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Fiber Bragg Grating Based Intensity Dependent Optical Notch Filter

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We have reported the results of an analytical study of nonlinear reflection characteristics of uniform fiber Bragg grating at high excitation intensity. The nonlinear coupled mode equations have been solved analytically to obtain the reflectivity of the grating in the nonlinear regime. The complete analysis is made considering the propagation of a CW laser beam at high input intensity. The study reveals that the stop band at the Bragg wavelength shifts towards higher wavelength side with increasing excitation intensity and splits into two equally intense bands at a specific input intensity. These intensity dependent features of Bragg grating can be used in the design of optical notch filter with huge potentiality in the current photonic technology.

Keywords: Fiber Bragg grating, self phase modulation, optical notch filter, modulation instability, nonlinear refraction.

1 INTRODUCTION

The explosive growth of the internet communications over recent years has placed increasing demands on both the channel capacity and narrowband filtering devices in optical networks. Fiber Bragg grating (FBG) is emerging as one of the essential components for many applications in the field of lightwave technology. Furthermore, optical filters based on this device play pivotal role in optical communication system due to their advantages such as narrow bandwidth, low insertion losses, high reflectivity and compactness. The low intensity applications of this periodic structure are numerous such
as in cases of optical filters, add/drop multiplexers, optical sensors and dispersion compensators, etc. [1-4]. In FBGs, the existence of stop bands for the particular frequency ranges makes them useful as band pass/band rejection filters [5]. FBG based optical filters have been widely investigated in photosensitive fibers [6], semiconductor waveguides [7] and polymer waveguides [8]. Kashyap et al. [9] have developed ultra-steep edge high rejection (>74 dB) filter using FBG.

Recently, there has been a great deal of interest in the optical properties of periodic structure with intensity dependent index of refraction. The response of the FBG becomes nonlinear when intense electromagnetic field interacts with it. The nonlinear properties of periodic structures were first investigated by Winful et al. [10], who showed the possibility of optical bistability, multistability and switching behaviour. The nonlinear switching behaviour as well as optical bistability has been numerically analyzed in different kinds of FBGs [11-14]. Melloni et al. [15] experimentally demonstrated low power cross phase modulation based optical switching in FBG. Agrawal et al. [2] have discussed at length both the linear and nonlinear optical properties of FBG over a wide range of excitation intensity. Their numerical analysis showed that the stop band experiences red shift in wavelength at high excitation intensity and suggested the utility of FBGs in bistable optical switches.

In the present study, we have investigated analytically the optical properties of FBG in nonlinear regime by solving nonlinear coupled mode equations (NLCMEs). The reflectivity of the FBG has been obtained considering the propagation of continuous wave. The occurrence of self phase modulation in the grating has also been discussed. We have shown that the FBG can be used as a tunable notch filter in which the transmission wavelength can be changed by varying the intensity of the input beam.

2 SOLUTIONS OF NONLINEAR COUPLED MODE EQUATIONS

Fiber Bragg grating is a periodic structure that has alternated high and low refractive indices along the length of the core of the single mode optical fiber. When light propagates through such a periodic structure, it couples forward and backward propagating modes and mode coupling occurs at the Bragg wavelength $\lambda_B = 2n\Lambda$ (where $n$ is the linear model index and $\Lambda$ is the grating period). As a result, a narrow band of wavelengths satisfying the phase matching condition is reflected back and remaining wavelengths are transmitted showing that FBG acts as a band rejection/pass filter at low excitation intensity. The reflected band of frequencies is commonly referred to as a stop gap or stop band. The central wavelength of the stop gap is known as Bragg wavelength and corresponds to the condition where the wavelength of the light inside the structure is equal to one half the period of the dielectric layer. This condition is known as the Bragg resonance. Complete study
of wave propagation in FBG is generally described by the coupled mode theory [2].

At moderately large input intensity, the refractive index of the grating element becomes intensity dependent which causes change in the grating characteristics and is defined as

\[ n(\omega, z) = n_{\text{eff}}(\omega) + n_2|E|^2 + n_g(z) \]  

(1)

where \( n_{\text{eff}} \) is the average linear refractive index, \( n_2 \) is the nonlinear Kerr coefficient, \( n_g(z) \) accounts for periodic index variation inside the grating. Also, \( E(r, \omega) \) is the electric field of the pump and is given by [2]

\[ E(r, \omega) = F(x, y) A_f(z, \omega) \exp(i\beta_B z) + F(x, y) A_b(z, \omega) \exp(-i\beta_B z) \]  

(2)

Here, \( \beta_B = \pi/\Lambda \) is the Bragg wave number, \( A_f \) and \( A_b \) are the envelope function of the forward and backward travelling waves, which are assumed to be slowly varying in space and time. Transverse variation for these two counterpropagating modes (\( A_f, A_b \)) is governed by the same modal distribution \( F(x, y) \) in a single mode fiber. In order to examine the characteristics of the outgoing backward or forward propagating modes, following pair of nonlinear coupled mode equations have been used [2]:

\[ \frac{\partial A_f}{\partial z} + \beta_1 \frac{\partial A_f}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_f}{\partial t^2} + \alpha A_f = i\delta A_f + i\kappa A_b + i\gamma (|A_f|^2 + 2|A_b|^2) A_f, \]  

(3)

and

\[ \frac{\partial A_b}{\partial z} + \beta_1 \frac{\partial A_b}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_b}{\partial t^2} + \alpha A_b = i\delta A_b + i\kappa A_f + i\gamma (|A_b|^2 + 2|A_f|^2) A_b. \]  

(4)

In these equations, \( \beta_1 \) and \( \beta_2 \) are first- and second-order dispersion parameter related to the group velocity \( v_g \) and \( \alpha \) is the loss coefficient. \( \delta, \kappa \) and \( \gamma \) are detuning parameter, coupling coefficient and nonlinear parameter, respectively, and are given by

\[ \delta = 2\pi n \left( \frac{1}{\lambda} - \frac{1}{\lambda_B} \right), \quad \kappa = \frac{2\pi n_g}{\lambda_B} \quad \text{and} \quad \gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}} \]

where \( A_{\text{eff}} \) is the effective core area. We consider normalized intensity such that the power flow is assumed to be through unit area of cross section. In the steady state CW nonlinear regime, \( \beta_1 \) and \( \beta_2 \) are neglected in equations (3) and (4). For typical grating lengths (< 1 m), the loss coefficient \( \alpha \) can also be
neglected. With these assumptions, NLCMEs take the simplified forms:

\[
i \frac{\partial A_f}{\partial z} + \delta A_f + \kappa A_b + \gamma (|A_f|^2 + 2|A_b|^2) A_f = 0, \tag{5}
\]
and

\[
- i \frac{\partial A_b}{\partial z} + \delta A_b + \kappa A_f + \gamma (|A_b|^2 + 2|A_f|^2) A_b = 0. \tag{6}
\]

The preliminary solution of NLCMEs was reported by Agrawal et al. [2] where they predicted the red/blue shift in the stop band depending on the positive/negative nonlinearity of refractive index. In the following analysis we have obtained the analytical solution of NLCMEs by neglecting higher order terms of backward propagating mode. The solutions of equations (5) and (6) are obtained as

\[
A_f(z) = A_1 \exp(iSz) + A_2 \exp(iTz), \tag{7}
\]
and

\[
A_b(z) = B_1 \exp(iSz) + B_2 \exp(iTz) \tag{8}
\]
with

\[
S = -\gamma I_0 + \frac{\sqrt{\gamma^2 l_0^2 + 4q_{nl}^2}}{2} \quad \text{and} \quad T = -\gamma I_0 - \frac{\sqrt{\gamma^2 l_0^2 + 4q_{nl}^2}}{2}.
\]

Here, \( I_0 = |A_f|^2 + |A_b|^2 \) is the input intensity at Bragg wavelength \( \lambda_B \). The coefficients \( A_1, A_2, B_1 \) and \( B_2 \) are interdependent and satisfy the following four relations:

\[
(\delta - S + \gamma |A_f|^2) A_1 = -\kappa B_1, \tag{9a}
\]
\[
(S + \delta + \gamma |A_b|^2 + 2\gamma |A_f|^2) B_1 = -\kappa A_1, \tag{9b}
\]
\[
(\delta - T + \gamma |A_f|^2) A_2 = -\kappa B_2, \tag{9c}
\]
and

\[
(T + \delta + \gamma |A_b|^2 + 2\gamma |A_f|^2) B_2 = -\kappa A_2. \tag{9d}
\]

Equations (9) are satisfied for nonzero values of \( A_1, A_2, B_1 \) and \( B_2 \) if the possible values of \( q_{nl} \) obey the nonlinear dispersion relation

\[
q_{nl}^2 = q^2 + \delta X + Y, \tag{10}
\]

With \( q = (\delta^2 - \kappa^2)^{1/2} \) being the linear dispersion parameter for the Bragg grating and the parameter \( X \) and \( Y \) are defined as

\[
X = \gamma (I_0 + 2|A_f|^2) \quad \text{and} \quad Y = \gamma^2 |A_f|^2 (I_0 + |A_f|^2) \tag{11}
\]
Equations (10) and (11) exhibit the intensity dependent modification in the dispersion parameter. Consequently we find that for frequency detuning $\delta$ lying in the range $-\kappa < \delta < (1 + \frac{\kappa}{2} + \frac{\sqrt{\kappa}}{2})^{1/2} < \kappa$, $q_{nl}$ becomes purely imaginary and most of the incident field will be reflected due to the fact that the grating will not support the propagating wave. In the absence of nonlinearity ($\gamma = 0$), the stop band extends for $-\kappa < \delta < \kappa$ and equations (9) and (10) resemble to the standard solutions of the coupled mode equations for Bragg grating in linear regime. One can thus expect a red shift in the stop band frequency due to the nonlinearity.

On substituting parameter $q_{nl}$ in equations (7) and (8), the fields of forward and backward propagating modes take the form

$$A_f(z) = A_1 \exp(iqz) \exp \left\{ i \left( \frac{4\delta X + 4Y + \gamma^2 I_0^2}{8q} - \gamma I_0 \right) z \right\}$$

$$+ A_2 \exp(-iqz) \exp \left\{ -i \left( \frac{4\delta X + 4Y + \gamma^2 I_0^2}{8q} - \gamma I_0 \right) z \right\},$$

and

$$A_b(z) = B_1 \exp(iqz) \exp \left\{ i \left( \frac{4\delta X + 4Y + \gamma^2 I_0^2}{8q} - \gamma I_0 \right) z \right\}$$

$$+ B_2 \exp(-iqz) \exp \left\{ -i \left( \frac{4\delta X + 4Y + \gamma^2 I_0^2}{8q} - \gamma I_0 \right) z \right\}.$$ (13)

In the above equation, the phase of the forward and backward propagating wave is represented by

$$\phi(z) = \pm \left\{ q + \left( \frac{4\delta X + 4Y + \gamma^2 I_0^2}{8q} - \gamma I_0 \right) z \right\}.$$ (14)

Hence, equation (14) shows that the phase $\phi(z)$ of the reflected as well as transmitted wave gets modulated due to the effect of nonlinearity at high excitation intensity and this self phase modulation of the waves is proportional to the excitation intensity $I_0$.

