Chapter 1

Introduction

The ever-increasing need for communication capacity has brought the field of optics into focus for the past decades. Fiber optics is the only technique known today to have the power to meet the strong demands for flexibility and high bandwidth posed by the rapidly growing communication networks. The optical systems were primarily used in point-to-point long distance links [1, 2]. In the future, fiber optic networks will be routed directly into neighborhoods, households, and even to the back of each computer [3, 4]. In the more distant future, it is possible that even the signals bouncing between the different components inside the computer will be transmitted and received optically [5].

Today, optical network and signal processing requires converting the optical information into electrical signals, processing in the electronic domain, and converting back to the optical domain before retransmission. Such an operation requires detection, retiming, reshaping, and regeneration at each switching and routing point. This necessitates complex and expensive electronic and electro-optical hardware at each routing and switching node. The ability to perform signal processing operations entirely within the optical domain would eliminate the requirement of optical-electrical-optical conversions, while providing the
agility and speed inherent to optical elements. As optical fiber gradually replaces copper cables, it will become necessary for many of the electronic network components to be replaced by equivalent optical components: such as splitters, filters, routers, switches etc.

In order to have all such optical components to be compact, manufacturable, low-cost, and integratable, it is highly desirable that they be fabricated in an optical fiber. The inline optical fiber based component demonstrated the tremendous potential applications over the optical components fabricated on a semiconductor planar surface using lithographic techniques. Fiber Bragg grating offers one possible solution for constructing inline optical devices to fulfill above requirements. Advantages of fiber Bragg grating over competing semiconductor material based optical components include all-fiber geometry, low insertion loss, low absorption loss, low scattering loss, high return loss, and potentially low cost. Moreover, the most distinguishing feature of fiber Bragg grating is the flexibility it offers for achieving desired spectral characteristics. Numerous physical parameters can be varied including: induced index change, length, apodization, period chirp, fringe tilt and desired wavelength.

1.1 Fiber Bragg Grating Theory

A Fiber Bragg Grating (FBG) is a periodic variation of the refractive index within the core of the optical fiber optic along its length. The principal property of FBGs is that they reflect light in a narrow bandwidth that is centered about the Bragg wavelength $\lambda_B$. A narrow band of the incident optical field within the fiber is reflected by successive, coherent scattering from the index variations. When the reflection from a crest in the index modulation is in phase with the next one, we have maximum reflection. The fiber Bragg grating structure is simply an optical
diffraction grating, which means that when a light wave incident on the grating
at an angle $\theta_i$, it is diffracted at an angle $\theta_r$ such that

$$\sin \theta_i - \sin \theta_r = \frac{m\lambda}{n_0 \Lambda} \tag{1.1}$$

Where $\Lambda$ is the grating period, $\lambda$ is the wavelength of light inside the medium, $n_0$ is the average refractive index and integer $m$ determines the diffraction order. This phase matching condition can be written as [20]

$$k_i - k_d = mk_g \tag{1.2}$$

Where $k_i$ and $k_d$ are the wave vectors associated with the incident and diffracted light. The grating wave vector $k_g$ has magnitude $2\pi / \Lambda$ and points in the direction along which the refractive index of the medium changes in a periodic manner. In the case of single mode fiber, all three vectors lie along the fiber axis. As a result $k_d = -k_i$ and the diffracted light propagates backward. Thus as shown schematically in Fig 1.1, a fiber grating acts as a reflector for a specific wavelength of light for which the phase matching condition is satisfied.

![Figure 1.1: Schematic illustration of a fiber Bragg grating. Dark and light shaded regions within the fiber core show periodic variations of the refractive index.](image-url)
In equation (1.1), if $\theta_i = \pi/2$, $\theta_r = -\pi/2$ and $m=1$, a resonance condition is obtained at a particular wavelength, known as Bragg wavelength, which is given by

$$\lambda_B = 2 n_0 \Lambda$$

(1.3)

Where $\lambda_B$ is the Bragg wavelength, $\Lambda$ is the grating period and $n_0$ is the effective refractive index of the transmitting medium. This condition is known as the Bragg condition. Physically, the Bragg condition ensures that weak reflections occurring throughout the grating add up in phase to produce a strong reflection.

![Diagram showing the application of a FBG as an optical filter.](image)

Figure 1.2: shows the application of a FBG as an optical filter. Light waves at several different wavelengths are traveling through the optical fiber and entering into the FBG. One of the wavelengths ($\lambda_B$) is reflected back by the FBG.
1.2 Historic Introduction

The first observations of refractive index changes were noticed in germanium-doped silica fibre and were reported by Hill et al. in 1978 [6, 7]. During an experiment that was carried out to study the nonlinear effects in an optical fibre, a visible light from an argon ion laser of wavelength at 488 nm was launched into the core of a specially designed fibre (heavily doped with germanium) and under prolonged exposure, an increase in the attenuation of the fibre was observed. In this observation, it was determined that the intensity of the light back reflected from the fibre increased significantly with time during the exposure. The physical phenomena behind this was explained as follows: The incident laser light interfere with the Fresnel reflected beam and initially form a weak standing-wave intensity pattern. The high intensity points alter the index of refraction in the fibre core permanently, forming a refractive index grating that had the same spatial periodicity as the interference pattern. This photorefractive effect in optical fibres is called photosensitivity and the refractive index grating is known as Hill’s grating. It acts as a distributed reflector that couples the forward propagating light to the counter-propagating light beams. The coupling of the beams provides positive feedback, which enhance the strength of the back-reflected light, and thereby increases the intensity of the interference pattern, which in turn increases the index of refraction at the high intensity point. This process is continued until the reflectivity of the grating reached a saturation level. This grating has a very weak index modulation, which was estimated to be of the order of $10^{-6}$, resulting in a narrow-band reflection filter at the writing wavelength.

Photosensitivity in optical fibres remained dormant for several years after its discovery, mainly due to limitations of the writing technique. During that time,
two significant results were attained. The first one was demonstrated in 1981, by Lam & Garside [8], where it was demonstrated that the magnitude of the photoinduced refractive index modulation depends on the square of the writing power at the argon ion wavelength. This suggested a two-photon process as the possible mechanism of refractive index change. The second result was reported in 1985 by Parent et al. [9] that the photoinduced change in the refractive index was anisotropic, despite the significance of the result was not appreciated immediately. Anisotropy is an unusual property of photosensitivity in optical fibres. It was demonstrated that the reflectivity of internally written gratings is found to depend on the polarization of the reading light beam, i.e., the refractive index measured with light polarized parallel to the writing beam’s direction of polarization is slightly different than that measured for light polarized perpendicular to the writing beam polarization. This photoinduced refractive index change is called birefringence.

Despite the potentialities of this new technology, few advances were made because photosensibility was found in a limited number of optical fibres highly doped with germanium. Besides, the spectral response of Hill’s gratings was limited to the writing beam wavelength as well as the writing fabrication technique.

In 1989, Meltz et al. [10] presented a new fabrication technology of Bragg gratings in the core of a germanium doped optical fibre by exposing the fibre externally from the side to an interference pattern in the UV spectral region. To form the interference pattern within the core of the fibre, an UV light beam from a laser was split into two beams that were interacted in the fibre core. The UV writing wavelength range was chosen to be 240-250 nm (nearly half the wavelength at 488 nm in argon laser). This wavelength is close to the absorption peak at ~240nm of an oxygen deficiency in atomic structure of the
optical fibre. This oxygen-deficient germanium defect is thought to be responsible for the photosensitivity in germanium doped silica fiber. The choice of UV wavelength was based on the fact that photosensitivity is a two photon absorption process in the visible region, and thus should be a one photon absorption process in the UV region. The interaction of two beams in the core of the optical fibre resulted in an interference pattern that would be converted, by photosensitivity, in core's refractive index spatial modulation, giving rise to diffraction gratings. The new external fabrication technique depends not only on the wavelength of the light used for writing, but also on the angle between the two interfering light beams. Thus, gratings can be written at any wavelength by simply adjusting the incidence angle.

The process of one photon absorption resulted in the increase of the photosensitivity mechanism efficiency, essentially due to the direct excitation of the absorption line at 244nm characteristic of germanium doped silica. This was an important step towards the development of different UV writing techniques, making possible flexible fabrication of fibre Bragg gratings.

1.3 Photosensitivity in Optical Fibers

There is considerable evidence that photosensitivity of optical fibers is due to defect formation inside the core of Ge-doped silica fibers [11-13]. It is known the optical fiber core is often doped with germania to increase its refractive index and introduce an index step at the core-cladding interface. The Ge concentration is typically 3–5%. The presence of Ge atoms in the fiber core leads to formation of oxygendifficient bonds (such as Si–Ge, Si–Si, and Ge–Ge bonds), which act as defects in the silica matrix [14]. The most common defect is the GeO defect. It forms a defect band with an energy gap of about 5 eV (energy required to break the bond). Single-photon absorption of 244-nm radiation from an excimer laser (or
two-photon absorption of 488-nm light from an argon-ion laser) breaks these defect bonds and creates GeE' centers. Extra electrons associated with GeE' centers are free to move within the glass matrix until they are trapped at hole-defect sites to form color centers known as Ge(I) and Ge(II).

