi_i)$, $j=1, \ldots, v$, and $\left(1-a_j + \sum_{i=1}^{r} \theta_j \xi_i\right)$, $j=1, \ldots, \lambda$, to the left, of $L_j$ the various parameters being so restricted that these poles are all simple and none
of them coincide; and with the points \( z_i = 0, \forall i \in \{1, \ldots, r\} \), being tacitly excluded, the multiple integrals in (1.12.1) converges absolutely if

\[
(1.12.7) \quad |\arg z_i| < \frac{1}{2} \pi \Delta_i, \forall i \in \{1, \ldots, r\},
\]

where

\[
(1.12.8) \quad \Delta_i = - \sum_{j=k+1}^{A} \theta_j^{(i)} + \sum_{j=1}^{\psi_j^{(i)}} + \sum_{j=1}^{\phi_j^{(i)}} + \sum_{j=1}^{\psi_j^{(i)}} + \sum_{j=1}^{\phi_j^{(i)}} + \sum_{j=1}^{\phi_j^{(i)}} > 0,
\]

\[\forall i \in \{1, \ldots, r\}.\]

The above function is most generalized function of several complex variables and it will be used in the Chapters VII of our thesis.

1.13 Generalization and unified presentation of polynomials. The orthogonal and non-orthogonal polynomials may be generalized in four ways; (i) by suitable generating function (ii) by Rodrigues’ formula (iii) by recurrence relation or (iv) by differential equation. In the present thesis we shall make appeal to technique (i) only.

(i) By Defining Suitable Generating Function.

The name “Generating Function” was first introduced by Laplace [110] in 1812. If a function \( F(x,t) \) has a power series (not necessarily convergent) expansion in \( t \), and it is of the form

\[
(1.13.1) \quad F(x,t) = \sum_{n=0}^{\infty} a_n f_n(x) t^n,
\]

where \( a_n; n=0,1,2,\ldots \) be specified sequence independent of \( x \) and \( t \) then \( F(x,t) \) is called generating function of \( f_n(x) \).

In the study of polynomial sets, there is a great importance of generating functions. For the use of generating functions we may refer to Sheffer [130], Brenke [13], Rainville [124], [125], Huff [99], Truesdell [158], Palas [121], Boas and Buck [12], Zeitlin [161] and Gould-Hopper [90] etc., Recently Mittal ([116], [117]) and Panda [123] have also
discovered many interesting and useful generating functions and operational generating functions for a large number of special functions (polynomials) of Laguerre, Hermite, Bessel, Jacobi etc.

Singhal and Srivastava [133] studied a class of bilateral generating functions for certain classical polynomials. Also Srivastava-Lavoie [150] and Srivastava [143] presented a systematic introduction to and several applications of general method of obtaining bilinear, bilateral or mixed multilateral generating functions for a fairly wide variety of special functions in one, two or more variables, Bhargava [10] used their theorems for obtaining some bilinear, bilateral and mixed multilateral generating functions. For more details of Generating functions see Chandel-Yadava ([72],[74],[77]), Chandel-Sahgal [60] and Srivastava and Manocha [154].

In 1947, Fasenmyer [86] studied the polynomials (called Sister Celine’s polynomials) generated by

\[
(1.13.2) \left(1-t \right)^{-1} _p F_q \left[ \begin{array}{l}
 a_1, \ldots, a_p; -4xt \\
 b_1, \ldots, b_q; (1-t)^2 
\end{array} \right] = \sum_{n=0}^{\infty} F_{n+2} \left[ \begin{array}{l}
 -n, n+1, a_1, \ldots, a_p; \\
 1/2, b_1, \ldots, b_q; x 
\end{array} \right] t^n.
\]

Her polynomials include as special cases the Legendre polynomials \( P_n(1-2x) \), Jacobi polynomials, Rice’s \( H_n(p,q,x) \), Bateman’s polynomials \( Z_n(x) \) and \( F_n(x) \). For generalized Rice polynomials see Chandel and Pal [59]. Chandel ([18] to [21]) studied the generalized Laguerre polynomials \( f_n^c(x,r) \) (and the polynomials related to them) defined by

\[
(1.13.3) \quad \left(1-t \right)^c \exp \left[ -\left(r/(1-t) \right) t \right] = \sum_{n=0}^{\infty} f_n^c(x,r) t^n.
\]