In order to examine intensity dependent reflection characteristics of the grating we have obtained the analytical solution for the amplitude of forward and backward propagating modes using equations (7) and (8) as

$$A_f(z) = A_1 \exp(iSz) + r_{nl} B_2 \exp(iTz),$$

and

$$A_b(z) = B_2 \exp(iTz) + r_{nl} A_1 \exp(iSz).$$ (16)
The corresponding expression for the effective reflection coefficient \( r_{nl} \) in nonlinear regime is found to be

\[
r_{nl} = r \left[ 1 - \frac{(\delta X + Y)}{2q(\delta + q)} - \frac{\gamma}{\delta + q} \left( \frac{y q l_0^2}{8 q_{nl}} + |A_f|^2 \right) \right]
\]

with \( r \) being the linear effective reflection coefficient and is given by

\[
r = -\frac{\kappa}{\delta + q}
\]

From equation (17), we further notice that the effective nonlinear reflection coefficient will have diminishingly small values for

\[
\frac{(\delta X + Y)}{2q(\delta + q)} + \frac{\gamma}{\delta + q} \left( \frac{y q l_0^2}{8 q_{nl}} + |A_f|^2 \right) = 1.
\]

Consequently, for the above condition, the Bragg grating will be transparent. The condition of transparency can be satisfied at specific values of wavelengths of the incident beam. It is also clear from the above condition that one can control the pass band as well as stop band by choosing the proper intensity of the incident beam.

Applying the boundary conditions that light is incident only at the front end at \( z = 0 \) of the FBG as shown in Fig. 1, the nonlinear reflection coefficient \( r_{ng} \) for a grating of length \( L \) has been obtained by using equations (15) to (17) as

\[
r_{ng} = \frac{A_h(0)}{A_f(0)} = -\kappa \left[ \frac{\delta + \tau}{\gamma} \right] \left[ 1 - \exp(ikL) \right] \left[ \frac{\delta + \tau}{\gamma} \right]^2 - k^2 \exp(ikL)
\]

where

\[
\tau = \delta + \gamma |A_f|^2 \quad \text{and} \quad k = \sqrt{\gamma^2 l_0^2 + 4 q_{nl}^2}.
\]

and corresponding expression for the reflectivity \( R_{ng} (= |r_{ng}|^2) \) in the nonlinear regime is

\[
R_{ng} = \frac{2 \kappa^2 M_+ [\cosh \theta_i - \cos \theta_r]}{[M_+^2 + \xi^4] \cosh \theta_i + [M_+^2 - \xi^4] \sinh \theta_i - 2 \kappa^2 [M_+ \cos \theta_r + 2 \Psi \sin \theta_r]}
\]

\[\text{FIGURE 1}\]
Schematic of a FBG of length \( L \) illuminated by electromagnetic field amplitude \( A(z) \).
with

$$\theta_{\epsilon(i)} = k_r L(k_i L), \quad \Psi = \left(\frac{k_r}{2} + \tau\right)\left(\frac{k_i}{2}\right), \quad \text{and} \quad M_{\pm} = \left(\frac{k_r}{2} + \tau\right)^2 \pm \left(\frac{k_i}{2}\right)^2.$$ 

$k_r$ and $k_i$ being the real and imaginary part of the parameter $k$, respectively.

3 RESULTS AND DISCUSSIONS

In this section, we have discussed the effect of high excitation intensity on the reflectivity of FBG. Equations (17) to (21) reveal that the condition of the transparency of the Bragg grating gets modified at large excitation intensity. The reason for this can be attributed to the intensity dependence of the dispersion parameter $q$ which becomes nonlinear. These equations also exhibit the dependence of effective reflection coefficient ($r_{\text{eff}}$) as well as reflectivity ($R_{\text{ng}}$) under nonlinear regime on various physical parameters of the FBG. In order to examine these features, we have carried out numerical estimates for fiber Bragg gratings with values of $kL \approx 1, 2$ and $3$, respectively. The effective mode index $n_{\text{eff}}$ was chosen to be 1.46 and the grating period $\Lambda = 530.8 \text{ nm}$ (corresponding Bragg wavelength $\lambda_B = 1550 \text{ nm}$). The induced refractive index change $n_\text{g} = 5 \times 10^{-4}$ as well as nonlinear refractive index $n_2 = 2.6 \times 10^{-20} \text{ m}^2/\text{W}$ and effective area $A_{\text{eff}} = 80 \mu\text{m}^2$ were chosen [12, 13].

The intensity dependent changes in the reflectivity of FBG is plotted in Figs 2 to 4 using the above mentioned material parameters in equation (21).
for $\kappa L \approx 1.2$ and 3, respectively. In these figures, the curves $a, b, c, d$ and $e$ represents the reflectivity of fiber Bragg grating at incident intensity 0.001 kW/cm$^2$, 1.5 kW/cm$^2$, 3.5 kW/cm$^2$, 6.5 kW/cm$^2$ and 9.5 kW/cm$^2$, respectively. One can notice two important features from these figures. Firstly,
the Bragg wavelength $\lambda_B$ at which peak reflectivity occurs exhibits a red shift equivalent to $2n_2|E|^2A$. Secondly, one can observe that at higher input intensities ($\geq 1.5$ kW/cm$^2$), splitting in the stop band (reflection band) occurs.

At sufficiently high excitation intensity of 9.5 kW/cm$^2$, two equally intense reflected pulses are observed. The peak wavelengths and FWHM of these pulses and spacing between them depend upon the values of $kL$. It is also worthy to note that at an intensity of 9.5 kW/cm$^2$ the central peak splits while the side lobes become almost same as that under the linear regime. Such splittings have also been reported in optical fibers as well as in FBG at high excitation intensity due to the effect of modulational instability (MI) [16–21] which is a well known phenomenon in nonlinear wave propagation studies causing CW fields to be unstable.

In the context of optical fibers, the modulational instabilities rely on the interplay between the anomalous dispersion provided by the glass and the nonlinear refractive index, and can lead to the spontaneous break up of a CW beam into a periodic train of soliton like pulses. The number of peaks depends on nonlinear phase shift and increases linearly with it [16–18]. Similarly, in a nonlinear Bragg grating MI occurs through the interplay of the nonlinearity and the dispersion provided by the grating. The Bragg grating exhibits strong reflection and anomalous dispersion in a range of wavelengths close to the photonic bandgap. Such MI phenomena in the grating have been reported theoretically and numerically by Winful [19] and De Sterke et al. [20]. In both the analyses, it is shown that when a CW beam with sufficiently high input intensity is incident on a grating and tuned to a frequency within the range of the photonic bandgap, it is converted into a train of ultrashort optical pulses. These self pulsations were interpreted in terms of a MI. Eggleton et al. [21] have reported the experimental demonstration of the existence of Bragg grating solitons at high excitation intensities because of the MI occurring as an interplay between the nonlinearity in the glass and the group velocity dispersion (GVD) induced by the grating.

In the present analysis, GVD term has been neglected and hence we report that the cause of instability/splitting of the reflected beam is the self phase modulation provided by nonlinearity. The instability in the CW beam can be ascribed to the dispersion parameter $\eta$ and nonlinear parameter $\gamma$. Within the stop band, the term $\left(\frac{\delta}{\kappa} + \tau\right)$ in the equation (21) becomes negligibly small at high excitation intensity and results in the splitting of the reflection band. The instability occurs only when the input intensity exceeds a critical value of intensity such that 0.001 kW/cm$^2$. Above this critical intensity where effective nonlinear coefficient $\Gamma (= \gamma|Af|^2)$ exceeds the coupling coefficient $\kappa$ in the stop band (i.e. $\Gamma > \kappa$), $q_{nl}$ becomes imaginary. It is also interesting to note that one can obtain a purely imaginary value of $q_{nl}$ if

$$\kappa^2 > (\delta^2 + \delta X + Y)$$

(22)
The threshold condition for the propagating mode to become unstable is achieved for $k_1 = (S^2 + 5X + Y)$ yielding two roots of $\delta$ as

$$\delta = -\frac{X}{2} \pm \frac{1}{2} \left[ \frac{X^2 - 4(Y - \kappa)}{L} \right]^{1/2}.$$  \hspace{1cm} (23)

As a consequence, for the condition represented by (22), the propagating behaviour of the beam becomes completely unstable and undergoes pulse break up as shown in Figs 2 to 4.

In Fig. 5, the reflection spectra of the FBGs at input intensity of 9.5 kW/cm$^2$ for $\kappa L \approx 1, 2$ and 3 are plotted to exhibit that this features of splitting of reflection spectra into two pulses in FBG can be realized as a notch filter at wavelengths of 1571.9 nm, 1560.37 nm and 1558.45 nm with the width of the notch being $\Delta \lambda = 0.40$ nm, 0.32 nm and 0.28 nm, respectively. In all this three reflection spectra, the reflectivity at the resonance wavelength is almost unity.
This filter can play a vital role in modern optical communication systems like WDM and DWDM where optical filters with ultra narrow bandwidth in the range of 0.2 nm to 0.5 nm are required.

4 CONCLUSIONS

We have investigated analytically the intensity dependent reflection characteristics of uniform fiber Bragg grating by solving the nonlinear coupled mode equations. The theoretical expression for the reflectivity in the nonlinear regime of the fiber Bragg grating for steady state high intensity excitation by a CW beam has been derived. The shifting and splitting of the stop band have been found to occur as a consequence of the effects of intensity dependent refractive index change inside the grating and due to the effect of self phase modulation. The splitting of the reflection band at an intensity of 9.5 kW/cm² can be used as an optical notch filter with the width of the notch being \( \Delta \lambda = 0.40 \) nm, 0.32 nm and 0.28 nm, respectively.

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REFERENCES


Nonlinear Wave Propagation in Cubic-Quintic Fiber Bragg Grating

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Using nonlinear coupled mode equations the propagation of optical beam through fiber Bragg grating has been studied analytically taking into account the cubic-quintic nonlinear effects. Expression for the reflectivity of fiber Bragg grating is obtained considering the propagation of a quasi-CW laser beam at high input intensity. It is noticed that the quintic nonlinearity arising due to fifth-order optical susceptibility modifies the photonic band gap of Bragg grating and makes it transparent for entire band under high excitation intensity. From stability analysis, we find that the wave becomes unstable due to self phase modulation over a certain range of the detuning parameter.

Keywords: Cubic-quintic nonlinearity, fiber Bragg grating, self phase modulation, modulation instability, nonlinear refraction.

1 INTRODUCTION

Photosensitivity based optical fiber Bragg gratings (FBGs) are emerging as one of the essential components for a variety of all optical networks due to their low insertion losses, narrow band widths and high reflectivities. FBGs based optical components are currently showing considerable potential as passive integrated devices in photonics. Furthermore these features of Bragg
gratings make them useful in optical communication and sensor system. The
low intensity applications of this periodic structure are numerous such as
optical filters, add/drop multiplexers, optical sensors and dispersion compens­
sators etc. [1,2]. Recently, intensive research and development activities are
carried out on a variety of FBGs which includes uniform, chirped, tilted,
apodized to study their band pass and band rejection responses [3]. Although
most of the applications of FBGs are focused on their linear properties, but
inspired with the theoretical investigation of optical bistability, multistability
and switching in the nonlinear distributed feedback structures [4] recent theo­
retical as well as experimental results have demonstrated very interesting
applications of FBGs based on their nonlinear properties such as switching,
bistability, optical limiting, soliton propagation and pulse shaping through
dispersion compensation etc. [5-10].

Most of the previous investigations of nonlinear effects in FBGs were con­
centrated on the Kerr nonlinearity which is associated with third-order nonlin­
ear susceptibility $\chi^{(3)}$. It is worth mentioning here that fifth-order susceptibility
$\chi^{(5)}$ with high nonlinear index $n_4$ also becomes important at moderately high
input intensity in FBG. The materials exhibiting moderately large contribution
from third- and fifth-order susceptibility to self focusing (SF) and self defo­
cusing (SDF) phenomena are called as cubic-quintic nonlinear materials. The
effect of cubic-quintic nonlinearity on nonlinear wave propagation in optical
fiber has been studied by several workers [11,12]. Atai et al. [13] for the first
time considered the quintic nonlinearity in FBG and investigated the existence
and stability of solitons as a result of interplay between SF cubic and SDF
quintic nonlinearity. Porsezian et al. analyzed theoretically both the SF and the
SDF quintic effects on the photonic band gap (PBG) of the nonlinear disper­
sion curve [14]. More recently, Yosia et al. [18] have studied the phenomena of
modulation instability and double optical bistability in CQFBG.