![Diagram of GeO defect and GeE' hole center]

**Figure 1.3:** figure of bond structures for photo-sensitization and recombination process.

Such modifications in the glass structure change the absorption spectrum \( \alpha(\omega) \). However, changes in the absorption also affect the refractive index since \( \Delta \alpha \) and \( \Delta n \) are related through the Kramers–Kronig relation [15]

\[
\Delta n(\omega') = \frac{c}{\pi} \int_0^\infty \frac{\Delta \alpha(\omega)d\omega}{\omega^2 - \omega'^2}
\]

Even though absorption modifications occur mainly in the ultraviolet region, the refractive index can change even in the visible or infrared region. Moreover, since
index changes occur only in the regions of fiber core where the ultraviolet light is absorbed, a periodic intensity pattern is transformed into an index grating. Typically, index change $\Delta n \approx 10^{-4}$ in the 1.3 to 1.6 $\mu$m wavelength range, but can exceed 0.001 in fibers with high Ge concentration [16]. The presence of GeO defects is crucial for photosensitivity to occur in optical fibers. However, standard telecommunication fibers rarely have more than 3% of Ge atoms in their core, resulting in relatively small index changes. The use of other dopants such as phosphorus, boron, and aluminum can enhance the photosensitivity (and the amount of index change) to some extent, but these dopants also tend to increase fiber losses. It was discovered in the early 1990s that the amount of index change induced by ultraviolet absorption can be enhanced by two orders of magnitude ($\Delta n > 0.01$) by soaking the fiber in hydrogen gas at high pressures (200 atm) and room temperature [17]. The density of Ge–Si oxygen-deficient bonds increases in hydrogen-soaked fibers because hydrogen can recombine with oxygen atoms. Once hydrogenated, the fiber needs to be stored at low temperature to maintain its photosensitivity. However, gratings made in such fibers remain intact over long periods of time, indicating the nearly permanent nature of the resulting index changes [18]. Hydrogen soaking is commonly used for making fiber gratings. Some workers have reported that photosensitivity can be enhanced by doping of rare earth ions in fibers with holmium and thulium [19].
1.4 Fabrication Techniques of the Fiber Bragg Gratings

Fiber Bragg gratings are created by "inscribing" or "writing" systematic (periodic or aperiodic) variation of refractive index into the core of a special type of optical fiber using an intense ultraviolet (UV) source such as a UV laser. Fiber gratings can be made by using several different techniques, each having its own merits. The four major existing fabrication schemes for making optical fiber Bragg gratings are: 1) single-beam internal technique, 2) dual-beam holographic technique, 3) phase-mask photolithographic technique, and 4) point-by-point fabrication technique [14, 20]. The method that is preferable depends on the type of grating to be manufactured. Normally a germanium-doped silica fiber is used in the manufacture of fiber Bragg gratings. The germanium-doped fiber is photosensitive, which means that the refractive index of the core changes with exposure to UV light. The amount of the change depends on the intensity and duration of the exposure as well as the photosensitivity of the fibre. To write a high reflectivity fiber Bragg grating directly in the fiber the level of doping with germanium needs to be high.

1.4.1 Single-Beam Internal Technique

Over the years there have been several techniques developed to inscribe grating into optical fiber. The first of these was discovered by Hill et al in 1978 [6, 7] and is called the internal inscription technique (single-beam internal method). This method requires the use of an argon ion laser (514.5 or 488nm) to create a standing wave pattern in fiber core. The creation of the standing wave arises from
the incident light interfere with the reflection from the cleaved end of the fiber. The incident laser light interfered with the 4% reflection (from the cleaved end of the fiber) to initially form a weak standing wave intensity pattern within the core of the fiber. The points where constructive interference occurs have a higher intensity. These points create a permanent change of index of refraction and in turn, create a periodic perturbation to develop a Bragg grating. A schematic of this set-up is shown below in figure 1.4 with the argon light creating the standing wave pattern. While this method is one of the easier methods to create FBGs, the downfall of this technique is that Bragg wavelength is based upon incident laser wavelength and therefore fabrication techniques is limited. Also in order to obtain a useful reflection spectrum from this method, the grating needs to be quite long, typically on the order of a few tens of centimeters in length.

Figure 1.4: A typical apparatus used in generating self-induced Bragg gratings using an argon ion laser. Typical reflection and transmission characteristics of these types of gratings are shown in the graph.
1.4.2 Dual-Beam Holographic Technique

The dual-beam holographic technique, shown schematically in Fig. 1.5, makes use of an external interferometric scheme similar to that used for holography. Two optical beams, obtained from the same laser (operating in the ultraviolet region) and making an angle $2\theta$ are made to interfere at the exposed core of an optical fiber.

![Figure 1.5: Schematic illustration of the dual-beam holographic technique.](image)

A cylindrical lens is used to expand the beam along the fiber length. Similar to the single-beam scheme, the interference pattern creates an index grating. However, the grating period $\Lambda$ is related to the ultraviolet laser wavelength $\lambda_u$ and the angle $2\theta$ made by the two interfering beams through the simple relation

$$\Lambda = \frac{\lambda_u}{2 \sin \theta}$$

(1.4)

The most important feature of the holographic technique is that the grating period $\Lambda$ can be varied over a wide range by simply adjusting the angle $\theta$ (see figure). The wavelength $\lambda_a$ at which the grating will reflect light is related to $\Lambda$...
as $\lambda_B = 2n_0 \Lambda$. Since $\lambda_B$ can be significantly larger than $\lambda_m$, Bragg gratings operating in the visible or infrared region can be fabricated by the dual-beam holographic method even when $\lambda_m$ is in the ultraviolet region. In a 1989 experiment [10], Bragg grating reflecting 580-nm light was made by exposing the 4.4-mm long core region of a photosensitive fiber for 5 minutes with 244-nm ultraviolet radiation. Reflectivity measurements indicated that the refractive index changes were $\approx 10^{-5}$ in the bright regions of the interference pattern. Bragg grating formed by the dual-beam holographic technique were stable and remained unchanged even when the fiber was heated to 500°C.

Because of their practical importance, Bragg gratings operating in the 1.55-μm region were made in 1990 [21]. Since then, several variations of the basic technique have been used to make such grating in practical manner. An inherent problem for the dual-beam holographic technique is that it requires an ultraviolet laser with excellent temporal and spatial coherence. Excimer laser commonly used for this purpose have relatively poor beam quality and require special care to maintain the interference pattern over the fiber core over the duration of several minutes.

1.4.3 Phase-Mask Photolithographic Technique

This nonholographic technique uses a photolithographic process commonly employed for fabrication of integrated electronic circuits. The basic idea is to use a phase mask with a periodicity related to the grating period [22]. The phase mask acts as a master grating that is transferred to the fiber using suitable method. In one realization of this technique [23], the phase mask was made on a quartz substrate on which a patterned of chromium was deposited using electron-
beam lithography in combination with reactive-ion etching. Phase variations induced in the 242-nm radiation passing through the phase mask translate into a periodic intensity pattern similar to that produced by the holographic technique. Photosensitivity of the fiber converts intensity variations into an index grating of the same periodicity as that of the phase mask. The phase mask can also be used to form an interferometer using the geometry shown in figure 1.6.

![Figure 1.6: Schematic illustration of a phase mask interferometer used for making fiber gratings.](image)

The ultraviolet laser beam falls normally on the phase mask and is diffracted into several beams in the Raman–Nath scattering regime. The zeroth-order beam (direct transmission) is blocked or canceled by an appropriate technique. The two first-order diffracted beams interfere on the fiber surface and form a periodic intensity pattern. The grating period is exactly one-half of the phase mask period. In effect, the phase mask produces both the reference and object beams required for holographic recording. The chief advantage of the phase mask method is that the demands on the temporal and spatial coherence of the ultraviolet beam are
much less stringent because of the noninterferometric nature of the technique. In fact, even a nonlaser source such as an ultraviolet lamp can be used. Furthermore, the phase mask technique allows fabrication of fiber gratings with a variable period (chirped gratings) and can be used to tailor the periodic index profile along the grating length. It is also possible to vary the Bragg wavelength over some range for a fixed mask by using a converging or diverging wavefront during the photolithographic process [24]. On the other hand, the quality of fiber grating (length, informality, etc.) depends completely on the master phase mask, and all imperfections are reproduced precisely. Nonetheless, grating with 5-mm in length and 94% reflectivity were made in 1993, showing potential of this technique [23].