Agrawal [4] introduced the polynomials defined by

\[
(1.13.4) \quad \left(1-tr^q \right)^c \exp \left[ -\frac{r'xt}{(1-tr^q)^{r'}} \right] = \sum_{n=0}^{\infty} f_n^c(x;p,q,r) t^n
\]

and discussed the polynomials related to them.
Further Panda [123] generalized above polynomials through generating function:

\[(1-t)^{c} G\left(\frac{xt^s}{(1-t)^r}\right) = \sum_{n=0}^{\infty} g_n^c(x,r,s)t^n.\]

where \(c\) is an arbitrary parameter, \(r\) is any integer positive or negative, and \(s=1,2,3,\ldots\), and

\[G(z) = \sum_{n=0}^{\infty} \gamma_n z^n, (\gamma_0 \neq 0).\]

Further Sinha [135] (Also see Corrigendum due to Chandel [27]) studied special case of \(g_n^c(x,r)\), when \(\gamma_n = \frac{1}{n!}, \gamma_0 = 1.\)

For special interest Chandel and Bhargava ([40],[41]) studied an interesting special case of (1.13.5) when \(\gamma_n = \frac{(b)_n}{n!}.\)

\[(1-t)^{c}\left[1-xt^s/(1-t)^r\right]^{b} = \sum_{n=0}^{\infty} \Gamma_n^{(b,c)}(x,r,s)t^n\]

and introduced their associated polynomials. Chandel and Chandel [45] also introduced a new class of polynomials through their generating function

\[(1-t)^{c} G\left(\frac{xt}{(1-t^q)}\right) = \sum_{n=0}^{\infty} g_n^c(x,p,q,r,t^n.\]

and discussed their related polynomials.

The generalization of all polynomials of Louville [113] Legendre [112], Tchebycheff (see [156]), Gegenbaur [87(a)], Humbert [97], Pincherle (as stated in [97] and Kinney [104] led Gould [91] to define the polynomials, through generating function:

\[(c-mxt+yt^n)^p = \sum_{n=0}^{\infty} t^n P_n(m,x,y,p,C).\]

where \(m\) is positive integer and other parameters are unrestricted in general.

Srivastava [143] considered the class of generalized Hermite
polynomials defined by generating function

\[(1.13.9) \sum_{n=0}^{\infty} \gamma_n^{(m)}(x) \frac{t^n}{n!} = G(mxt - t^n).\]

For its special case \(G(z) = e^z\), see Chandel [24].

Chandel and Yadava [70] unified the study of above two classes (1.13.8) and (1.13.9) by considering the following generating function for certain polynomial systems:

\[(1.13.10) \quad G(C - mxt + yt^s) = \sum_{n=0}^{\infty} g_n(m, x, y, q, C) t^n.\]

Inspired by (1.13.6) and (1.13.8), Chandel and Bhargava [42] introduced a class of polynomials through generating function

\[(1.13.11) \quad \left[ (C - mxt + yt^m)^p \right] \left[ 1 - \frac{r' x t^s}{(C - mxt + y t^m)^p} \right]^{-q} = \sum_{n=0}^{\infty} B_n^{(p, q)}(m, x, y, r, s, C) t^n,
\]

where \(m, s\) are positive integers and other parameters are unrestricted in general. They also studied their related polynomials.

Further, to unify the study of four general classes (1.13.5), (1.13.7), (1.13.8) and (1.13.11) Chandel [28] introduced a class of polynomials through the generating function.

\[(1.13.12) \quad \left( C - mxt + y t^m \right)^p G \left[ \frac{r' x t^s}{(C - mxt + y t^m)^p} \right] = \sum_{n=0}^{\infty} R_n^{(p)}(m, x, y, r, s, C) t^n,
\]

and also discussed its special case when \(\gamma_n = (-1)^n / n!\).

Chandel and Dwivedi ([49],[50]) also considered polynomial systems through generating functions

\[
\left( C - mxt + y t^m \right)^p G \left[ \frac{r' z t^s}{(C - mxt + y t^m)^p} \right].
\]
and
\[ (C - mxt + y^m)^p \left[ \frac{r^xt^s}{(C - mxt + y^m)^r} \right]; \]
and discussed their special cases and related polynomials.