In this investigation, we present the filter characteristics of FBG in the
presence of cubic and quintic nonlinearity by solving cubic-quintic nonlinear
coupled mode equations (CQNLCMEs) analytically. The complete analysis
is made considering the propagation of a quasi-CW laser beam at high input
intensity. It is observed that at high excitation intensity, the nonlinear wave
propagating through cubic-quintic periodic structure become unstable above
certain threshold intensity resulting in the splitting of reflected beam. The
instability in the wave increases linearly with increasing excitation intensity.
It is also noticed that at this intensity the quintic nonlinearity modifies the
photonic band gap of Bragg grating and makes it transparent for entire band.

2 THEORETICAL MODEL

Wave propagation in periodic structures is characterized by the presence of
stop bands and pass bands. A wave whose wavelength lies within a stop band
is strongly reflected and a wave whose wavelength lies outside the stop band is transmitted showing that FBG acts as a band pass/rejection filter at low excitation intensity. The wavelength which is strongly reflected is called Bragg wavelength and given by $\lambda_B = 2n\Lambda$ (where $n$ is the linear refractive index and $\Lambda$ is the grating period). This phenomenon is described by coupled mode theory [16], and it provides accurate result for description of wave propagation through a grating.

We consider light propagation in one-dimensional cubic-quintic Bragg grating inscribed in single mode fiber. When the medium of grating is only Kerr type, the refractive index is given as

$$n(\omega, z) = n_{\text{eff}}(\omega) + n_2 |E|^2 + n_4(z)$$

where $n(\omega, z)$ is the total refractive index of the medium, $n_{\text{eff}}$ is effective refractive index of the medium, $n_2$ accounts for periodic index variation inside the grating, $n_4$ is the nonlinear Kerr coefficient and $E(r, \omega)$ is the electric field of the pump and is given by [16]

$$E(r, \omega) = F(x, y)A_f(z, \omega) \exp(i\beta_f z) + F(x, y)A_b(z, \omega) \exp(-i\beta_b z).$$

Here, $\beta_B = \pi/\Lambda$ is the Bragg wave number, $A_f$ and $A_b$ are the envelope functions of the forward and backward travelling waves, which are assumed to be slowly varying in both space and time. Transverse variation for these two counterpropagating modes is governed by the same modal distribution $F(x, y)$ in a single mode fiber. When the intensity of the input beam is sufficiently large or materials with higher order nonlinear coefficients are considered, the intensity dependent refractive index of the nonlinear optical material can be expressed as [17]

$$n(\omega, z) = n_{\text{eff}}(\omega) + \Delta n + n_4(z)$$

with

$$\Delta n = n_2 |E|^2 + n_4 |E|^4 + .......$$

Here, $n_4$ represents the quintic nonlinear coefficient. The coefficients $n_2$ and $n_4$ are related to the third- and fifth- order nonlinear susceptibilities through the relation $n_2 = 3\chi^{(3)}/(8n_0)$ and $n_4 = 5\chi^{(5)}/(16n_0)$. This type of nonlinearity can be obtained in many different optical materials such as semiconductors, semiconductor doped glasses [18,19], para-toluene sulfonate (or PTS) $\pi$-conjugated polymer [20], chalcogenide glasses [21] and some transparent organic materials [22]. When the nonlinear coefficients $n_2$ and $n_4$ possess same sign, the fifth- order nonlinearity will only add to the self focusing or
defocusing phenomena. On the other hand, when \( n_2 \) and \( n_4 \) have opposite signs then there will be a scope to reduce the extent of self focusing and defocusing in the media.

It is interesting to note that there is an upper limit to the refractive index change that can be induced optically [23]. The field strength at which saturation occurs depends on the particular physical processes that cause the nonlinear refractive index change. In order to take into account the saturation behavior, the nonlinear refractive index is modeled as [23-25]

\[
\Delta n = \frac{n_2 |E|^2}{1 + |E|^2 / I_{sat}}
\]

with \( I_{sat} \) as the characteristic saturation intensity. For small intensities \( |E|^2 \ll I_{sat} \), the above expression reduces to Kerr nonlinearity. On the other hand, at large intensity \( |E|^2 \gg I_{sat} \), the refractive index saturates and approaches its maximum value \( \Delta n_{sat} = n_s I_{sat} \). For considering cubic-quintic nonlinearity only in FBG, the nonlinear correction to the medium’s refractive index given in equation (4) is modeled as [14]

\[
\Delta n = \frac{n_2 |E|^2}{1 + (n_4 / n_2)|E|^2}
\]

which takes a simplified form for \( n_4 |E|^2 \ll n_2 \) as

\[
\Delta n = n_2 |E|^2 - n_4 |E|^4.
\]

From the above equation, it may be noted that the nonlinear coefficients \( n_2 \) and \( n_4 \) may have similar or opposite signs and can be realized in a semiconductor double doped optical fiber [18, 19] in which doping of silica glass with two appropriate semiconductor particles leads exactly to the cubic-quintic form of the refractive index with an effectively increased value of \( n_4 \) and decreased one of \( n_2 \) [17]. In these fibers, both the situations (i) \( n_2 > 0 \) and \( n_4 < 0 \) (ii) \( n_2 < 0 \) and \( n_4 > 0 \) are physically possible depending on the nature of dopant, operating wavelength and input intensity of the beam. In the presence of quintic nonlinearity such that \( n_2 > 0 \) and \( n_4 < 0 \), the wave propagation in a nonlinear periodic structure is governed by following cubic-quintic nonlinear coupled mode equations (CQNLCMEs) [14, 15]

\[
\frac{i}{i} \frac{\partial A_f}{\partial z} + \delta A_f + \kappa A_b + \gamma_1 (|A_f|^2 + 2|A_b|^2) A_f \\
- \gamma_2 (|A_f|^4 + 6|A_f|^2 |A_b|^2 + 3|A_b|^4) A_f = 0.
\]
and
\[ -i \frac{\partial A_n}{\partial z} + \delta A_n + \kappa A_j + \gamma_1 \left( |A_n|^2 + 2 |A_j|^2 \right) A_n \\
- \gamma_2 \left( |A_n|^2 + 6 |A_n|^2 |A_j|^2 + 3 |A_j|^4 \right) A_n = 0. \] (9)

Here, \( \delta \) and \( \kappa \) are detuning parameter and coupling coefficient, respectively. \( \gamma_1 \) and \( \gamma_2 \) are the nonlinear parameters related to the cubic and quintic nonlinear coefficient \( n_2 \) and \( n_4 \), respectively and are defined as

\[ \gamma_1 = \frac{2 \pi n_2}{\lambda_b}, \quad \gamma_2 = \frac{2 \pi n_4}{\lambda_b}, \text{ and } \delta = 2 \pi n \left( \frac{1}{\lambda^2} - \frac{1}{\lambda_b^2} \right). \]

The function in equations (8) and (9) are normalized so that the input intensity \( I_0 \) throughout this paper is expressed in GW/cm\(^2\). It may be noted here that the effective area of the mode is reflected in the intensity and to calculate the actual input power, one has to multiply the intensity by the effective area of the mode. Preliminary solution of NLCMEs was calculated by Agrawal et al. [16] where they predicted the red/blue shift in the stop band depending on the positive/negative nonlinearity of refractive index. In the following analysis, we have obtained the analytical solution of NLCMEs by neglecting higher order terms of backward propagating mode. The solutions of equations (8) and (9) are obtained as

\[ A_j(z) = A_1 \exp(iS'z) + A_2 \exp(iT'z), \] (10)

and

\[ A_n(z) = B_1 \exp(iS'z) + B_2 \exp(iT'z), \] (11)

with

\[ S' = -\left( \gamma_1 I_0 - F \right) + \frac{\sqrt{\left( \gamma_1 I_0 - F \right)^2 + 4q_{\text{in}}^2}}{2} \]

and

\[ T' = -\left( \gamma_1 I_0 - F \right) - \frac{\sqrt{\left( \gamma_1 I_0 - F \right)^2 + 4q_{\text{in}}^2}}{2}. \]

In the above equations, the parameter \( F \) is defined as

\[ F = 2 \gamma_2 I_0^2 - \gamma_3 \left( |A_n|^4 - 2 |A_j|^2 |A_n|^2 \right). \]
Here, \( I_0 = |A_0|^2 + |A_s|^2 \) is the input intensity at Bragg wavelength \( \lambda_B \). The coefficients \( A_1, A_2, B_1 \) and \( B_2 \) are interdependent and satisfy the following four relations

\[
\begin{align*}
\left( \delta - S' + \gamma_1 |A_j|^2 - \gamma_2 |A_j|^4 \right) A_1 &= -\kappa B_1, \\
\left( S' + \delta + \gamma_1 \left( I_0 - |A_j|^2 \right) - \gamma_2 \left( I_0 + 2 |A_j|^2 \right) \right) B_1 &= -\kappa A_1, \\
\left( \delta - T' + \gamma_1 |A_j|^2 - \gamma_2 |A_j|^4 \right) A_2 &= -\kappa B_2, \\
\left( T' + \delta + \gamma_1 \left( I_0 + |A_j|^2 \right) - \gamma_2 \left( I_0 + 2 |A_j|^2 \right) \right) B_2 &= -\kappa A_2.
\end{align*}
\]

Equations (12) are satisfied for nonzero values of \( A_1, A_2, B_1 \) and \( B_2 \) if the possible values of \( q_{nl} \) obey the dispersion relation in nonlinear cubic-quintic regime at high excitation intensity expressed by

\[
q_{nl}^2 = q^2 + \delta (X - U) + (Y + V - W),
\]

with \( q = (\delta^2 - \kappa^2)^{1/2} \) as the linear dispersion relation for the Bragg grating [19] and the parameters \( U, V, W, X \) and \( Y \) are defined as

\[
\begin{align*}
U &= \gamma_2 \left( I_0^2 + 3 |A_j|^4 + 4 |A_j|^2 |A_s|^2 \right), \\
V &= \gamma_2^2 \left( |A_j|^4 |A_s|^4 + 6 |A_j|^2 |A_s|^4 + 3 |A_j|^4 \right), \\
W &= \gamma_1 \gamma_2 \left( |A_j|^4 |A_s|^2 + 5 |A_j|^2 |A_s|^4 + |A_j|^4 + 6 |A_j|^4 |A_s|^2 \right), \\
X &= \gamma_1 \left( I_0^2 + 2 |A_j|^2 \right), \text{ and } Y = \gamma_1^2 |A_j|^2 \left( I_0 + |A_j|^2 \right)
\end{align*}
\]

At moderately high input intensity, the quintic nonlinear parameter \( \gamma_2 \) in equation (13) is negligible and the equation reduces to the case of dispersion relation in nonlinear Kerr medium [26] as

\[
q_{nl}^2 = q^2 + \delta X + Y.
\]
On substituting parameter $q_{cq}$ in equations (10) and (11), the fields of forward and backward propagating modes take the form

$$A_1(z) = A_1 \exp(iqz) \exp \left[ i \left( \frac{4\delta(X-U) + 4(Y + V - W) + F^2}{8q} - F \right) z \right]$$

$$+ A_2 \exp(-iqz) \exp \left[ -i \left( \frac{4\delta(X-U) + 4(Y + V - W) + F^2}{8q} - F \right) z \right],$$

and

$$A_2(z) = B_1 \exp(iqz) \exp \left[ i \left( \frac{4\delta(X-U) + 4(Y + V - W) + F^2}{8q} - F \right) z \right]$$

$$+ B_2 \exp(-iqz) \exp \left[ -i \left( \frac{4\delta(X-U) + 4(Y + V - W) + F^2}{8q} - F \right) z \right].$$

In the above equations, the phases of the forward and backward propagating waves are represented by

$$\varphi(z) = \pm \left( q + \frac{4\delta(X-U) + 4(Y + V - W) + F^2}{8q} - F \right) z.$$  \hspace{1cm} (18)

Equation (18) shows that the phases $\varphi(z)$ of the reflected as well as transmitted waves get modulated due to the effect of nonlinearity at high excitation intensity and this self phase modulation varies with increasing excitation intensity $I_0$.