1.4.4 Point-by-Point Fabrication Technique

One of the more flexible methods in inscribing Bragg gratings into optical fibers is the point-by-point fabrication technique. The point-by-point technique [25] for fabricating Bragg gratings is accomplished by inducing a change in the index of refraction a step at a time along the core of the fibre. A focused single pulse from an excimer laser produces each grating plane separately. A single pulse of UV light from an excimer laser passes through a mask containing a slit. A focusing lens images the slit onto the core of the optical fiber from the side as shown in Fig. 1.7, and the refractive index of the core increases locally in the irradiated fiber section. The fiber is then translated through a distance $\Lambda$ corresponding to the grating pitch in a direction parallel to the fiber axis, and the process is repeated to form the grating structure in the fiber core. Essential to this technique is a very stable and precise submicron translational system.
The advantages of this procedure are that it allows a great deal of flexibility in the grating, including the length, pitch and response. Therefore, with the same setup it is possible to create different grating structures without having to replace any component. There are few practical limitations of this technique. First, only short fiber gratings (<1cm) are typically produced because of time-consuming nature of point-to-point method. Second, it is hard to control the movement of a translation stage accurately enough to make this scheme practical for long gratings. Third, it is not easy to focus the laser beam to a small spot size that is only a fraction of the grating period. Typically, the grating period required for first order reflection at 1550 nm is approximately 530nm. Because of the submicron translation and tight focusing required, first order 1550 nm Bragg gratings have yet to be demonstrated using the point-by-point technique. Malo et al. [25] have only been able to fabricate Bragg gratings, which reflect light in the 2nd and 3rd order, that have a grating pitch of approximately 1 µm and 1.5 µm, respectively.
1.5 Fiber Bragg Grating Model

In this section, we describe the theoretical model of fiber Bragg gratings (FBG) that is used in the thesis. The results from coupled-mode theory are briefly reviewed to analyze the reflection and transmission spectra of fiber gratings. Finally, we summarize the main properties of grating in linear regime such as photonic bandgap, reflection characteristics, transmission characteristics, phase and strength of the structure.

1.5.1 Coupled Mode Theory

In the previous section, we described the theory and fabrication techniques of fiber Bragg grating. We now turn to study the spectral characteristics and wave propagation in FBG. There are two important methods which have been adopted to study and analyze the reflection and transmission properties of FBG. The one method referred as Bloch formalism, used commonly for describing motion of electrons in semiconductors- is applied to Bragg gratings. Another method is known as coupled mode theory in which the forward and backward propagating waves are treated independently, and the Bragg grating provides a coupling between them. However, in this thesis we take coupled mode theory only into consideration since it is straightforward, intuitive and it accurately models the optical properties of fiber Bragg gratings. Coupled-mode theory is described in a number of texts; detailed analysis can be found in [20, 26-30]. The notation in this section follows most closely that of G. P Agrawal [20]. Throughout this thesis, we assume that the fiber is lossless and single mode in the wavelength range of interest. Moreover, we assume that the fiber is weakly guiding, i.e. the difference
between the refractive indices in the core and the cladding is very small. Then the electric and magnetic fields are approximately transverse to the fiber axis, and we can ignore all polarization effects due to the fiber structure and consider solely the scalar wave equation.

According to the coupled mode theory, the total field at any value of \( z \) can be written as a superposition of the two interacting modes and the coupling process results in a \( z \)-dependent amplitude of the two coupled modes. It is assumed that any point along the grating within the single-mode fiber has a forward propagating mode and a backward propagating mode. Thus the total field within the core of the fiber is given by

\[
\tilde{E}(z, \omega) = F(x, y)\left(\tilde{A}_f(z, \omega) \exp( i\beta_f z) + \tilde{A}_b \exp(-i\beta_b z)\right)
\]  

(1.5)

where \( \tilde{A}_f \) and \( \tilde{A}_b \) represents the amplitudes of the forward and backward propagating modes, respectively, \( \beta_b = \pi / \Lambda \) is the Bragg wave number for the first order grating. It is related to the Bragg wavelength through the Bragg condition \( \lambda_b = 2n_0\Lambda \) which can be used to define the Bragg frequency \( \omega_B = \pi c / n_0\Lambda \) and \( F(x, y) \) is the transverse modal field distribution. The total field given by Equation (1.5) has to satisfy the wave equation given by

\[
\nabla^2 \tilde{E} + n^2(\omega, z)\omega^2 / c^2 \tilde{E} = 0,
\]

(1.6)

In the above formula, \( n(\omega, z) \) denotes the total refractive index variation along the FBG and is given by

\[
n(\omega, z) = n_g(\omega) + n_2 |\tilde{E}|^2 + n_\gamma(z).
\]

(1.7)

Here, \( n_g \) is the average refractive index of the grating, \( n_2 \) is the Kerr coefficient and \( n_\gamma(z) \) is the periodic refractive index variation and \( \tilde{E} \) is the electric field propagating inside the grating. Substituting Equation (1.5) and Equation (1.7) into Equation (1.6) and considering a slowly-varying envelope approximation, we can obtain the following coupled mode equations in frequency domain [20]:

22
\[
\frac{\partial A_f}{\partial z} = i\left[\delta(\omega) + \Delta \beta\right]A_f + i\kappa A_b, \quad (1.8)
\]

and
\[
-\frac{\partial A_b}{\partial z} = i\left[\delta(\omega) + \Delta \beta\right]A_b + i\kappa A_f. \quad (1.9)
\]

Where \(\delta\) is a measure of detuning from the Bragg frequency and is defined as
\[
\delta(\omega) = n_0 c \left[\omega - \omega_B\right] = \beta(\omega) - \beta_B = 2\pi n_0 \left(\frac{1}{\lambda} - \frac{1}{\lambda_B}\right) \quad (1.10)
\]

The nonlinear effects are included through \(\Delta \beta\) defined as [31]
\[
\Delta \beta = k_0 \frac{\int \int_{-\infty}^{\infty} \Delta n|F(x, y)|^2 \, dx \, dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^2 \, dx \, dy} \quad (1.11)
\]

Here, \(k_0\) is the free space wave-vector and \(\kappa\) is the coupling coefficient which governs the grating-induced coupling between the forward and backward propagating waves. For a first-order grating, \(\kappa\) is given by
\[
k = k_0 \frac{\int \int_{-\infty}^{\infty} n_x |F(x, y)|^2 \, dx \, dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^2 \, dx \, dy} \quad (1.12)
\]

In this general form, \(\kappa\) can include transverse variations of \(n_g\) occurring when the photoinduced index change is not uniform over the core area. For a transversely uniform grating
\[
\kappa = \frac{\pi n_g}{\lambda_B} \quad (1.13)
\]

as can be inferred from Eq. (1.12) by taking \(n_g\) as constant and using \(k_0 = 2\pi/\lambda\) [20]. In time domain the coupled mode equations (1.8) and (1.9) can be given as
\[
\frac{\partial A_f}{\partial z} + \beta_1 \frac{\partial A_f}{\partial t} + i \beta_2 \frac{\partial^2 A_f}{\partial t^2} + \frac{\alpha}{2} A_f = i \delta A_f + i \kappa A_b + i \gamma |A_f|^2 + 2|A_b|^2 A_f
\]  
(1.14)

and

\[
- \frac{\partial A_b}{\partial z} + \beta_1 \frac{\partial A_b}{\partial t} + i \beta_2 \frac{\partial^2 A_b}{\partial t^2} + \frac{\alpha}{2} A_b = i \delta A_b + i \kappa A_f + i \gamma |A_b|^2 + 2|A_f|^2 A_b.
\]  
(1.15)

Here, \( \beta_1 = 1/v_g \) is related inversely to the group velocity, \( \beta_2 \) governs the group-velocity dispersion (GVD), and the nonlinear parameter \( \gamma \) is related to nonlinear refractive index \( n_2 \) as \( \gamma = 2m_2/\lambda_n \) in case of FBG having unit core area.