To further generalize (1.13.10), Chandel and Yadava [71] introduced some polynomial system of several variables by means of generating function

\[
(1.13.13) \quad G(a_0 + a_1x_1t + \ldots + a_nx_n t^n) = \sum_{n=0}^{\infty} A_{n,m}^{n_1, \ldots, n_m} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} t^n
\]

and discussed their special cases.

To further generalize (1.13.12) and the polynomials of Chandel and Dwivedi ([49],[50]), Chandel and Yadava [71] introduced a polynomial system of several variables through generating function

\[
(1.13.14) \quad \left( a_0 + a_1x_1t + \ldots + a_nx_n t^n \right)^p \left[ \frac{r^xt^s}{(a_0 + a_1x_1t + \ldots + a_nx_n t^n)^r} \right]
= \sum_{n=0}^{\infty} B_{n,m,p,r,s}^{n_1, \ldots, n_m} \begin{bmatrix} x_1, x_1 \\ \vdots \\ x_m \end{bmatrix} t^n
\]

and discussed their special cases.

Recently Chandel, Agrawal and Kumar [34] introduced a multivariable analogue of Gould-Hopper's polynomials [90], defined by generating function

\[
(1.13.15) \quad \sum_{m_1, \ldots, m_n=0}^{\infty} H_{m_1, \ldots, m_n}^{(b,m,v,p)}(x_1, \ldots, x_n) \frac{t_1^{m_1}}{m_1!} \ldots \frac{t_n^{m_n}}{m_n!}
\]
\[= \exp \left[ h(t_1^m + \ldots + t_n^m) \right] \left[ 1 + v(x_1 t_1 + \ldots + x_n t_n)^m \right] \]

and discussed is generalization through generating function

\[
(1.13.16) \quad \exp \left[ h(t_1^m + \ldots + t_n^m) \right] G[v(x_1 t_1 + \ldots + x_n t_n)]
\]

\[= \sum_{m_1, \ldots, m_n = 0}^{\infty} S_{m_1, \ldots, m_n}^{(h, m, v)} (x_1, \ldots, x_n) \frac{t_1^{m_1}}{m_1!} \ldots \frac{t_n^{m_n}}{m_n!}
\]

Recently Chandel and Sahgal [61] introduced a multivariable analogue of Panda's polynomials [123], through generating function

\[
(1.13.17) \quad (1-t_1)^{c_1} \ldots (1-t_m)^{c_m} \left[ 1 - \frac{x_1 t_1^{r_1}}{(1-t_1)^{r_1}} \ldots \frac{x_m t_m^{r_m}}{(1-t_m)^{r_m}} \right]^{-b}
\]

\[= \sum_{n_1, \ldots, n_m = 0}^{\infty} G_{n_1, \ldots, n_m} (b, c_1, \ldots, c_m; n_1, \ldots, n_m; x_1, \ldots, x_m) t_1^{n_1} \ldots t_m^{n_m}
\]

where \( b, c_1, \ldots, c_m \) are any parameters, \( r_1, \ldots, r_m \) are any integers positive or negative while \( s_1, \ldots, s_m \) are positive integers.

They also considered its generalization through generating function.

\[
(1.13.18) \quad (1-t_1)^{c_1} \ldots (1-t_m)^{c_m} G \left[ \frac{x_1 t_1^{r_1}}{(1-t_1)^{r_1}} + \ldots + \frac{x_m t_m^{r_m}}{(1-t_m)^{r_m}} \right]
\]

\[= \sum_{n_1, \ldots, n_m = 0}^{\infty} G_{n_1, \ldots, n_m} (c_1, \ldots, c_m; n_1, \ldots, n_m; x_1, \ldots, x_m) t_1^{n_1} \ldots t_m^{n_m}
\]

and discussed other special cases.