3. NONLINEAR DISPERSION CURVES IN CQFBG MEDIUM

To understand the behaviour of dispersion relation obtained in equation (13), let us first consider the low power regime. For $\gamma_1 = \gamma_2 = 0$, the dispersion relation (13) becomes $q_{\alpha_1}^2 (= q_1^2) = \delta^2 - \kappa^2$ which is a linear dispersion relation [16]. If the detuning frequency $\delta$ of the incident light falls in the range $-\kappa < \delta < \kappa$, $q_{cq}$ becomes purely imaginary and most of the incident field will be reflected due to the fact that the grating will not support the propagating wave. This range referred to as the photonic bandgap (PBG) or stop band which is similar to the electronic energy band gap occurring in crystals. In order to understand the effect of nonlinearity on the dispersion curves, we have chosen two cases (i) $\gamma_1 = 0, \gamma_2 = 0$ and (ii) $\gamma_1 = 0, \gamma_2 \neq 0$. For the first case, the medium is Kerr type and equation (13) reduces to $q_{\alpha}^2 = q_1^2 + \delta X + Y$. 
We find that the range of the stop band extends to $-\kappa < \delta \left(1 + \frac{X}{\delta} + \frac{Y}{\delta^2}\right)^{1/2} < \kappa$ and equations (12) and (13) resemble to the standard solutions of the coupled mode equations for Bragg grating in Kerr regime [26]. In the second case, the medium is cubic-quintic and we find that the frequency detuning $\delta$ lies in the range $-\kappa < \delta \left(1 + \frac{X}{\delta} + \frac{Y}{\delta^2} - \frac{U}{\delta} + \frac{W - V}{\delta^2}\right)^{1/2} < \kappa$. When $\left(\frac{X}{\delta} + \frac{Y}{\delta^2}\right) > \left(\frac{U}{\delta} + \frac{W - V}{\delta^2}\right)$ the PBG will shift towards the lower branch of the dispersion curve. On the other hand, at $\left(\frac{X}{\delta} + \frac{Y}{\delta^2}\right) = \left(\frac{U}{\delta} + \frac{W - V}{\delta^2}\right)$, the effect of cubic nonlinearity is compensated by quintic nonlinearity. If one chooses $\left(\frac{X}{\delta} + \frac{Y}{\delta^2}\right) < \left(\frac{U}{\delta} + \frac{W - V}{\delta^2}\right)$ the PBG will shift towards upper branch of the dispersion curve. Thus we find that both cubic as well as quintic nonlinearities modify the range of stop band of the FBG and shift the photonic bandgap. To verify the above behavior of PBG, we have plotted detuning parameter $\delta$ as a function of $q_{eq}$ in Figure 1 for input intensities 10 GW/cm$^2$ and 15 GW/cm$^2$ using quasi-CW laser source having wavelength band from 1547 nm to 1553 nm and the pulse width is chosen in the 1 ns-100 ps range. It may be noted here that a similar order of magnitude of the pump intensity was considered by Lee and Agrawal [8] to be obtained from a quasi CW-laser of 1 ns pulse duration. Also, Taverner et al [27] used a quasi-CW Diode-seeded LA-EDFA chain radiation source at 1536 nm in their experimental work to demonstrate all-optical AND gate in an apodized FBG. Almost around the same time, 

![Figure 1](image-url)

**FIGURE 1**

Nonlinear dispersion curves showing variation of $\delta$ with $q_{eq}$ for input intensities 10 GW/cm$^2$ (a) and 15 GW/cm$^2$ (b). Dotted curves show the linear case, $(\gamma_0, \gamma_2 = 0)$, dashed curves show the nonlinear cubic case $(\gamma_0 \neq 0, \gamma_2 = 0)$ and solid lines show the cubic-quintic case $(\gamma_0, \gamma_2 \neq 0)$. 

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these authors [28] also reported their experimental results on nonlinear optical switching using the same pulsed radiation source at an intensity $\sim 15$ GW/cm$^2$. In the light of the availability of such quasi-CW high intensity radiation sources, we have also concentrated our attention to a similar experimental situation with excitation intensity in the same range (viz., 10 to 35 GW/cm$^2$). Also, we consider following FBG parameters of chalcogenide glass with effective mode refractive index $n_{eff} = 2.45$, Bragg wavelength $\lambda_B = 1550$ nm, induced refractive index change $n_g = 5 \times 10^{-4}$, nonlinear cubic and quintic coefficients $n_2 = 2.7 \times 10^{-17}$ m$^2$/W and $n_4 \approx 1 \times 10^{-31}$ m$^4$/W$^2$, respectively [15,19].

In Figure 1, dotted curves correspond to the linear case whereas dashed and solid curves show cubic and cubic-quintic responses, respectively. From this Figure 1, we may infer that self focusing cubic nonlinearity shifts the photonic bandgap in lower branch of the dispersion curve (dashed curves) while self defocusing quintic nonlinearity counterbalance the effect of cubic nonlinearity. Consequently, the stop band returns toward the center frequency (solid curve). Our results are in agreement with the results reported by Agrawal et al. [16], Who suggested that at certain threshold intensity, PBG shifts to either upper or lower branches of the dispersion curves depending on the sign of the cubic nonlinearity.

4. FILTER CHARACTERISTICS OF FBG IN CQ MEDIUM

In order to examine intensity dependent reflection characteristics of the grating in CQ medium we have obtained the analytical solution for the amplitude of forward and backward propagating modes using equations (10) and (11) as

$$A_f(z) = A_i \exp(iS'z) + r(q)B_i \exp(iT'z)$$

and

$$A_b(z) = B_i \exp(iT'z) + r(q)A_i \exp(iS'z).$$

The corresponding expression for the effective reflection coefficient ($r(q)$) in nonlinear cubic quintic regime is obtained by using equation (12b) as

$$r(q) = r \left[ 1 - \frac{\{\delta(X-U)+(Y+V-W)\}}{2q(\delta+q)} \right]$$

$$\quad - \frac{1}{\delta+q} \left[ \gamma_1 |A_i|^2 - \gamma_2 \left( |I_0| + 2 |A_i| \right)^2 + F + \left( \gamma_1 I_0 - F \right)^2 \right]$$

with $r$ being the linear effective reflection coefficient and is given by
From equation (21), we further notice that the effective nonlinear reflection coefficient will have diminishingly small values for

\[ r = -\frac{\kappa}{\delta + q} \]  

For the above condition, the Bragg grating will be transparent. The condition of transparency can be satisfied at specific values of wavelengths lying in the range \(-\kappa < \delta \left(1 + \frac{X}{\varepsilon} + \frac{Y}{\delta} - \left(\frac{U}{\delta} + \frac{V - W}{\varepsilon^2}\right)^{1/2}\right) < \kappa\). It is also clear from the above condition that one can control the pass band as well as stop band by choosing the proper intensity of the incident beam.

Applying the boundary conditions that light is incident only at the front end at \(z = 0\) of the FBG as shown in Figure (2), the nonlinear reflection coefficient \(r_{eq}\) for a grating of length \(L\) has been obtained by using equations (19) to (21) as

\[ r_{eq} = \frac{-\kappa \left[\frac{k}{2} + \tau\right] \left[1 - \exp(ikL)\right]}{\left[\frac{k}{2} + \tau\right]^2 - \kappa^2 \exp(ikL)} \]  

where \(\tau = \delta + \gamma_1 |A_j|^2 - \gamma_2 |A_j|^4\) and \(k = \sqrt{(\gamma_1 I_0 - F)^2 + 4q_{eq}^2}\)

and corresponding expression for the reflectivity \(R_{eq} = |r_{eq}|^2\) in the cubic-quintic nonlinear regime is

\[ R_{eq} = \frac{2\kappa^2 M_\ast [\cosh\theta - \cos\theta]}{\left(M_\ast^2 + \kappa^4\right) \cosh\theta + \left(M_\ast^2 - \kappa^4\right) \sinh\theta - 2\kappa^2 \left(M_\ast \cos\theta + 2\Psi \sin\theta\right)} \]  

\[
\text{FIGURE 2}
\text{Schematic of a FBG of length L illuminated by electromagnetic field of amplitude A(z).}
\]
In the above equation,

\[ \theta_{\text{int}} = k_r L(k_r L), \quad \Psi = \left( k_r + \tau \right) \left( k_i / 2 \right), \quad \text{and} \quad M_A = \left( k_r / 2 + \tau \right)^2 + \left( k_i / 2 \right)^2. \]

\( k_r \) and \( k_i \) being the real and imaginary part of the parameter \( k \), respectively.

Equations (19) to (25) reveal that the condition of the transparency of the Bragg grating gets modified at large excitation intensity. For cubic nonlinearities, the transparency band is large while with quintic nonlinearity it reduces. The reason for this can be attributed to the intensity dependence of the dispersion parameter \( q \) which becomes nonlinear. These equations also exhibit the dependence of effective reflection coefficient \( r(q) \) as well as reflectivity \( R_{\text{BRG}} \) under nonlinear regime on various physical parameters of the CQFBG. In order to examine these features, we have carried out numerical estimates for fiber Bragg gratings using material parameters mentioned in section 3 with values of \( n_L \approx 2 \) and plotted the intensity dependent changes in the reflectivity of FBG in Figure 3 at various input intensities of 0.01 GW/cm\(^2\) (a); 5 GW/cm\(^2\) (b); 15 GW/cm\(^2\) (c); 25 GW/cm\(^2\) (d) and 35 GW/cm\(^2\) (e) for cubic and cubic-quintic nonlinearities, respectively. Doted curves of this Figure show the role of cubic nonlinearity while solid curves represent the cubic-quintic nonlinearity in the medium.

It is clear from the doted curve in Figure 3 that at higher input intensity, the Bragg wavelength \( \lambda_B \) exhibits red shift given by \( 2n_2 |E|^2 \Lambda \) from the Bragg wavelength of 1550 nm due to the contribution of intensity dependent refractive index and from the solid curve, we find that the introduction of SDF quintic nonlinearity counterbalances the red shift of the Bragg wavelength \( \lambda_B \) arising due to the SF cubic nonlinearity. One may infer from this Figure that, the large shift as expected from a cubic nonlinear situation reduces substantially due to the correction in the Bragg wavelength \( \lambda_B \) by an amount of \( 2n_2 |E|^2 - n_4 |E|^4 \Lambda \).

One can also notice a few more important features from Figure 3 such as

(i) At an input intensity of 0.01 GW/cm\(^2\) (Figure 3(a)), the behavior of FBG is linear with Bragg wavelength equivalent to \( 2n_2 |E|^2 - n_4 |E|^4 \Lambda \). (ii) For input intensity lying between 5 and 15 GW/cm\(^2\), the Bragg wavelength shows red shift as well as splitting of the stop band (Figure 3(b) and 3(c)). It is to be pointed out here that such splitting of the stop band at high excitation intensity has been reported earlier by Eggleton et al. [6] who assigned the features to the effect of modulational instability occurring as an interplay between nonlinearity in the glass and group velocity dispersion (GVD) introduced by the grating. But, in the present analysis, GVD term has been neglected and hence we report that the cause of instability/splitting of the reflected beam is the self phase modulation (SPM) arising due to the optical nonlinearity. The instability in the CW beam can be ascribed to the dispersion parameter \( q \) and nonlinear parameters \( \gamma_1 \) and \( \gamma_2 \). Within the stop band, the term \( \left( k_r / 2 + \tau \right)^2 \) in
equation (25) becomes negligibly small at high excitation intensity and results in the splitting of the reflection band. The instability occurs only when the input intensity exceeds a critical value of intensity such that 0.01 GW/cm².