1.5.2 Solution of Coupled Mode Equation in Linear Case

Bragg gratings are useful because of their frequency or time-dependent nature. We will now examine this frequency dependence by solving the coupled mode equations which were developed in the previous section. In this section, we will focus on the linear case in which the nonlinear effects are negligible. For such case we neglected the nonlinear parameter \( \gamma \). After setting the nonlinear contribution \( \Delta \beta \) to zero in Equations (1.8) & (1.9), we obtain

\[
\frac{\partial A_f}{\partial z} = i \delta A_f + i \kappa A_b
\]  
(1.16)

and

\[
- \frac{\partial A_b}{\partial z} = i \delta A_b + i \kappa A_f
\]  
(1.17)

To solve these equations, let us differentiate Equations (1.16) & (1.17) with respect to \( z \)
\[
\frac{\partial^2 A_f}{\partial z^2} = i\delta \frac{\partial A_f}{\partial z} + i\kappa \frac{\partial A_b}{\partial z}
\] (1.18)

and

\[
-\frac{\partial^2 A_b}{\partial z^2} = i\delta \frac{\partial A_b}{\partial z} + i\kappa \frac{\partial A_f}{\partial z}
\] (1.19)

Substituting \(\frac{\partial A_b}{\partial z}\) and \(\frac{\partial A_f}{\partial z}\) in Equation (1.18) & (1.19) from equation (1.17) and (1.16), we found the differential equations in the form

\[
\frac{\partial^2 A_f}{\partial z^2} + (\delta^2 - \kappa^2) A_f = 0
\] (1.20)

and

\[
\frac{\partial^2 A_b}{\partial z^2} + (\delta^2 - \kappa^2) A_b = 0
\] (1.21)

Let \((\delta^2 - k^2) = q^2\)

\[
\frac{\partial^2 A_f}{\partial z^2} + q^2 A_f = 0
\] (1.22)

and

\[
\frac{\partial^2 A_b}{\partial z^2} + q^2 A_b = 0
\] (1.23)

A general solution of these linear equations takes the form

\[
A_f(z) = A_1 \exp(iqz) + A_2 \exp(-iqz)
\] (1.24)

and

\[
A_b(z) = B_1 \exp(iqz) + B_2 \exp(-iqz)
\] (1.25)

These equations show that \(z\) dependent parts of the forward and backward waves in the FBG are exponential with the propagation constant \(q\). This parameter \(q\) represents the linear dispersion relation of fiber Bragg grating and defined as

\[
q = \pm \sqrt{\delta^2 - k^2}
\] (1.26)

The constant \(A_1, A_2, B_1, B_2\) in Equations (1.24) & (1.25) are interdependent and by using Equations (1.24) & (1.25) we find that these constants satisfy the following four relations:
(q - \delta)A_1 = \kappa B_1 ; \quad (q + \delta)B_1 = -\kappa A_1 \quad (1.27)
(q - \delta)B_2 = \kappa A_2 ; \quad (q + \delta)A_2 = -\kappa B_2 \quad (1.28)

One can eliminate $A_2$ and $B_1$ by using Equations (1.24) to (1.28) and write the general solution in terms of an effective reflection coefficient $r(q)$ as

\[ A_f(z) = A_1 \exp(iqz) + r(q)B_2 \exp(-iqz) \quad (1.29) \]
and

\[ A_b(z) = B_2 \exp(-iqz) + r(q)A_1 \exp(iqz) \quad (1.30) \]

where

\[ r(q) = \frac{q - \delta}{\kappa} = -\frac{\kappa}{q + \delta} \quad (1.31) \]

is the effective reflection coefficient of the fiber Bragg grating. The $q$ dependence of $r(q)$ and the dispersion relation (1.26) indicate that both the magnitude and phase of the backward reflection depend on the frequency $\omega$.

1.5.3 Concept of Photonic Bandgap

The dispersion relation of Bragg gratings exhibits an important property known as the photonic bandgap as seen clearly in Fig. 1.8, where detuning parameter $\delta$ is plotted as a function of $q$ for both an uniform medium (dashed line) and a periodic medium (solid line). For the uniform medium the slope is constant, and thus the dispersion is negligible. By introducing a grating, the dispersion relation is modified and if the frequency detuning $\delta$ of the incident light falls in the range $-\kappa < \delta < \kappa$, $q$ becomes purely imaginary. Most of the incident field is reflected in that case since the grating does not support a propagating wave. The range $|\delta| \leq \kappa$ is referred to as the photonic bandgap. For this range of detuning, light cannot propagate through the grating and undergoes strong reflection. It is also called the stop band since light stops transmitting through the grating when its frequency falls within the photonic bandgap.
Figure 1.8: Dispersion curves showing variation of $\delta$ with $q$ and the existence of the photonic bandgap for a fiber grating.

If the frequency detuning $\delta$ of the incident light falls outside the range $-\kappa < \delta < \kappa$ i.e. (outside the stop band but close to its edges) then light will be transmitted through the grating. the effective propagation constant of the forward and backward propagating waves from Eqs. (1.5) and (1.24) is $\beta_e = \beta_\phi \pm q$, where $q$ is given by Eq. (1.26) and is a function of optical frequency through $\delta$. This frequency dependence of $\beta_e$ indicates that a grating will exhibit dispersive effects even if it was fabricated in a nondispersive medium. Expanding the parameter $\beta_e$ in a Taylor series around the carrier frequency $\omega_0$ of the wave, we get

$$\beta_e = \beta_0^\varepsilon + (\omega - \omega_0) \beta_1^\varepsilon + \frac{1}{2} (\omega - \omega_0)^2 \beta_2^\varepsilon + \frac{1}{6} (\omega - \omega_0)^3 \beta_3^\varepsilon + \ldots$$  (1.32)
where $\beta_{m}^{g}$ with $m = 1, 2, 3...$ is defined as [20]

$$\beta_{m}^{g} = \frac{d^{m}q}{d\omega^{m}} \approx \left(\frac{1}{v_{g}}\right)^{m} \frac{d^{m}q}{d\delta^{m}}. \quad (1.33)$$

Here, derivatives are evaluated at $\omega = \omega_{g}$. The superscript g correspond the grating.

![Figure 1.9](image)

**Figure 1.9**: Curve showing variation of group velocity $V_{G}$ with $\delta$ in a fiber Bragg grating.

In Eq. (1.33), $v_{g}$ is the group velocity of wave in the absence of the grating i.e ($\kappa = 0$). Consider first the group velocity of the wave inside the grating as $V_{G} = 1/\beta_{m}^{g}$ which can be calculated using Equation (1.33) and (1.26) as

$$V_{G} = \pm v_{g} \sqrt{1 - \kappa^{2}/\delta^{2}} = c/n_{0} \sqrt{1 - \kappa^{2}/\delta^{2}} \quad (1.34)$$

where the choice of $\pm$ signs depends on whether the wave is moving in the forward or the backward direction. Far from the band edges ($|\delta| >> \kappa$), optical wave is unaffected by the grating and travels at the group velocity expected in the absence of the grating. However, as $|\delta|$ approaches $\kappa$, the group velocity decreases...
and becomes zero at the two edges of the stop band where $|\delta|=\kappa$. The group velocity curve of Equation (1.34) is illustrated in Fig. 1.9, for both an uniform medium (dashed line) and a periodic medium (solid line). Figure shows that close to the photonic bandgap, an optical wave experiences considerable slowing down inside a fiber Bragg grating.

### 1.5.4 Spectral Response of Bragg Grating

The reflection and transmission coefficient of fiber Bragg grating can be calculated by using Eqs. (1.29) and (1.30) and applying the appropriate boundary conditions as

$$\begin{align*}
A_b(z=0) &= 1 \\
A_r(z=L) &= 0
\end{align*}$$

(1.35)

where $L$ is the length of the grating. Equation (1.35) implies that the incident wave has unit amplitude at $z=0$ and the amplitude of the reflected wave at $z=L$ is zero because there is no reflected wave beyond $z=L$. The boundary conditions applying on the FBG structure is shown in Figure 1.10.

![Figure 1.10: Schematic of a FBG of length L illuminated by electromagnetic field of amplitude $A(z)$.](image)

We defined the reflection coefficient of the FBG by the ratio of the amplitude of reflected wave at $z=0$ to the amplitude of incident wave at $z=0$ as
If we use the boundary condition $A_b(L) = 0$ in Eq. (1.30),

$$B_2 = -r(q)A_1 \exp(2iqL)$$ (1.37)

Using Equation (1.31) and Equation (1.37) in Eq. (1.36), we obtained the reflection coefficient as

$$r_g = \frac{i\kappa \sin(qL)}{q \cos(qL) - i\delta \sin(qL)}$$ (1.38)

The corresponding expression for the reflectivity $R_g = |r_g|^2$ in the linear regime is found as

$$R_g = \frac{\kappa^2 \sin^2(qL)}{q^2 \cos^2(qL) + \delta^2 \sin^2(qL)}$$ (1.39)

At resonance there is no detuning i.e $\delta = 0$, hence reflectivity is maximum. The expression for the reflectivity becomes:

$$R_{g_{\text{max}}} = |r_g(\delta = 0)|^2 = \tanh^2(\kappa L)$$ (1.40)

Varying some parameters such as grating length and magnitude of induced index change, it is possible to obtain narrow-band transmission as well as high reflectivity at the Bragg wavelength. Optimization of these parameters is fundamental when the objective is to use fibre Bragg gratings in band-pass filtering applications such as wavelength multiplexing/demultiplexing and add/drop optical filter.