Very recently, motivated by (1.13.12) and (1.13.18), Sengal [157] considered the polynomials \( \{ R_{n_1, \ldots, n_m} ([m], [x], [t], [r], [s], [C]) / n_i = 0, 1, 2, \ldots; i = 1, \ldots, n \} \) of several variables defined through generating function

\[
(1.13.19) \quad \left( C_1 - m_1 x_1 t_1 + y_1 t_1^{n_1} \right) \ldots \left( C_n - m_n x_n t_n + y_n t_n^{n_n} \right)^r
\]
(22)

\[
G \left[ \frac{t_1^{n_1} x_1^{s_1}}{(C_1 - m_1 x_1 + y_1 t_1^{m_1})^{n_1}} + ... + \frac{r_n^{s_n} x_n^{s_n}}{(C_n - m_n x_n + y_n t_n^{m_n})^{n_n}} \right]
\]

\[= \sum_{n_1, ..., n_n=0}^{\infty} R^{(p_1, ..., p_n)}_{n_1, ..., n_n} \left[ x_1^{n_1} [x \mid y \mid x \mid s \mid c] t_1^{n_1} ... t_n^{n_n} \right],\]

where \( m_i, s_i \ (i=1, ..., n) \) are positive integers and other parameters are unrestricted in general.

Recently, Chandel and Sahgal [62] introduced a multivariable analogue of Gould-Hopper’s polynomials [90] and Gould polynomials [91] through generating relation

(1.13.20)

\[
\sum_{n_1, ..., n_n=0}^{\infty} \mathcal{P}^{(m_1, ..., m_n; M_1, ..., M_r; h_1, ..., h_r; p)}_{n_1, ..., n_n}(x_1, ..., x_m) \frac{t_1^{n_1}}{n_1!} ... \frac{t_r^{n_r}}{n_r!}
\]

\[= \left( 1 + m_1 x_1 t_1 + h_1 t_1^{M_1} + ... + m_r x_r t_r + h_r t_r^{M_r} \right)^p,
\]

where \( M_1, ..., M_r \) are positive integers and \( m_1, ..., m_n, \) \( h_1, ..., h_r \) are any numbers real or complex independent of variables \( x_1, ..., x_r \). They also gave following generalization of (1.13.20) through generating relation

(1.13.21)

\[
\sum_{n_1, ..., n_n=0}^{\infty} \mathcal{G}^{(m_1, ..., m_n; M_1, ..., M_r; h_1, ..., h_r)}_{n_1, ..., n_n}(x_1, ..., x_m) \frac{t_1^{n_1}}{n_1!} ... \frac{t_r^{n_r}}{n_r!}
\]

\[= G(m_1 x_1 t_1 + h_1 t_1^{M_1} + ... + m_r x_r t_r + h_r t_r^{M_r}).\]

Tiwari [156a] gave another multivariable analogue of Gould and Hopper’s polynomials defined by generating relation

(1.13.22)

\[
\sum_{n_1, ..., n_n=0}^{\infty} \mathcal{H}^{(h_1, ..., h_r; m_1, ..., m_n; v_1, ..., v_r; p)}_{n_1, ..., n_n}(x_1, ..., x_m) \frac{t_1^{n_1}}{n_1!} ... \frac{t_r^{n_r}}{n_r!}
\]

\[= \exp[h_1 t_1^{m_1} + ... + h_r t_r^{m_r}] [(1 + v_1 x_1 t_1 + ... + v_r x_r t_r]^p,
\]
where all \(|t_i|<1\), \(h_p, v_i, k_i\) and \(p\) are any real or complex numbers independent of all variables \(x_i, \ldots, x_r\), while all \(m_i\) are non-negative integers; \(i=1, \ldots, r\).

She also gave its generalization, defined by generating relation.

\[
\sum_{n_1, \ldots, n_r=0}^{\infty} R^{(h_1, \ldots, h_r; m_1, \ldots, m_r; v_1, \ldots, v_r)}(x_1, \ldots, x_m) \frac{t_1^{m_1}}{n_1!} \cdots \frac{t_r^{m_r}}{n_r!} = \exp[h_1 t_1^{m_1} + \ldots + h_r t_r^{m_r}] G[v_1 x_1 t_1 + \ldots + v_r x_r t_r]
\]

where

\[
G(z) = \sum_{n=0}^{\infty} \gamma_n z^n \quad \gamma_n \neq 0
\]

Further motivated by above works Chandel and Tiwari [64] introduced another generalized multivariable analogue of Coulb and Hoppers polynomials [90], defined by generating relation:

\[
\sum_{n_1, \ldots, n_r=0}^{\infty} S^{(h_1, \ldots, h_r; m_1, \ldots, m_r; v_1, \ldots, v_r; p)}(x_1, \ldots, x_m) \frac{t_1^{m_1}}{n_1!} \cdots \frac{t_r^{m_r}}{n_r!} = [1 + v_1 x_1 t_1 + \ldots + v_r x_r t_r] p G(h_1 t_1^{m_1} + \ldots + h_r t_r^{m_r})
\]

where \(|t_i|<1\) and all parameters \(h_p, v_i, p\) are unrestricted in general but independent of all \(x_i\), while \(m_i\) are non-negative integers; \(i=1, \ldots, r\) and \(g(z)\) is given by (1.13.24).