It is also interesting to note that one can obtain a purely imaginary value of qcc if

\[ \kappa^2 > \delta^2 + \delta(X-U)+(Y+V-W). \]  

(26a)

The threshold condition for the propagating mode to become unstable is achieved for \( \kappa^2 = \delta^2 + \delta(X-U)+(Y+V-W) \) yielding two roots of \( \delta \) as
As a consequence, for the condition represented by equation (26), the propagating behavior of the beam becomes completely unstable and undergoes break up as shown in Figure 3. (iii) For input intensity larger than 15 GW/cm², the incorporation of quintic nonlinearity counterbalances the cubic nonlinearity which results in the returning of the Bragg wavelength towards 1550 nm as can be seen from the comparison of the dotted and solid curves in Figure 3 (d). (iv) At around 35 GW/cm², solid curve in Figure 3 (e) shows that the reflectivity of FBG becomes negligibly small and it becomes transparent for entire band (1549 nm -1553 nm). It is expected that at sufficiently high input intensity, large amount of negative nonlinearity is introduced in the medium which not only compensates the effect of cubic nonlinearity but also changes the phase of the reflected beam. As a result, the phase matching condition is not satisfied by the entire band and hence all the incident wavelengths of light is transmitted through the fiber Bragg grating.

5. CONCLUSIONS

To conclude, we have analyzed the influence of the quintic nonlinearity on the dispersion curve as well as on the reflectivity of the FBG. Filter characteristics of CQFBG is investigated analytically by solving the CQ nonlinear coupled mode equations. The expression for the reflectivity of the CQFBG in the quasi-CW regime has been derived. The results show that for appropriate high input intensities quintic nonlinearity affects the stop band of Bragg grating to the extent that the grating becomes transparent for the entire incident wavelength band. The shifting and breaking of the stop band is observed due to the phenomena of SPM in the grating. These nonlinear features of CQFBG can be exploited for generation of solitons as well as for optical switching in lightwave communication system with better precision than those available in the literature.

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Optical multistability in nonlinear fiber Bragg grating

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Using the optical Kerr effect in nonlinear coupled mode theory, the occurrence of optical multistability has been analytically investigated in nonlinear fiber Bragg grating for a quasi-CW laser beam. The expression for the transmittivity is obtained in nonlinear regime. It is observed that multi stable features occur near the stopband when operating wavelength is chosen in the vicinity and inside the stopband of the Bragg grating. The effect of intensity of the incident light and detuning wavelength on the multistable features of fiber Bragg grating is also studied in the present work.

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1. Introduction

The study of optical bistability is one of the most important areas of research in nonlinear optics due to its potential applications in all-optical computing and optical signal processing. Optical bistability (OB) characterizes as an optical system which exhibits two possible output intensities for the same input intensity. Such bistable devices have extensive interesting applications such as optical transistor, differential amplifier, optical switch, optical limiters, optical clipper, optical discriminator and optical memory elements. The principle of optical bistability was first put forward by Szoke et al [1] suggesting that bistability would occur at exact resonance if a Fabry-Perot resonator is filled with a saturable absorber in which the absorption coefficient is a decreasing function of local intensity. Later on McCall [2] numerically proved that under suitable conditions the same system can show differential gain with transistor action. The result of this work was experimentally demonstrated by Gibbs et al [3] using Na vapor filled Fabry-Perot cavity. Almost simultaneously, Felber and Marburger [4] gave the simplest explanation of dispersive optical bistability in a Fabry-Perot resonator where the optical cavity is filled with a material whose refractive index is intensity dependent. Smith et al [5] demonstrated that if a Fabry-Perot resonator contains an electro-optic element then multistability can be observed instead of bistability which has vital role in multilevel optical logic and many state optical memory operations. Later on Okada and Takizawa [6] examined theoretically and experimentally optical multistable characteristics in mirrorless electro-optic device. Miller et al [7] demonstrated optical bistability, multistability, differential gain, limiter and optical transistor in semiconductor InSb Fabry-Perot devices. Lee et al [8] experimentally realized hybrid optical multistability using a semiconductor light emitting device, a photodiode and transistor, where they presented graphical solution as well as a stability analysis to explain the occurrence of optical multistability.

The optical bistability and multistability of periodic media in the form of distributed feedback structure in integrated optics was first investigated by H. G. Winful et al. in 1979 using III-V semiconductor material [9]. Another theoretical demonstration of optical bistability in semiconductor periodic structure was reported by He and Cada [10], where they have calculated for the first time nonlinear reflectivity spectrum and obtained large OB in the vicinity of its stopband due to the optical resonance effect. Herbert and Malcuit [11] described first experimental observation of optical bistability and multistability in nonlinear periodic structure.

The multilevel optical logic operations based on multistability is important to reduce complexity of devices and interconnections since it increases the information capacity of each line and each storage element in an optical communication system as compared to the binary logic operations. The opportunities provided by fiber Bragg grating are of enormous importance for further development of fiber optic communication systems [12]. The nonlinear nature of the grating allows dynamic tuning of the band gap, the study of optical bistability and multistability in FBG has been of considerable significance in recent days. Wabnitz [13] analyzed numerically the nonlinear propagation of counterpropagating pulses in a nonlinear fiber Bragg grating and discussed bistable switching of intense optical pulses. Broderick [14] presented theoretically and numerically all-optical switching characteristics in nonlinear fiber Bragg grating using cross phase modulation. Mellon et al [15] demonstrated experimentally all-optical switching phenomena in phase shifted FBG based on a cross phase modulation induced by an intense pump pulse on a low intensity probe. Ogusu and Kamizono [16] investigated the effect of the material response time on optical bistability in a nonlinear fiber...
Bragg grating and found that switch-on time depends on the material response time and the switch-off time is almost independent of it. Lee and Agrawal [17] considered both the uniform and phase shifted grating and compared their performance numerically as a nonlinear switch when optical pulses are sent to the grating. Recently, Yosia et al. [18] have observed double optical bistability in nonlinear π-phase shifted chalcogenide fiber Bragg grating (c-FBG) and suggested all optical transistor operation in such device.

In the present work, we have studied analytically the phenomena of optical multistability in fiber Bragg grating using coupled mode theory. We have solved nonlinear coupled mode equations (NLCMEs) in a simplest way and obtained the solutions for forward and backward propagating field amplitudes. The expression for transmittivity of fiber Bragg grating for a quasi-CW laser beam is obtained and optical multistability behavior is studied. The numerical results based on our analysis show that the multistable phenomenon is strongly dependent upon the applied input electric field intensity as well as on the wavelength of the incident light.

2. Theoretical model

A fiber Bragg grating couples forward and backward propagating waves with wavelength λ close to the Bragg wavelength λB. We have assumed that Bragg grating have Kerr type response so that the nonlinear refractive index is given by

\[ n(\omega, z) = n_{\text{eff}}(\omega) + n_z |E|^2 + n_x(z) \]  

where \( n_{\text{eff}} \) is the average refractive index of the grating, \( n_z \) is the Kerr coefficient and \( n_x(z) \) is the periodic index variation and \( E \) is the electric field propagating inside the grating and is written as [12]

\[ E(z, \omega) = A_f(z, \omega) \exp(i \beta_f z) + A_b(z, \omega) \exp(-i \beta_b z) \]  

Here, \( A_f \) and \( A_b \) are the amplitudes of the forward and backward propagating waves, respectively and \( \beta_f = \pi / \Lambda \) is the Bragg wave number. Using standard assumptions of a slowly-varying envelope approximation, we have used the following pair of NLCMEs [12]:

\[ i \frac{\partial A_f}{\partial z} + \delta A_f + \kappa A_b + \gamma(A_f^2 + 2|A_f|^2) A_f = 0, \]  

and

\[ -i \frac{\partial A_b}{\partial z} + \delta A_b + \kappa A_f + \gamma(A_b^2 + 2|A_b|^2) A_b = 0. \]  

Here, δ, κ and γ are detuning parameter, linear coupling coefficient and nonlinear coefficient, respectively, and are defined as

\[ \delta = 2n_{\text{eff}} \left( \frac{1}{\lambda} - \frac{1}{\lambda_B} \right), \quad \kappa = \frac{2\pi n_{\text{eff}}}{\lambda_B}, \quad \text{and} \quad \gamma = \frac{2\pi n_{\text{eff}}}{\lambda_B}. \]  

In the following analysis we have solved the above NLCMEs analytically by neglecting higher order terms of backward propagating mode and solutions are obtained as [19]

\[ A_f(z) = A_1 \exp(i Sz) + A_2 \exp(i Tz), \]  

and

\[ A_b(z) = B_1 \exp(i Sz) + B_2 \exp(i Tz). \]  

with

\[ S = -\frac{\gamma A_0 + \sqrt{\gamma^2 A_0^2 + 4q^2}}{2} \]  

and

\[ T = -\frac{\gamma A_0 - \sqrt{\gamma^2 A_0^2 + 4q^2}}{2}. \]  

Here, \( I_0 = |A_0|^2 \) is the input intensity at Bragg wavelength \( \lambda_B \) and \( q_{\text{nl}} \) is the nonlinear dispersion relation in nonlinear Kerr regime and is defined as

\[ q_{\text{nl}}^2 = q^2 + \delta X + Y, \]  

where \( q = (\delta^2 - \kappa^2)^{1/2} \) is the linear dispersion parameter and parameters X and Y are defined as

\[ X = \gamma \left( I_0 + 2|A_f|^2 \right) \]  

and

\[ Y = \gamma^2 |A_f|^2 \left( I_0 + |A_f|^2 \right). \]  

In the presence of Kerr nonlinearity the photonic band gap becomes

\[ -\kappa < \delta \left( 1 + \frac{X}{\delta^2} + \frac{Y}{\delta^2} \right)^{1/2} < \kappa, \]  

\( q_{\text{nl}} \) becomes purely imaginary and most of the incident field will be reflected due to the fact that the grating will not support the propagating wave. It is clear from expression of nonlinear photonic band gap that the intensity of the input beam modifies the dispersion parameter and such modification affects the reflection and transmission characteristics of the grating. On substituting parameter \( q_{\text{nl}} \) in equations (5) and (6), the fields of forward and backward propagating modes take the form

\[ A_f(z) = A_1 \exp(i Sz) + t_{ab} B_2 \exp(i Tz) \]  

and

\[ A_b(z) = B_2 \exp(i Tz) + t_{ab} A_1 \exp(i Sz). \]
Here, $t_{nl}$ is the effective transmission coefficient in nonlinear regime and is found as

$$t_{nl} = \frac{\kappa}{S + \delta + \gamma |A_0|^2}$$  \hspace{1cm} (11)

Applying the proper boundary conditions, the nonlinear transmission coefficient ($t_{ng}$) for a grating of length $L$ has been obtained by using equations (9) to (11) as

$$t_{ng} = \frac{A_f(z = L)}{A_f(z = 0)} = \frac{\psi^2 - \kappa^2}{\psi^2 - \kappa^2} \exp(i\delta L)$$  \hspace{1cm} (12)

where,

$$\psi = \left(\frac{k}{2} + \tau\right), \tau = \delta + \gamma |A_0|^2$$

and

$$k = \sqrt{\gamma^2 \kappa^2 + 4 \alpha c_{nl}}.$$  \hspace{1cm} (11)

The corresponding expression for the transmittivity $T_{ng} = |t_{ng}|^2$ in the nonlinear regime is

$$T_{ng} = \frac{1}{1 + \left(\frac{4 \psi^2 \kappa^2}{(\psi^2 - \kappa^2)^2}\right) \sin^2(\Phi)}$$  \hspace{1cm} (13)

With $\Phi = kL/2$. It is interesting to compare the transmittivity of nonlinear fiber Bragg grating obtained in Equation (13) with transmittivity ($\tau$) of standard electrooptic nonlinear Fabry-Perot device obtained by Smith et al in 1978 [7] as

$$\tau = \frac{1}{1 + \left(\frac{4R}{(1-R)^2}\right) \sin^2(\Phi)}$$  \hspace{1cm} (14)

A comparison of Equation (13) and (14) shows that the FBG is equivalent to an optical resonator with mirror reflectivity (transmissivity) $R = \kappa^2 \psi^2 (1-R) = \psi^2 - \kappa^2$ and phase shift $\Phi = kL/2$.