Figure 1.11 shows the spectral response of the Bragg grating for the reflected wave considering five different Bragg gratings with value of $\kappa L = 0.5, 1.0, 2.0, 4.0$ and $8.0$. In all these cases we assumed effective index $n_0 \approx 1.451$, grating index $n_g \approx 0.5 \times 10^{-3}$ and Bragg wavelength $\lambda_B \approx 1550$ nm. From this figure, we can identify
two different operating regimes. Firstly, when the product $\kappa L$ is small compared to 1, the reflectivity of the Bragg gratings is minimum. Since the grating begins and ends abruptly and extends for a length $L$, the spectral response has a characteristic "sinc" shape whose bandwidth is inversely proportional to the grating length $L$. We refer to gratings with $\kappa L < 1$ as weak Bragg gratings, because in general they only reflect a fraction of the incident light. The peak reflectivity in this regime depends upon the value of $\kappa$, but the overall spectral shape and bandwidth is determined only by the grating length. A weak Bragg grating does not make a suitable add/drop filter, because it only partially reflects the input signal. However, there are cases where the "sinc" shaped spectral response of a weak grating is desirable. In many binary communications systems, the encoded signal has precisely at the same "sinc"-shaped spectral response. Thus, the weak Bragg grating provides a convenient implementation of an all-optical filter, which should provide the optimal signal-to-noise ratio for detecting the signal in the presence of white background noise [32-34]. Secondly, for gratings where the product $\kappa L$ exceeds 1, the spectral response has a plateau-like shape, as shown in Fig. 1.11. In this regime, the grating has a very high reflectivity within a band of frequencies called the stopband.

Outside of the stopband, the spectral response shows a series of ripples or sidelobes. The sidelobes quickly decay as we move away from the Bragg condition until the structure is effectively transparent. If the grating is made longer without changing the value of $\kappa$, the bandwidth remains unchanged, but the peak reflectivity rises closer to 1, the spectrum becomes more square, and the sidelobes get closer together. An undesirable feature seen in Figs. 1.11 is the presence of multiple sidebands located on each side of the stop band. These sidebands originate from weak reflections occurring at the two grating ends where the refractive index changes suddenly compared to its value outside the grating region. Even though the change in refractive index is typically less than
1%, the reflections at the two grating ends form a Fabry–Perot cavity with its own wavelength-dependent transmission. An apodization technique is commonly used to remove the sidebands [32].

\[
\beta \quad p \quad p \quad p \quad P \quad P \quad P \quad P \quad O \quad O \quad O \quad O \quad O \quad O \quad O \quad O
\]

\[\kappa L = 8.0\]
(Strong Bragg Grating)

\[\kappa L = 4.0\]

\[\kappa L = 2.0\]

\[\kappa L = 1.0\]

\[\kappa L = 0.5\]
(Weak Bragg Grating)

Figure 1.11: Calculated reflection spectral response as a function of wavelength for five different fiber Bragg gratings, with increasing value of \(\kappa L\).
Fig. 1.12 shows the dependence of peak reflectivity \( R_{\text{max}} \) on the grating length and refractive index change. It is clear, that it is possible to reach the same peak reflectivity with shorter gratings using fiber with high \( n_g \) values. That is very useful to find effective length of grating.

**Figure 1.12**: Peak reflectivity \( R_{\text{max}} \) as a function of Bragg grating length \( L \), calculated for different values of grating index \( n_g \) (1) \( 0.25 \times 10^{-3} \) (dashed dot curve), (2) \( 0.5 \times 10^{-3} \) (dashed curve), (3) \( 0.75 \times 10^{-3} \) (dotted curve) and (4) \( 1 \times 10^{-3} \) (solid curve).

### 1.5.5 Estimation of Bandwidth of Bragg Grating

Figure 1.11 shows the typical spectral response of an FBG. The peak reflectivity \( R_{\text{max}} \) at the Bragg wavelength \( \lambda_B \) is the most important parameter of interest in FBG design. For a given \( R_{\text{max}} \), \( \kappa L \) may be calculated from Equation (1.40). The quantity \( \kappa L \) is required in order to obtain the bandwidth (BW) of FBG's reflection.
spectrum (the difference in wavelengths between the first two reflection minima on either side of the main reflection peak). The reflection bandwidth, $\Delta \lambda$, for a uniform Bragg grating can be defined as [33]

$$\Delta \lambda = \frac{\lambda_B^2}{n_0 \pi L} \sqrt{\pi^2 + (\kappa L)^2}$$

(1.41)

<table>
<thead>
<tr>
<th>S. No.</th>
<th>FBG Length (mm)</th>
<th>Grating index ($n_*$)</th>
<th>$\approx \kappa L$</th>
<th>Max Reflectivity (%)</th>
<th>Bandwidth (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>$0.5 \times 10^{-3}$</td>
<td>0.5</td>
<td>22</td>
<td>3.35</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>$0.5 \times 10^{-3}$</td>
<td>1.0</td>
<td>59</td>
<td>1.75</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>$0.5 \times 10^{-3}$</td>
<td>2.0</td>
<td>93</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>4.0</td>
<td>$0.5 \times 10^{-3}$</td>
<td>4.0</td>
<td>99</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>8.0</td>
<td>$0.5 \times 10^{-3}$</td>
<td>8.0</td>
<td>100</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 1.1: Calculated bandwidth of a fiber Bragg grating reflector at different values of $\kappa L$.

It is important to note that for a given value of maximum reflectivity of fiber Bragg grating ($R_{\text{max}}$) and bandwidth ($\Delta \lambda$), the value of $\kappa L$ from Equation (1.40) can be substituted in (1.41) to determine the desired length $L$ of the FBG; thereafter, refractive index modulation $n_g$ can be estimated from Equation (1.13). All the calculations are given in Table-1.1 where the value of $n_0$ and $\lambda_B$ are assumed as 1.451 and 1550 nm, respectively. Table-1.1 reveals that, we can obtain a desired bandwidth of fiber Bragg grating by choosing proper values of grating length and grating index $n_g$.
1.5.6 Phase Response of Bragg Grating

In previous sections we have calculated the amplitude and bandwidth of fiber Bragg grating from the coupled mode theory. From Equation (1.38), it is observed that the reflection coefficient of FBG is a complex value and it can be written in phasor form as [34]

$$r_g(\omega) = |r_g| \times \exp[ i \phi_L(\omega)]$$  \hspace{1cm} (1.42)

where $|r_g|$ and $\phi(\omega)$ are amplitude and phase of FBG, respectively. In order to characterize a FBG completely, it is essential to know its amplitude and phase response. The amplitude and phase of the FBG can be calculated as

$$|r_g|^2 = R_g = (X_L^2 + Y_L^2)$$  \hspace{1cm} (1.43)

and

$$\phi_L = 2 \tan^{-1} \left( \frac{Y_L}{\sqrt{X_L^2 + Y_L^2 + X_L}} \right).$$  \hspace{1cm} (1.44)

Here, $X_L$ and $Y_L$ represent the real and imaginary part of the complex reflection coefficient and it is obtained using Equation (1.38) as

$$X_L = -\frac{\kappa \delta \sin^2(qL)}{q^2 \cos^2(qL) + \delta^2 \sin^2(qL)}, \quad Y_L = \frac{q \kappa \sin(qL) \cos(qL)}{q^2 \cos^2(qL) + \delta^2 \sin^2(qL)}$$  \hspace{1cm} (1.45)

The phase of the reflected beam at length $L$ in linear case can be written using tangent half angle formula, Equation (1.44) and (1.45) as

$$\phi_L = 2 \tan^{-1} \left( \frac{q \sin(2qL)}{2 \sqrt{\sin^2(qL) \sqrt{\delta^2 - \kappa^2 \cos^2(qL)} - 2 \delta \sin^2(qL)}} \right).$$  \hspace{1cm} (1.46)
Figure 1.13 shows the variation in (a) the real part ($X_L$) (b) the imaginary part ($Y_L$) and (c) the phase ($\phi_L$) as a function of wavelength for two values of grating strength $\kappa L = 1$ (dotted curve) and $\kappa L = 2$ (solid curve). It is clear that in the region outside of the stop band, the phase of the light changes according to the unperturbed material refractive index whereas inside the stop band, the phase decreases slowly with increasing strength of the grating.
1.6 Fiber Bragg Grating Under Nonlinear Regime