Motivated by above work, in the present thesis we shall introduce multidimensional polynomials through their generating functions and study them in Chapters 2,3,4,5 and 6.

**1.15 Applications of Special Functions.** For applications of Special Functions in mathematical physics for mixed boundary value problems one may refer to Sneddon [134]. Chandel [29] discussed a mixed boundary value problem on heat conduction and determined the temperature at any point on the surface of sphere by solving dual series
equations involving the Legendre polynomials, Chandel-Bhargava [44], Chandel-Dwivedi [51] and Chandel-Yadava [73] discussed a problem on heat conduction employing generalized Kampé de Fériet function of Srivastava-Daoust [147], Srivastava's hypergeometric function of three variables [140], and multiple hypergeometric function of Srivastava-Daoust [148] respectively.


Further Chandel, Agrawal and Kumar [38] made application of Lauricella's $F_D^{(a)}$ in determining velocity coefficient of chemical reaction.

Chandel and Tiwari [65] employed multile hypergeometric function of Srivastava and Daoust ([147],[148]) to solve two boundary value problems on (1) heat conduction in a rod (ii) deflection of vibrating string under certain conditions. Recently, Chandel and Singh [67(a)] employed multivariable polynomials of Srivastava [146] and multivariable $H$-function of srivastava and Panda ([151],[152]) to solve two boundary
value problems under certain conditions.

Very recently Chandel and Sengar [66] have discussed two boundary value problems on heat conduction involving the product of multivariable $H$-function of Srivastava-Panda ([151],[152]) and several generalized polynomials of Srivastava [142] and their special cases have been discussed. Further Chandel and Sengar [67] have also discussed a problem on heat conduction under Robin condition involving the product of multivariable $H$-function ([151],[152]) and several generalized polynomials of Srivastava [142].

Motivated by above work, in the present thesis in the Chapter-7, we shall make applications of generalized polynomials of several variables of Srivastava [146], multivariable $H$-function of Srivastava and Panda ([151],[152]) in boundary value problems. In Chapter 8, we shall also employ Hermite polynomials, Srivastava polynomials of several variables [146] along with multiple hypergeometric function of several variables of Srivastava and Daoust [148] in a problem of heat conduction.

Chapter-2 of present thesis introduces generalized multivariable analogue of Gould and Hopper's polynomials defined through generating function.

Chapter-3 introduces generalized multivariable analogue of Gould and Gould-Hopper polynomials defined through generating function.

In Chapter-4, we introduce and study generalized polynomials system of several variables defined through generating function.

In Chapter-5, A multivariable analogue of Chandel polynomials is introduced and studied, while in Chapter-6, a multivariable analogue of Chandel and Yadava, polynomial is introduced.

In Chapter-7 application of the generalized Hermite and Srivastava polynomials of several variables of Srivastava and multivariable $H$-function of Srivastava and Panda are shown in boundary value problems.
In Chapter 8, we shall employ Hermite polynomials, Srivastava polynomials of several variables along with multiple hypergeometric function of Srivastava-Daoust in a problem of Heat conduction.

REFERENCES


(27)


(29)


[78] Chandel, R.C.S. and Vishwakarma, P.K., Fractional derivatives of


[87(a)] Gegenbauer, L., Uber de Bessels Chen functionen, Sitzungsberichte der mathematishnatuwissens Chaffichen classe der kaiserlichen, Academiedue Wissen Schaflen zu wien Zweite Abteilung, 69 (1874), 1-11.


(32)


[98] Humbert, P., La fonction $W_{k,\mu_1,\ldots,\mu_n}(x_1,\ldots,x_n)$, *C.R. Acad. Sci. Paris*, 171 (1920), 428-430.


(35)