3. Results and discussions

On the basis of the theoretical formulations developed in the preceding sections (Equation 13), we have demonstrated the optical multistability behavior of fiber Bragg grating in nonlinear Kerr regime by plotting the transmitted intensity with input intensity in Fig. 1. The tunable quasi-CW laser source in C-band (1535 - 1565 nm) is assumed as the light source. It may be noted here that a similar order of magnitude of the pump intensity was considered by Lee and Agrawal [17] to be obtained from a quasi-CW-laser of 1 ns pulse duration. Also, Taverner et al [20] used a quasi-CW Diode-seeded LA-EDFA chain radiation source at 1536 nm in their experimental work to demonstrate all-optical AND gate in an apodized FBG. All the results presented here are for chalcogenide FBG having effective index $n_{eff} = 2.45$, change in grating index $n_g = 3 \times 10^{-3}$, nonlinear Kerr coefficient $n_2 = 2.7 \times 10^{-17}$ m$^2$/W. The length of the grating $L = 2$ cm and Bragg wavelength $\lambda_0 = 1550$ nm were chosen [18]. We have considered the chalcogenide glass FBG because it reduces the required input intensity to observe nonlinear effects as compared to silica FBG due to high value of nonlinear Kerr coefficient $n_2$ in such glasses.

The plot of the transmitted intensity as a function of the input intensity is given in Fig. 1 for three different values of incident wavelengths such as $\lambda = 1549.75$ nm (Fig. 1a), $\lambda = 1549.80$ nm (Fig. 1b), $\lambda = 1549.85$ nm (Fig. 1c) and $\lambda = 1549.90$ nm (Fig. 1d). All the incident wavelengths considered above are lying inside the stop band of the fiber Bragg grating. At low intensity these wavelength are reflected by the grating as a result the transmission of the structure is low. As the intensity of the input beam is increased the average refractive index of the grating will increases and the stop band appear to shift towards higher wavelengths side resulting in an increase in the transmission of those wavelengths which were reflected by the grating at low intensity.

Fig. 1. Transmitted vs. incident intensity for a nonlinear fiber Bragg grating for different values of detuning wavelengths.
It is observed from Fig. 1, that when the wavelength of the incident light is tuned deeper into the stop band (very near to the Bragg resonance, Fig. 1d) the transmitted intensity shows strong oscillatory behavior. The occurrence of oscillatory behavior can be explained as follows: It is well established that reflection spectrum of FBG shows the presence of multiple sidelobes with decreasing intensity located at each side of the stop band. These sidelobes originate from the weak reflections occurring at the two grating ends where refractive index changes suddenly compared to its value outside the grating region due to which a Fabry-Perot cavity with its own wavelength dependent transmission is formed. As the input intensity increases the feedback path of the Fabry-Perot cavity increases. As a result the phase shift of the incident light is increases due to self phase modulation. This causes the strong periodic sidelobes showing multistability in the transmission. Our observations are consistent with the stability analysis of Sterke [26] who suggested that at high excitation intensity there are many regions where the high transmission states are predicted due to temporal fluctuations which become chaotic. He found that as the wavelength of the incident beam is tuned deeper and deeper into the stopgap, the system tends to become more and more unstable giving rise to many stable and unstable states at the output. Multistable behavior in nonlinear fiber Bragg grating can also be considered in terms of many gap solitons formation inside the stopband of the grating [21-24]. In 1998 Broderick et al. [25] has observed experimentally five gap soliton at a particular input intensity when the wavelength of the incident beam is tuned inside the photonic bandgap of fiber Bragg grating. They suggested that the bistable switching is associated with the formation of gap soliton inside the grating.

4. Conclusion

We have investigated the phenomenon of optical multistability in nonlinear chalcogenide fiber Bragg grating by incorporating Kerr effect in the coupled mode analysis. The expression for the intensity dependent transmittivity is obtained by solving nonlinear coupled mode equations analytically for quasi-CW laser beam. The theory demonstrates that the optical multistability takes place when operating wavelengths are chosen inside the stopband of fiber Bragg grating at suitable incident intensity. We believe the present analytical study will be useful in future experimental work for the exploration of optical multistability in nonlinear fiber Bragg grating and for the development of nonlinear optical components for all-optical signal processing.

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OPTICAL LIMITING IN NONLINEAR FIBER BRAGG GRATING

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Nonlinear refraction based optical limiting phenomena is investigated analytically in the transmission mode of a fiber Bragg grating using coupled mode theory. A simple expression for intensity dependent transmittivity has been obtained by incorporating the Kerr effect in nonlinear coupled equations. The transmission is studied as a function of the incident intensity at various incident wavelengths. We have found that the transmittance of the grating gradually increases as a ramp function and becomes constant after a critical intensity when the wavelength of the incident light is tuned above the Bragg wavelength.

Keywords: Optical limiting; Fiber Bragg grating; nonlinear refraction; nonlinear coupled mode equation.

1. Introduction

Optical power limiters (OPL) have tremendous potential as simple yet effective devices for controlling the optical power to protect the sensitive optical components from high-power laser radiation. Nowadays, optical fiber network technology demands the all-optical fiber based nonlinear devices to control the intensity of light in an efficient manner. In this aspect, optical fiber limiters have received significant attention. Optical limiting (OL) is a nonlinear phenomenon in which the transmittance of the device decreases with increased incident light intensity. An ideal optical
limiter has a linear transmittance at low input intensities but above the threshold intensity, its transmittance becomes constant. In modern optical communication system, it is demonstrated that optical limiting not only controls the optical power but can also be used for pulse shaping, pulse smoothing, pulse compression and sensor applications. Optical limiting results from irradiancedependent nonlinear optical properties of materials. The incoming intense light alters the refractive and absorptive properties of the materials resulting in a greatly reduced transmitted intensity and therefore, it is important to determine the magnitude of the nonlinearity of materials to select suitable materials as optical limiting media. After the pioneer work of Siegman in 1962, optical limiting (OL) is extensively investigated both theoretically and experimentally in a wide range of materials via different nonlinear optical mechanisms such as reverse saturable absorption, nonlinear refraction, induced scattering, thermal Blooming and multiphoton absorption.

Recently, optical signal processing using nonlinear periodic structures has become the interest of the researchers for current photonic network technology. Nonlinear periodic structures work on the principle of nonlinear refractive index change and distributed Bragg reflection. These structures also offer many structural and material degrees of freedom for achieving desired optical signal processing functionality. From the available literature, one may find that nonlinear periodic structures support optical switching, optical bistability and solitonic propagation of pulses. Herbert et al., for the first time, experimentally observed optical power limiting in nonlinear periodic structures using a thermal nonlinearity in a dye-doped colloidal crystal. Brzozowski and Sargent have analyzed broad-band optical limiting behavior in disordered nonlinear structures that are periodic on average. They have shown that highly disordered structures exhibit true optical limiting over a spectral range much greater than the limiting bandwidth of perfectly periodic nonlinear media. The same authors provided the solutions of coupled-mode equations for one-dimensional (1D) periodic medium composed of layers with an identical linear refractive index and alternating opposite Kerr coefficients. Pelinovsky et al. analyzed both theoretically and numerically the all-optical limiting in nonlinear periodic structure when the Kerr nonlinearity is compensated exactly across the alternating layers. Using the coupled mode theory, Sheriff et al. investigated numerically the optical limiting and intensity dependent diffraction in low-contrast nonlinear photonic crystals periodic in one, two and three dimensions. Dong and Hu studied the propagation of coherent light through a passive optical power limiter consisting of two alternating layers with different linear and nonlinear refractive indices and obtained general formulas for the transmittance and the optical limiting output power.

The coupled mode theory developed in reference yields the relationship between the incident and transmitted intensities for distributed feedback structure which consist of alternating layers of materials possessing opposite Kerr nonlinearities. The physical structure of such periodic devices seems to be complicated from fabrication point of view. Nowadays, optical signal processing operations demand
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a compact optical fiber based systems to maintain data rate speed and couple light in and out of optical fibers in an efficient way. Fiber Bragg grating offers one possible solution for constructing inline optical devices to fulfill the above requirements. Advantages of fiber Bragg grating over competing semiconductor material-based distributed waveguide components include all-fiber geometry, low insertion loss, low absorption loss, low scattering loss, high return loss and potentially low cost. Moreover, the most distinguishing feature of fiber Bragg grating is the flexibility they offer in achieving desired spectral characteristics. The less-loss optical transmission at 1550 nm wavelength and other advantages of fiber Bragg grating have stimulated the present authors to examine analytically the occurrence of optical limiting in optical fiber Bragg gratings in place of the conceptualized distributed feedback systems introduced by several authors.

With this in mind, we have made our efforts to investigate analytically the occurrence of optical limiting action in an optical fiber Bragg gratings using the basic set of nonlinear coupled mode equations and derived the solutions for both forward and backward propagating field amplitudes. We have obtained an expression of transmittivity for a quasi CW laser beam that gives physical insight of optical limiting and relate the response of the FBG as limiter to its physical parameters. The investigation is based on the nonlinear refraction mechanism which describes an intensity dependent refractive index modulation. To study the optical limiting, we have compared the limiting action for four different incident wavelengths. All the wavelengths are considered slightly above the Bragg wavelength. The results of our study show that after a certain threshold intensity, transmittance of the grating become constant and FBG works as a perfect optical limiter.

2. Theoretical Model

A fiber Bragg grating is a periodic structure of alternating layers of high and low refractive indices along the length of the core of the single mode optical fiber whose spatially intensity dependent refractive index may be expressed as

\[ n(\omega, z) = n_{\text{eff}}(\omega) + n_2|E|^2 + n_g(z) \]  

(1)

where \( n_{\text{eff}} \) is the unperturbed refractive index, \( n_2 \) is the nonlinear Kerr coefficient and \( n_g(z) \) is the periodic index variation inside the grating. In order to obtain an analytical expression for forward- and backward-propagating fields inside the structure, we express the electric field \( \vec{E} \) within the structure as

\[ \vec{E}(r, \omega) = F(x, y)\vec{A}_f(z, \omega) \exp(i\beta_f z) + F(x, y)\vec{A}_b(z, \omega) \exp(-i\beta_f z). \]  

(2)

Here, \( \beta_f = \pi/\Lambda \) is the Bragg wave number, \( \vec{A}_f \) and \( \vec{A}_b \) are the field amplitudes of the forward and backward travelling waves respectively. Transverse variation for these two counter-propagating modes is governed by the same modal distribution \( F(x, y) \) in a single mode fiber. In order to examine the characteristics of
counter-propagating modes, the following pair of nonlinear coupled mode equations (NLCMEs) for one-dimension has been used:

\[ i \frac{\partial A_f}{\partial z} + \delta A_f + \kappa A_b + \gamma (|A_f|^2 + 2|A_b|^2) A_f = 0, \]  
\[ -i \frac{\partial A_b}{\partial z} + \delta A_b + \kappa A_f + \gamma (|A_b|^2 + 2|A_f|^2) A_b = 0. \]

Here, \( \delta, \kappa \) and \( \gamma \) are detuning parameter, linear coupling coefficient and nonlinear parameter, respectively, and are defined as

\[ \delta = 2 \pi n_{\text{eff}} \left( \frac{1}{\lambda} - \frac{1}{\lambda_B} \right), \quad \kappa = \frac{\pi n_2}{\lambda_B} \quad \text{and} \quad \gamma = \frac{2 \pi n_2}{\lambda_B}. \]