It was explained in the preceding sections that linear fiber Bragg grating has multiple potential applications in optical communication and in all-optical signal processing. Although most of the applications of FBGs are focused on their linear properties, but in nonlinear regime where the FBG is operated at high excitation intensity it offers the potential for the development of high quality nonlinear optical components such as optical switches, tunable filters and optically bistable devices. In view of the fast switching nonlinear photonic devices, the use of fiber Bragg grating deserves particular attention because of its compact structure. Recent theoretical as well as experimental results have demonstrated very interesting applications of FBGs based on their nonlinear optical properties such as switching, bistability, optical limiting, soliton propagation and pulse shaping through dispersion compensation etc [20]. The field of nonlinear optics explores and exploits the modification of the optical properties of a material system in the presence of light [35]. It is one of the frontiers of research activities in science and technology and plays pivotal role in the emerging photonics technology [36-37]. Today, nonlinear optics has grown in different dimensions which has not only changed the facet of the technology with the discovery of new phenomena; but has also improved our basic understanding about the interaction of radiation with matter. Many nonlinear optical (NLO) and optoelectronic devices are commercially available for applications in different branches of science and technology. In this section, a detailed study on the subject of nonlinear fiber Bragg gratings is presented. In the beginning some basic concepts of nonlinear optics, nonlinearity in optical fiber, nonlinear optics in fiber Bragg gratings are given and than a detailed survey on past research on nonlinear fiber Bragg gratings has been summarized.
1.6.1 Introduction to Nonlinear Optics

Before the inception of lasers, it was assumed that optical parameters, such as refractive index and absorption coefficient of medium are independent of the intensity of the incident light propagating through the medium. It was because of the fact that, the field amplitude of ordinary light sources is of the order of \(10^6\) V/m which is much smaller than the inter atomic and atomic field strength (which is eventually of the order of \(10^9-10^{12}\) V/m) and hence no nonlinear effects can be seen using such conventional light sources.

Nonlinear optical phenomena arise when certain materials are subjected to intense electromagnetic radiation (of the order of or higher than that of the atomic or inter atomic field). To elaborate the same, let us consider a dielectric material in the presence of an intense electromagnetic field. The field causes the electrons within the material to become polarized with respect to the nuclei, which causes an overall polarization within the dielectric. The polarization effect responds linearly to the incident radiation if the field strength is low. The proportional low field relationship describes the area of linear optics and can be represented by

\[
\vec{P} = \varepsilon_0 \chi \vec{E}
\]

(1.47)

where \(\vec{P}\) is the polarization field vector, \(\vec{E}\) is the electromagnetic field vector, and \(\varepsilon_0\) is the permittivity of free space. \(\chi\) is the linear susceptibility of the material which gives rise to the linear optical effects such as refraction, dispersion and birefringence.

The invention of laser by T. H. Maiman [38] in 1960 enabled us to examine the behavior of light in optical material at higher intensities than it was possible before. If the intensity of the laser radiation is sufficiently high, the material response no longer remains linear and the optical susceptibility \((\chi)\) no longer
remains a constant and it becomes a function of the field strength $E$ of the light wave. Under these circumstances using the first approximation, $\chi$ can be expressed as [39],

$$\chi(E) = \chi^{(1)} + \chi^{(2)}E + \chi^{(3)}E^2 + \ldots \ldots$$ (1.48)

where $\chi^{(1)}$, $\chi^{(2)}$ and $\chi^{(1)}$,... are the parameters of the active medium characterizing its first, second and third order polarizability, respectively. Thus induced polarization can be written as

$$\vec{P} = \varepsilon_0 \chi \vec{E} = \varepsilon_0 \left( \chi^{(1)} + \chi^{(2)}E + \chi^{(3)}E^2 + \ldots \ldots \right) \vec{E},$$ (1.49)

The field of nonlinear optics comprises of many fascinating phenomena in terms of various orders of the nonlinear susceptibilities. Phenomena that are based on the second order nonlinear relation between $\vec{P}$ and $\vec{E}$ are frequency doubling of a monochromatic wave (i.e. second harmonic generation; SHG), the mixing of two monochromatic waves to generate third wave whose frequency is either sum or difference of the frequencies of the original waves (i.e. optical frequency conversion).

When the light used is in the form of powerful laser beams, some materials manifest marked changes in their optical properties. This in turn causes modifications in the frequency, phase or amplitude of the light interacting with the material. These phenomena are studied under the general subject area of nonlinear optics (NLO). New materials and devices with desirable nonlinear optical properties have been the subject of intense interest recently; because these materials and devices may provide an efficient means of modifying the nature of an incident light beam. Materials with significant NLO properties may therefore be of technological importance for most of the applications such as optical processing, switching, frequency generation, data storage, optical communication, optical computing etc.
1.6.2 Nonlinearity in Optical Fiber

The ever-increasing need for communication capacity has brought the field of optics into focus for the past decades. Fibre optics is the only technique known today to have the power to meet the strong demands for flexibility and high bandwidth posed by the rapidly growing communication networks. An optical fibre has promising features, such as very less power loss and good robustness, when used as information carriers.

The nonlinear effects in optical fiber originate from the polarizability of the molecules under applied optical field. The polarizability of molecules is related to anharmonic motion of bound electrons under the influence of an applied field [40]. When light passes through a dielectric medium such as an optical fiber, there is induced electric polarization arising from the influence of the applied field on the electric dipoles. The total polarization $\vec{P}$ induced by electric dipole is given by Equation (1.47) and (1.49). The linear susceptibility $\chi^{(1)}$ represents the dominant contribution to $\vec{P}$. Its effects are included through the refractive index $n$ and the attenuation coefficient $\alpha$. The second-order susceptibility $\chi^{(2)}$ is responsible for such nonlinear effects as second-harmonic generation (SHG), sum-frequency generation, parametric oscillations and optical rectification [41]. It is nonzero only for media that lack inversion symmetry at the molecular level. Since SiO$_2$ is a symmetric molecule, $\chi^{(2)}$ vanishes for silica fiber. Thus optical fibers do not normally exhibit second-order nonlinear effects. The lowest-order nonlinear effects in optical fibers are thus due to third-order susceptibility $\chi^{(3)}$, which is responsible for phenomena such as third-harmonic generation, four-wave mixing, and nonlinear refraction [42]. Most of the nonlinear effects in optical fibers therefore originate from nonlinear refraction, a phenomenon referring to the
The intensity dependence of the refractive index leads to a large number of interesting nonlinear effects; the two most widely studied are known as self-phase modulation (SPM) and cross-phase modulation (XPM). Self-phase modulation refers to the self-induced phase shift experienced by an optical field during its propagation in optical fibers. Its magnitude can be obtained by noting that the phase of an optical field changes by

\[ \phi = nk_0 L = \phi_L + \phi_{NL} = (n_0 + n_2 |E|^2) k_0 L \]  

where \( k_0 = 2\pi / \lambda \) and \( L \) is the fiber length. The intensity-dependent nonlinear phase shift, \( \phi_{NL} = n_2 k_0 L |E|^2 \), is due to SPM. Among other effects, SPM is responsible for spectral broadening of ultrashort pulses [43] and formation of optical solitons in the anomalous-dispersion regime of fibers.

### 1.6.3 Nonlinear Optics in Fiber Bragg Grating

In the previous sections we described the fundamental concepts of fiber Bragg gratings, nonlinear optics in photonic applications and Kerr nonlinearity in optical fibers. We are now interested in the Kerr nonlinearity in fiber Bragg gratings and related nonlinear component for all-optical communication system. As
mentioned earlier in section 1.1, the fiber Bragg grating possesses forbidden frequency gaps or stop gaps. These gaps are located around Bragg frequencies, which are determined by the modulation period and the average refractive index. Waves with frequencies lying in the stop gap are reflected and those frequencies which lie outside of the stopband are transmitted from the grating. The location of the gap has critical dependence on the refractive index and hence the Kerr nonlinearity (leading to intensity-dependent refractive index) plays a crucial role in this context. In nonlinear regime, the center frequency \( \omega_0 \) (or Bragg frequency), the spectral width and depth of the stopband are not fixed but are a function of the intensity \( I \). Considering nonlinearity in a Bragg structure, the position of center frequency given in Eq. (1.3) and the bandwidth (both width and depth) of the stopband in Eq. (1.43) may potentially change with incident intensity due to the intensity dependent refractive index. Therefore, the transmission of light through a nonlinear periodic structure is both wavelength and intensity-dependent, as illustrated in Figure 1.14. The Kerr coefficient is assumed to be positive in this case. The Figure demonstrates that, the Bragg resonance frequency \( \omega_0 \) shifts to lower frequencies with increasing intensity; while the width of the stopband increases with increasing intensity.

The above description of the dynamic movement of the stopband with intensity shows nonlinear Bragg structures as excellent candidate for all-optical communication devices. Figure 1.14 also illustrates the switching capability of such structures. The frequency component at \( \omega_1 \) is transmitted at low intensities, but is reflected strongly at higher intensities; while the frequency component at \( \omega_2 \) is strongly reflected at low intensities, but is scarcely reflected at high intensities. Because of the combined effect from nonlinearity and periodicity, light waves at frequencies \( \omega_1 \) and \( \omega_2 \) detuned themselves from the
Bragg condition at high intensities, changing their transmission characteristics. This property can be used to realize an optical switch.