In the following analysis, we have solved the above NLCMEs analytically by neglecting higher order terms of backward propagating mode and the solutions are obtained as

\[ A_f(z) = A_1 \exp(iS_1 z) + A_2 \exp(iS_2 z). \]  
\[ A_b(z) = B_1 \exp(iS_1 z) + B_2 \exp(iS_2 z). \]

with

\[ S_1 = -\frac{\gamma I_0 + \sqrt{\gamma^2 I_0^2 + 4q_{nl}^2}}{2} \quad \text{and} \quad S_2 = -\frac{\gamma I_0 - \sqrt{\gamma^2 I_0^2 + 4q_{nl}^2}}{2}. \]

Here, \( I_0 = |A_f|^2 + |A_b|^2 \) is the input intensity at Bragg wavelength \( \lambda_B \), \( q_{nl} \) is the nonlinear dispersion parameter in Kerr medium and is defined as

\[ q_{nl}^2 = q^2 + \delta X + Y \]

where \( q = (\delta^2 - \kappa^2)^{1/2} \) is the linear dispersion parameter for the Bragg grating and \( X \) and \( Y \) are defined as

\[ X = \gamma (I_0 + 2|A_f|^2) \quad \text{and} \quad Y = \gamma^2 |A_f|^2 (I_0 + |A_f|^2). \]

Equations (7) and (8) exhibit the intensity dependent modification in the dispersion parameter. On substituting parameter \( q_{nl} \) in Eqs. (5) and (6), the fields of forward and backward propagating modes take the forms

\[ A_f(z) = A_1 \exp(iS_1 z) + t_{nl} B_2 \exp(iS_2 z) \]

and

\[ A_b(z) = B_2 \exp(iS_2 z) + t_{nl} A_1 \exp(iS_1 z). \]

In the above equations, \( t_{nl} \) is the effective transmission coefficient under nonlinear regime and on mathematical simplification, one finds

\[ t_{nl} = \frac{\kappa}{S_1 + \delta + \gamma (I_0 + |A_f|^2)}. \]
Applying the proper boundary conditions, the nonlinear transmission coefficient \( t_{ng} \) for a grating of length \( L \) has been obtained by using Eqs. (9) to (11) as

\[
t_{ng} = \frac{A_f(z = L)}{A_f(z = 0)} = \frac{(k'^2 - \kappa^2) \exp(iS_1L)}{k'^2 - \kappa^2 \exp(iKL)}
\]  

with

\[
k' = \left( \frac{k}{2} + \tau \right), \quad \tau = \delta + \gamma |A_f|^2 \quad \text{and} \quad k = \sqrt{\gamma^2 |f_0|^2 + 4\tau^2_{nl}}.
\]

Quite interestingly, the above formalism yields the expression for the transmittivity \( T_{ng} = |t_{ng}|^2 \) of the FBG acting like a nonlinear optical material as

\[
T_{ng} = \frac{1}{1 + F'\sin^2(\Phi'/2)}
\]

where \( F' = 4k'^2\kappa^2/(k'^2 - \kappa^2)^2 \) is the effective finesse, \( \Phi' = kL \) is the phase shift and \( L \) is the length of the fiber Bragg grating. Equation (13) can be compared with the well-known expression for the transmittivity in a standard nonlinear Fabry-Perot device\(^25\) given by

\[
T = \frac{1}{1 + F\sin^2(\Phi/2)}
\]

where \( F = 4R/(1 - R)^2 \) is the finesse of the resonator, \( R(T + R = 1) \) is the mirror reflectivity, \( \Phi = 2\pi n_0 L(\lambda/2)^{-1} \) is the round trip phase shift, \( \lambda \) is the wavelength of the radiation and \( L \) is the length of the cavity. Thus, the FBG can be considered to be equivalent to an optical resonator with mirror reflectivity \( R = k'^2\kappa^2 \), transmittivity \( T(= 1 - R) = k'^2 - \kappa^2 \) and the phase shift \( \Phi = \Phi' = kL \). It appears worthy to note that the transmittivity of the Bragg grating is calculated by rationalized Eq. (12) and introducing the finesse of the so-called Fabry-Perot interferometer as \( T_{ng} = |t_{ng}|^2 \), similar to the definition of reflectivity \( R_{ng} = |r_{ng}|^2 \) as followed by Agrawal\(^25\). Subsequently, the comparison of Eqs. (13) and (14) enables one to infer on the bistable response and the optical limiting achieves in the chalcogenide glass optical fiber. We observe that the finesse of fiber Bragg grating cavity is intensity dependent and increases with increasing input intensity whereas finesse of nonlinear Fabry-Perot device is constant and depends on the ratio of mirror reflectivity and transmittivity.

3. Results and Discussions

On the basis of the theoretical formulations developed in the preceding sections (Eq. 13), we have demonstrated optical limiting behaviour of chalcogenide fiber Bragg grating in nonlinear Kerr regime by plotting the transmitted intensity \( (I_T = I_0 \times T_{ng}) \) as a function of input intensity \( (I_0) \) in Fig. 1. The response of the FBG limiter was investigated for four different incident wavelengths such as 1550.5 nm (doted curve), 1551 nm (dashed curve), 1551.5 nm (dashed-doted
Fig. 1. Optical limiting behavior of chalcogenide FBG with four detuning wavelengths: 1550.5 nm (dotted curve), 1551 nm (dashed curve), 1551.5 nm (dashed-dotted curve) and 1552 nm (solid curve). (curve) and 1552 nm (solid curve), considering the reflectivity of the grating as 90%, corresponding to value of $\kappa L = 3.65$. The incident wavelength used here are slightly greater than the Bragg wavelength of the Bragg grating. The tunable quasi-CW laser source in C-band (1535 - 1565 nm) can be selected as the light source. It may be noted that a similar order of magnitude of the pump intensity obtainable from a quasi CW-laser of 1 ns pulse duration was considered by Lee and Agrawal.12 Also, Taverner et al.26 used a quasi-CW Diode-seeded LA-EDFA chain radiation source at 1536 nm in their experimental work to demonstrate all-optical AND gate in an apodized FBG. Looking to the potentiality of the chalcogenide glass as FBG materials27-29 for optical limiting, we have made the numerical analysis of the occurrence of optical limiting in a chalcogenide glass with physical parameters such as effective index $n_{\text{eff}} = 2.45$, change in grating index $n_g = 6 \times 10^{-1}$, nonlinear Kerr coefficient $n_2 = 2.7 \times 10^{-17}$ m²/W, length of the grating $L = 3$ mm and Bragg wavelength $\lambda_B = 1550$ nm.

Figure 1 illustrates distinctly the optical limiting actions of fiber Bragg grating as studied by varying the incident intensity up to 900 MW/cm². It is clear from this figure that for very low excitation intensity, the transmitted intensity increases with incident intensity as a ramp function. Once this input intensity exceeds certain limit, the transmitted intensity begins to roll off and become constant. Such nonlinear characteristics of the grating are the signature of the well-known phenomenon known as optical limiting action and the system demonstrating this behaviour is called an optical limiter. The physical mechanism behind the limiting action of the grating can be explained as follows: A low excitation intensity, the FBG is highly reflective for wavelengths falling in the range $-\kappa < \delta < \kappa$ and transmissive for $-\kappa > \delta > \kappa$. The detuning parameter $\delta$ represents the difference between the frequency of the applied signal and the Bragg frequency of the grating. At high excitation intensity, detuning parameter $\delta$ as defined earlier becomes intensity
dependent and lies in the range $-\kappa < \delta (1 + \frac{x}{x} + \frac{y}{y})^{1/2} < \kappa$. The nonlinear dispersion parameter $q_n t$ becomes purely imaginary and most of the incident field will be reflected due to the fact that the grating will not support the propagating wave. If the frequencies of the illuminated beam are chosen outside of the stopband, the incident field will be transmitted and transmittance initially increases with increasing incident intensity. As the input intensity is further increased, the average refractive index of the structure increases with intensity which shifts the Bragg wavelength to a longer wavelength side and changes the width and depth of the photonic band gap. This average increase in refractive index due to the presence of Kerr nonlinearity also modulates the phase of the reflected and transmitted waves. As a result, new frequency components are continuously generated in the nonlinear medium.

It is very important to note that the stable optical limiting can be achieved when the frequency of the incoming beam completely lies within the stopband. It means that the photonic band gap should widen with increasing excitation intensity rather than shifting of the stopband. In order to obtain true optical limiting, Brzozowski and Sargent gave a theoretical model to analyze the passive optical limiter in nonlinear distributed structure which consist of alternating layers of materials possessing opposite Kerr nonlinearities. This arrangement kept average refractive index constant with increasing intensity but the width of the stopband increased such that the Bragg frequency remained unchanged. In our analysis, we have chosen strong grating with $kL \approx 3.65$. The width of photonic band gap of such grating increases with increasing intensity. As a result, most of the newly generated frequency components are reflected back to the input.

Fig. 2. Threshold limiting intensity at $dl_T/dl_0 = 0$ for four detuning wavelengths: 1550.5 nm (doted curve), 1551 nm (dashed curve), 1551.5 nm (dashed-doted curve) and 1552 nm (solid curve).
generated frequency components due to SPM are accommodated in the widened stop band. Hence, very small change in transmitted intensity with increasing intensity is observed after certain threshold input intensity. These observations lead one to infer that the grating acts as a limiter in the transmission regime. The value of threshold intensity at which the grating starts behaving as a limiter is called threshold limiting intensity \( I_{\text{limiting}} \) and is obtained by plotting the derivative \( dI_T/dI_0 \) as a function of the input intensity in Fig. 2.

Figure 2 manifests that for positive value of the derivative \( dI_T/dI_0 \), the corresponding transmitted intensity as plotted in Fig. 1 increases linearly with incident intensity. The region for which \( dI_T/dI_0 > 0 \) can be designated as the switching region of the limiter. For negative values of \( dI_T/dI_0 \), the transmitted intensity is fairly constant. Such a region can be considered as the clamped region of the device. The value of input intensity at which \( dI_T/dI_0 \) becomes zero can be termed as the threshold limiting intensity. A close look at Figs. 1 and 2 enables one to conclude that \( dI_T/dI_0 = 0 \) and \( I_{\text{limiting}} \) occur at the same value of the input intensity.

4. Conclusions

To conclude, we have analyzed optical limiting phenomena in nonlinear fiber Bragg grating by incorporating optical Kerr effect in the coupled mode analysis. The intensity dependent transmittivity of fiber Bragg grating is obtained analytically. We have found that for \( (\pi n_g L/\lambda_B) \approx 3.65 \), the transmittance of the grating becomes constant at a certain threshold intensity. This shows that the grating works as an optical limiter and can be a potential candidate for all-optical limiting applications in tomorrow's optical computing and communication systems.

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Optical Bistability in Kerr Fiber Bragg Grating

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Reflective optical bistability in a Kerr fiber Bragg grating has been analysed by solving nonlinear coupled mode equations for a quasi-CW laser beam. The input-output characteristic of the reflected light from fiber Bragg grating exhibits hysteresis loop. It has been found that the width of the hysteresis loop and switching intensity are strongly dependent on the detuning wavelength of the incident light.

Keywords: fiber bragg grating, nonlinear coupled mode equations, optical bistability, optical switching and Kerr nonlinearity.