In addition to switching, nonlinear Bragg structures have been either theoretically predicted and experimentally demonstrated to provide optical bistability, optical limiting, pulse compression, and optical logic operations. In the next section, a review of the research work investigating the behavior of Bragg structures with a third order nonlinearity is presented.
1.6.4 Previous Research on Nonlinear Periodic Structure (Bragg Structure)

As explained in the preceding section, nonlinear periodic structures can potentially achieve multiple optical signal processing functions. Considerable past research efforts have investigated various nonlinear periodic signal processing devices. To date, theoretical and experimental research works on nonlinear periodic structures have been concentrated on: steady state bistable response, the presence of stationary gap solitons; studies of reflection and transmission properties of, Bragg grating structures; propagation of Bragg solitons, pulse compression; and experimental demonstration of nonlinear stopband shifting.

1.6.4.1 CW Wave Propagation in Nonlinear Periodic Structure

Most of the earlier research work has concentrated on bistable phenomena using CW wave propagation in periodic structure. In a bistable nonlinear element the value of transmittance depends on whether the incident intensity is increasing or decreasing, i.e. the transmittance depends on its previous state and the strength of incident illumination. This behaviour is illustrated in Figure 1.15. The hysteresis loop present in the transfer characteristics of bistable elements enables steady-state optical memory operation.
Figure 1.15: The transmitted versus incident intensity characteristic of a bistable optical element exhibiting a hysteresis characteristic.

In the late 1970s and early 1980s a number of research groups predicted and demonstrated steady-state optical bistability in a nonlinear Fabry-Perot interferometer. In a number of publications the groups of Smith and Gibbs reported bistability in an electrooptically biased crystal in free-space [44] and integrated configurations [45]. They determined threshold conditions for bistability in terms of the incident power and strength of the nonlinearity [46], and demonstrated optical bistability in dielectric [47] and semiconductor [46, 48] materials. These achievements were summarized in [49] and a book [50], both written by Gibbs and published in 1985.

The nonlinear properties of periodic media were first investigated theoretically by Winful et al. in 1979 [51]. They reported that such media exhibit bistability, multistability and switching behaviour. In the subsequent years they studied the analyses of the effects of linear absorption on the response of nonlinear periodic structure [52], the response of a periodic nonlinear element to non-monochromatic illumination [53], a combined distributed feedback Fabry-Perot interferometer
structure [54], and coupling between various modes in a nonlinear fiber Bragg grating [55].

In 1992 He et al. reported experimental demonstration of optical bistability in nonlinear periodic structures. The optical element analyzed consists of 30 GaAs/AlGaAs Bragg periods. Strong bandedge nonlinearities were used in the spectral region of 875 to 885 nm. A shift in the reflectivity peak with increasing intensity was observed when the sample was illuminated with 10 μs square pulses [56].

In 1993 Herbert et al. experimentally demonstrated bistability and multistability in a colloidal crystal exhibiting electrostrictive nonlinearity illuminated with continuous-wave light at 514.5 nm [57].

In 1996 Li et al. discussed the dependence of the strength of bistability on the sign of the real part of nonlinearity. It was concluded that for a positive Kerr nonlinearity the transmission is severely suppressed near the low-energy end of the stopband, while for a negative Kerr nonlinearity the transmittance increases [58].

Lidorikis et al. in 1997 have studied the strength of the bistability with respect to the spectral position from the center of the stopband in colloidal crystal [59].

Bistable structures in general also support stationary gap solitons. Stationary gap solitons are fully-transmissive continuous-wave states whose electric field envelope distribution within a nonlinear periodic structure resembles the sech²(z) shape of a temporal soliton. In a stationary gap soliton the intensity inside the structure is higher than the incident intensity. This is in contrast to the fully-transmissive states at wavelengths that are far from the Bragg resonance in which the intensity distribution is uniform across the structure. Stationary gap solitons arise under monochromatic continuous-wave illumination of a nonlinear periodic structure at a wavelength lying within the initial built-in photonic stopband. Nonlinear refraction changes the position and shape of the stop-band.
The transmittance of the structure can change from low to high as the stopband is shifted entirely away from the spectral position of the light [60]. The concept of a stationary gap soliton was first introduced in 1987 by Chen and Mills in superlattice [61]. Later on Mills and Trullinger [62] analytically predicted the existence of stationary gap solitons in periodic structure.

A simple and intuitive TLM algorithm has been developed by Janyani et al to model and discussed many nonlinear phenomena such as bistable switching, soliton propagation and optical limiting in FBGs [63]. Yosia et al achieved three unique nonlinear switching behaviors of probe transmission with operating intensity in bistability region by cross phase modulation. They have investigated that the bistability threshold between the low and high state inside hysteresis loop of FBG is equivalent to the unstable state [64]. The same author in 2007 has presented numerically the optical bistability phenomena in third-, fifth- and seventh-order nonlinear periodic media and shown all-optical transistor operation based on the unstable-state principle [65]. In the same year Parini et al [66] have presented a systematic analysis of self pulsing instabilities in a FBG. They have found that inside the stop band the upper branch of the bistable response can be partially stable above a critical detuning. Recently, Tian [67] have investigated numerically the switching characteristics of linearly chirped nonlinear Bragg gratings using the reversely recursive transmission matrix method. The results show that, introducing chirp in fiber gratings is helpful to widen the bistable wavelength range; The positive chirp will be helpful to decrease the switching threshold, flatten the upper branch of the hysteresis, while the negative chirp may enable the switching contrast and the hysteresis loop width to obtain the optimization, but the switching-on threshold can have the drop. Zang et al [68] demonstrated a new optical bistability device by using two Fiber Bragg Gratings, in which an erbium-doped fiber (EDF) is inserted to form a nonlinear Fabry–Perot cavity (EDF FBG/F–P).
The operation principle of this device is described by the resonant nonlinearity theory combining with the transfer matrix method. Their results show that EDF FBG/F-P device has an evident merit in reducing the threshold switching power to 7 mW, resulting in a reduction about 6 orders, compared with that of single FBG device.

1.6.4.2 Pulse Propagation in Nonlinear Periodic Structure

The study of pulse propagation in nonlinear periodic structure was made by Winful and Sipe in 1985 and 1988 [69, 70] and by de Sterke and Sipe in 1988 [71]. They showed that near the edge of a stopband in a Bragg periodic structure, the nonlinear Schrödinger equation can be solved to yield soliton solutions for the propagation of optical pulses.

Bragg solitons are solitary waves: they propagate without changing their shape. In the case of a nonlinear periodic structure, solitonic propagation occurs due to the balance of the effects of grating dispersion and nonlinear self-phase modulation.

In 1989 Christodoulides and Joseph [72] and Aceves and Wabnitz [73] analyzed propagation of pulses with carrier frequencies close to the centre of the stopband and with power spectra within the stopband. Since the nonlinear Schrödinger equation assumes weak coupling between counterpropagating modes it cannot be used to describe the scenario in which a significant amount of pulse intensity is continuously transferred back and forth between the counterpropagating modes. Coupled mode theory that allows strong coupling was used to analyze such a system. It was shown that soliton solutions exist and that the velocity of these solitons can vary from zero to the speed of light. The slower
speed of the soliton corresponds to a greater rate of transfer of energy between forward and backward modes during pulse propagation. Because the spectrum of these solitons lies entirely within the stopband, they were later named gap solitons [74].

The experimental work on propagation of pulses in nonlinear periodic structures includes demonstration of pulse switching and solitonic propagation.

In 1992 Sankey et al. [75] reported all-optical pulse switching in a corrugated silicon-on-insulator waveguide. They reported that the reflectance experienced by the nanosecond pulses increases due to the shifting of stopband at high intensity.

Eggleton et al. [74] reported direct observation of Bragg soliton propagation in fiber Bragg gratings in 1996 using a Q-switched YLF laser producing 60 and 90 ps pulses at a wavelength of 1064 nm. Formation of solitons was observed for pulses with spectra overlapping the edge of the linear stopband and for pulses with spectra significantly overlapping the center of the stopband.

The formation of gap solitons was also observed by Miller et al. in 1999 in an AlGaAs waveguide [76]. At moderate incident powers, transmission of soliton shaped pulses was observed for pulses with spectra at the centre of the bandgap at 1.5 µm. For higher powers the solitons split into several shorter pulses [76].

In 1997 Broderick et al. demonstrated experimentally pulse switching in a nonlinear fiber Bragg grating using a pushbroom effect at 1.55 µm [77]. A strong pump pulse spectrally detuned a part of the continuous-wave probe out of the stopband. This detuned part of the probe had the time duration comparable to the length of the pump pulse and emerged out of the grating as a new probe pulse. This switching mechanism has not been attributed to the shifting of the built-in stopband but entirely to cross-phase modulation [77].

In a similar experiment in 1997 Broderick et al. demonstrated a reflection-based modification of the pushbroom effect [78]. In this experiment the wavelength of a continuous-wave probe was initially outside of the grating
stopband. A pulsed pump at a frequency far outside the bandgap was used to shift, through the cross-phase-modulation.

The propagation of a nonlinear ultrashort pulse in a photonic bandgap structure is investigated by Chi et al [79] using the finite-difference time-domain method. Their results show that an ultrashort pulse near the bandgap edge can propagate through a nonlinear fiber Bragg grating, even if the broadband spectrum of this ultrashort pulse overlaps the whole forbidden band of the grating. In 2005 Porsezian et al [80] investigated the phenomena of modulation instability and its associated gain spectra in Bragg grating structure for generating the ultrashort pulses. They have analyzed modulation instability under the influence of non-Kerr nonlinearity. In the same year, Yosia et al [81] numerically investigated the nonlinear switching characteristics and pulse propagation in the phase shifted cubic-quintic grating. They have found double optical bistability in a certain negative detuning window close to the Bragg wavelength. In 2010, Dasanayaka and Atai [82] have investigated the existence and stability of Bragg grating solitons in a cubic–quintic medium with dispersive reflectivity. They have found that the model supports two disjoint families of solitons. One family can be viewed as the generalization of the Bragg grating solitons in Kerr nonlinearity with dispersive reflectivity. On the other hand, the quintic nonlinearity is dominant in the other family. Stability regions are identified by means of systematic numerical stability analysis. In the case of the first family, the size of the stability region increases up to moderate values of dispersive reflectivity. However for the second family (i.e. region where quintic nonlinearity dominates), the size of the stability region increases even for strong dispersive reflectivity.
1.6.4.3 Shifting of the Stopband in Nonlinear Periodic Structure

Complementary to work on the steady-state response of nonlinear periodic structures and propagation of solitonic pulses, research was carried out on periodic structures in which the induced nonlinear index change is large, i.e. $\Delta n > 0.01$. The focus of this work was not the demonstration of bistable or solitonic behaviour but rather an observable movement of a photonic stopband.

An experimental demonstration of observable stopband shift in a 30 Bragg period GaAs/AlGaAs stack was reported in 1992 by He et al. In their experiment a 4 nm shift of the center of the stopband was observed at high levels of illumination when 1 µs square pulses produced by a dye laser were excited [56].

In 1992, Herbert et al. reported a power-dependent shift in the stopband of a three dimensional dye-doped colloidal crystal. A decrease in transmittance through the crystal was observed under continuous-wave illumination of the Ar Ion beam at 514.5 nm [57].

In 1992 Scalora et al have shown that depending on the spectral position of the probe beam, a strong pump beam would move the stopband towards or away from the weak probe beam, thereby altering probe transmission [83].

An experimental demonstration of nonlinear stopband shifting using short pulses was reported in 1997 by Pan et al. Intensity-dependent coherent scattering from a colloidal crystal infiltrated with optically linear liquid was described. The index of refraction of the liquid was slightly higher than that of the photonic crystal spheres. Under the illumination with 3.5 ns pulses at 514 nm, the negative thermal nonlinearity of the dye-doped spheres increased the contrast of the grating. A maximum increase in the reflectance was estimated at 2 % [84].
A theoretical paper was published in 1999 by Tran in which the nonlinear response of a structure with a very sharp stopband was studied. He studied about optical switching when the frequency of light was aligned with the edge of the stopband and with the maximum of one of the sidelobes in the reflectance spectrum [85].

A novel approach to fabricate nonlinear periodic structures was presented in 2001 and 2002 by a group of researchers from the Naval Research Laboratory [86, 87]. A sheet made out of two layers of two different polymers, each few tens of nanometers thick, was folded upon itself multiple times to generate a periodic structure of 4096 layers. The layers were not uniform in thickness. This introduced a disorder which resulted in a broadband response. Increasing reflection in the visible region was observed upon steady-state illumination. This was attributed to the nonlinear intensity-dependent refractive index contrast between the two constituent materials.

An experimental demonstration of ultrafast stopband shifting in the silicon two-dimensional photonic crystal was reported by Leonard et al. [88]. In this experiment a 300 fs pump probe having wavelength of 800 nm was used. A 20 nm shift of the edge of the stopband towards shorter wavelengths was observed [88].
1.7 The Need for Additional Research

Despite vast accomplishments in the research on nonlinear periodic structures, there are many opportunities to study the nonlinear phenomena in low loss optical fiber Bragg grating which can be exploited in ultrafast all-optical communication system.

While going through the available literature it is felt that the theoretical work developed so far by using coupled mode theory appears to be of complicated in nature in drawing any analytical inference to study the spectral characteristics of fiber Bragg grating under nonlinear regime. To achieve this, a new approach and simple analytical solution of coupled mode equations for fiber Bragg grating is needed to describe and summarize the conditions for intensity-domain optical stability which allows prediction of the nonlinear spectral characteristics. Accordingly, in the present research work an analytical solution of coupled mode equations have been obtained in a simplifying manner to study the various nonlinear optical properties of fiber Bragg grating. The effect of nonlinearity on the optical spectral characteristics of nonlinear fiber Bragg grating is studied and optical switching, optical bistability, optical multistability, optical limiting, optical pulsation and optical phase modulation in nonlinear fiber Bragg grating has been demonstrated theoretically.
1.7 Plan of the Thesis

The aim of the present research work is to study the nonlinear optical effect in fiber Bragg grating. The attention was paid to study analytically the reflection, transmission and phase properties of fiber Bragg grating in nonlinear cubic regime. The reflectivity of fiber Bragg grating was also studied analytically in the presence of both cubic and quintic nonlinear regime. The nonlinear coupled mode theory is used in the present analytical investigations.

In Chapter 2, the filter characteristics and optical bistable phenomena of nonlinear fiber Bragg grating has been reported. The nonlinear coupled mode equation has been solved analytically and obtained the expression for reflectivity in nonlinear Kerr regime. Our results suggest that at sufficient high input intensity the reflection spectrum splits into two intense bands and FBG works as a tunable notch filter in which the transmission wavelength can be changed by varying the intensity of the input beam. We have also studied optical bistability in reflection mode of FBG by plotting the reflected intensity with incident intensity.

The work reported in this Chapter is split into two parts: First part is related to filter characteristics of the FBG which has been published in Journal of Nonlinear Optics Quantum Optics, 41, 252-264 (2010) and in the second part its bistable characteristics is studied which has also been published in Journal of Nonlinear Optics Quantum Optics, 44, 15-24 (2012).

In chapter 3, we have analyzed the influence of the quintic nonlinearity on the dispersion curve as well as on the reflectivity of the FBG. Filter characteristics of cubic-quintic fiber Bragg grating is investigated using cubic-quintic nonlinear coupled mode equations. The results show that for appropriate high input intensities quintic nonlinearity shifts the stop band of Bragg grating such that the grating becomes transparent for the entire wavelength band of the incident
beam. The shifting and breaking of the stop band is observed due to the phenomena of SPM in the grating.

The above works has been published in *Journal of Nonlinear Optics Quantum Optics*, **41**, 313-327 (2010).

The study of optical multistable and limiting behavior in transmission mode of nonlinear fiber Bragg grating has been reported in Chapter 4. The expression for transmittivity is obtained incorporating the Kerr nonlinearity in coupled mode equations. The theory demonstrates that the optical multistability and limiting takes place when operating wavelengths are chosen lower and upper the Bragg wavelength, respectively.

The work described in this chapter has two parts; the first one is related with multistable behavior of FBG. This work was published in *Journal of Optoelectronics and Advanced Materials – Rapid Communication*, **6**, 25-28 (2012) and the second one describes the optical limiting behavior which has been published in *Journal of Nonlinear Optics Physics and Materials*, **21**, 1250017.1-1250017.10 (2012).

Chapter 5 is devoted to study the phase response of fiber Bragg grating at high excitation intensity. The transmission coefficient under nonlinear regime is derived and real, imaginary and phase part of the transmitted wave is determined using coupled mode theory. The result shows that the nonlinear phase increases stepwise with increasing input intensity and such phase modulation of the beam distorts the transmitted field to an extent such that the multiple switching states and optical pulsation occur.

The part of this work is presented in *International Conference on Optical Engineering 2012*, VTU, Belgaum.

Chapter 6 gives the important conclusions on the work described in the present thesis and the future scope of work in the area of nonlinear FBG.
References


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