1. INTRODUCTION

Optical bistability is one of the most interesting phenomena in a nonlinear optical medium which has received increasing interests due to their crucial role in realizing the future of all-optical signal processing, optical logics, optical switching and optical memory technology. A system is said to be optically bistable if it exhibit two output states for the same input intensity over some range of input values. Optical bistability in a nonlinear medium was first analyzed in a Fabry-Perot resonator [1] where the optical cavity is filled with a material whose refractive index is intensity dependent. Under the action of an
intense optical beam, the material exhibit a nonlinear response to the incident beam in the sense that the transmitted intensity is a nonlinear function of the incident intensity. Based on such nonlinear response, this device can be used as differential amplifier, optical switch, optical limiter, optical clipper, optical discriminator and an optical memory element depending on the chosen nonlinear medium and proper feedback conditions. The optical bistability of periodic media in the form of distributed feedback structure in integrated optics was first investigated by Winful et al. in 1979 using III-V semiconductor material [2]. The results of this investigation were confirmed experimentally in a variety of geometries such as distributed feedback laser amplifiers, semiconductors and colloidal crystal [3-6]. Winful and his coworkers [2] have demonstrated that a distributed feedback structure with a nonlinear refractive index is bistable with the feedback mechanism being distributed throughout the nonlinear medium. However, in recent years significant growth of interest among researchers has been witnessed in the application of the concept of optical nonlinearities in solids including Fabry-Perot etalons to the optical fibers with huge potentialities in optical communication technologies. The observation of optical switching and bistability in low loss optical fiber opened a new vista of optical fiber long haul communication [7-9].

Nowadays, signal processing operations entirely within the optical domain has eliminated the requirement of optical-electrical-optical conversions, while providing the agility and speed inherent to optical elements. Also, the explosive growth of the internet communications over recent years has placed increasing demands on both the channel capacity and fast speed optical devices in optical fiber networks. For achieving high speed data rate through optical fiber communication system there is a need of a compact optical fiber based component to maintain data rate speed and couple light in and out of optical fibers in efficient way. Fiber Bragg grating offers one possible solution for constructing inline optical devices to fulfill above requirements. Advantages of fiber Bragg grating over competing semiconductor material based optical components include all-fiber geometry, low insertion loss, low absorption loss, low scattering loss, high return loss, and potentially low cost. Moreover, the most distinguishing feature of fiber Bragg grating is the flexibility they offer for achieving desired spectral characteristics. Numerous physical parameters can be varied including: induced index change, length, apodization, period chirp, fringe tilt and desired wavelength. The nonlinear nature of the grating allows dynamic tuning of the band gap which shows many interesting nonlinear optical phenomena. Several numerical and experimental activities have been focused on the investigation of the optical switching and bistability in such periodic structure [10-17].

The loss less optical transmission at 1550nm wavelength and other advantages of fiber Bragg grating have stimulated the present authors to examine analytically the occurrence of optical bistability in optical fibers using the fiber Bragg gratings in place of the conceptualized distributed feedback systems introduced by Winful et al [2]. Moreover, the coupled mode theory
Optical Bistability of Kerr materials developed in reference [2] yielding finally the relation between the incident and transmitted intensities in terms of elliptic functions appears to be of complicated nature in drawing any analytical inference. Accordingly, we have addressed ourselves to the analytical investigations of the occurrence of optical bistability in an optical fiber with fiber Bragg gratings from the basic set of nonlinear coupled mode equations and derived the solutions for both forward and backward propagating field amplitudes which were taken as assumed solutions by Winful et al [2].

2. THEORETICAL MODEL

Linear theory of fiber Bragg grating shows that the refractive index of the grating is not dependent on the electric field associated with the propagating wave inside the structure. We are interested in exploring how the reflection of such structure is modified when the spatially varying index of refraction is intensity dependent. We assume that at high excitation intensity, the refractive index of the grating becomes intensity dependent and expressed as

\[ n(\omega, z) = n_{\text{eff}}(\omega) + n_2 |E|^2 + n_1(z) \]  

where \( n_{\text{eff}} \) is the average refractive index change of the fiber mode, \( n_2 \) is the nonlinear Kerr coefficient which is related to the third order nonlinear optical susceptibility \( \chi^{(3)}; n_1(z) \) is the periodic index variation inside the grating that is responsible for the coupling between forward and backward propagating fields in the structure. The electric field \( E \) propagating inside the grating and is written as a sum of propagating and counter-propagating waves near the Bragg frequency as \[ E(r, \omega) = F(x, y) A_f(z, \omega) \exp(i \beta_B z) + F(x, y) A_b(z, \omega) \exp(-i \beta_B z). \]  

Here, \( A_f \) and \( A_b \) are the amplitudes of the forward and backward propagating waves, \( \beta_B = \pi / \Lambda \) is the Bragg wave number related with Bragg wavelength through the Bragg condition \( \lambda_B = 2n \Lambda \). Transverse variation for these two counterpropagating waves are governed by the same modal distribution \( F(x, y) \) in a single mode fiber. Under slowly varying envelope approximation, the following pair of nonlinear coupled mode equations describes the propagation of forward and backward propagating waves in the periodic structure [15]:

\[ i \frac{\partial A_f}{\partial z} + \delta A_f + \kappa A_b + \gamma (|A_f|^2 + 2|A_b|^2) A_f = 0, \]  

and

\[ -i \frac{\partial A_b}{\partial z} + \delta A_b + \kappa A_f + \gamma (|A_b|^2 + 2|A_f|^2) A_b = 0. \]
Here, $\delta$, $\kappa$, and $\gamma$ are detuning parameter, linear coupling coefficient and non-linear parameter, respectively, and are defined as

$$\delta = 2\pi n_g \left( 1 - \frac{1}{\lambda} \right), \quad \kappa = \frac{2\pi n_s}{\lambda} \quad \text{and} \quad \gamma = \frac{2\pi n_s}{\lambda}.$$

In the following analysis we have solved the above NLCEMEs analytically by neglecting higher order terms of backward propagating mode and solutions are obtained as [19]

$$A_f(z) = A_1 \exp(iSz) + A_2 \exp(iTz),$$

and

$$A_b(z) = B_1 \exp(iSz) + B_2 \exp(iTz).$$

with $S = -\gamma I_0 + \sqrt{\gamma^2 I_0^2 + 4q_{nl}^2}$ and $T = -\gamma I_0 - \sqrt{\gamma^2 I_0^2 + 4q_{nl}^2}$.

Here, $I_0 = |A_f|^2 + |A_b|^2$ is the input intensity at Bragg wavelength $\lambda_B$, $q_{nl}$ is the nonlinear dispersion relation in cubic medium and is defined as

$$q_{nl} = q^2 + \delta X + Y.$$  \hspace{1cm} (7)

where $q = (k^2 - \kappa^2)^{1/2}$ being the linear dispersion parameter for the Bragg grating and the parameter $X$ and $Y$ are defined as

$$X = \gamma \left( I_0 + 2|A_f|^2 \right) \quad \text{and} \quad Y = \gamma^2 |A_f|^2 \left( I_0 + |A_f|^2 \right).$$  \hspace{1cm} (8)

Equations (7) and (8) exhibit the intensity dependent modification in the dispersion parameter. Consequently we find that for frequency detuning $\delta$

lying in the range $-\kappa < \delta < \kappa$, $q_{nl}$ becomes purely imaginary and most of the incident field will be reflected due to the fact that the grating will not support the propagating wave. In the absence of nonlinearity ($\gamma = 0$), the stop band extends for $-\kappa < \delta < \kappa$ and equations (9) and (10) resemble to the standard solutions of the coupled mode equations for Bragg grating in linear regime. The range $|\delta| \leq \kappa$, is referred to as the photonic bandgap or stopband since light stops transmitting through the grating when frequency of the incident light falls within it. When input intensity is increases the photonic bandgap shift towards the lower frequencies side due to increase the average index of the grating. This movement in photonic bandgap decreases the coupling between the forward and backward propagating waves and result in an increase in transmission. On substituting parameter $q_{nl}$ in equations (5) and (6), the fields of forward and backward propagating modes take the form
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\[ A_f(z) = A_1 \exp(iSz) + r_{nl} B_2 \exp(iTz) \] (9)

and

\[ A_n(z) = B_1 \exp(iTz) + r_{nl} A_1 \exp(iSz). \] (10)

The corresponding expression for the effective reflection coefficient \( r_{nl} \) in nonlinear regime is found to be

\[ r_{nl} = -\frac{k}{S + \delta + \gamma(I_0 + |A_f|^2)} \] (11)

Applying the boundary conditions that light is incident only at the front end at \( z = 0 \) of the FBG as shown in Fig. 1, the nonlinear reflection coefficient \( r_{nl} \) for a grating of length \( L \) has been obtained by using equations (9) to (11) as

\[ r_{nl} = \frac{A_n(0)}{A_f(0)} \frac{-\kappa \Psi[1 - \exp(i2\Phi)]}{\psi^2 - \kappa^2 \exp(i2\Phi)} \] (12)

where \( \Phi = kL/2 \), \( \Psi = \left( \frac{k}{2} + \tau \right) \), \( \tau = \delta + \gamma |A_f|^2 \) and \( k = \sqrt{\gamma^2 I_0^2 + 4 g_{nl}^2} \).

The corresponding expression for the reflectivity \( R_{nl} \) in the nonlinear regime is

\[ R_{nl} = \frac{4\kappa^2 \psi^2 \sin^2 \Phi}{\left[ \psi^2 - \kappa^2 \right]^2 + 4\kappa^2 \psi^2 \sin^2 \Phi} \] (13)

It can be noted that when the nonlinear refractive index vanishes from the expression (13), we recover the usual expression for the reflectivity of a fiber Bragg grating in linear regime \( R = \tanh^2(\kappa L) \).

Equation (13) can also be compared with the reflectivity of nonlinear electro-optic Fabry-Perot devices using reflected light-feedback [20]. They obtained the resonator reflectivity \( \rho \) as

\[ \rho = \frac{4R \sin^2 \Phi}{T^2 + 4R \sin^2 \Phi} \] (14)

**FIGURE 1**
Schematic of a FBG of length L illuminated by electromagnetic field amplitude \( A(z) \).
where $R(T)$ represented the mirror reflectivity (transmittivity). A comparison of Equation (13) and (15) shows that the FBG is equivalent to an optical resonator with mirror reflectivity, $R = \kappa \Psi^2$ transmissivity $T = \Psi^2 - \kappa^2$ and phase shift $\Phi = kL/2$.

3. RESULTS AND DISCUSSIONS

In this section, we have discussed the bistable behavior of fiber Bragg grating in Kerr regime by plotting the reflected intensity as a function of input intensity in Figures 2 and 3. We have considered chalcogenide fiber Bragg grating

![Graph showing reflected vs. incident intensity for a nonlinear FBG with $kL=1$ for different values of detuning $\delta L$.]
FIGURE 3
Reflected vs. incident intensity for a nonlinear FBG with detuning wavelength $\lambda = 1551$ nm for different values of $\kappa L$.

having $n_{eff} = 2.45$, $\lambda_B = 1550$ nm, $n_2 = 2.7 \times 10^{-17}$ m$^2$/W, $n_g = 1 \times 10^{-4}$ [14].
We have chosen a tunable quasi-CW laser source in C-band (1535 - 1565 nm). It may be noted here that a similar order of wavelength at high intensity was considered by Lee and Agrawal [8] to be obtainable from a quasi CW-laser of 1 ns pulse duration. Also, Taverner et al [21] used a quasi-CW Diode-seeded LA-EDFA chain radiation source at 1536 nm in their experimental work to demonstrate all-optical AND gate operation in an apodized FBG.
The plot of the reflected intensity as a function of the input intensity is given in Fig. 2 for three different values of detuning $\delta L = -16$ at $\lambda = 1550.5 \text{ nm}$ (Fig. 2a), $\delta L = -32$ at $\lambda = 1551 \text{ nm}$ (Fig. 2b) and $\delta L = -48$ at $\lambda = 1550.5 \text{ nm}$ (Fig. 2c). All the incident wavelengths are considered outside of the stop band of the fiber Bragg grating. In linear case, these wavelengths are completely transmitted through the FBG. As the intensity of the input beam increases, the stop band of the Bragg grating detunes itself towards the higher wavelength side from its original position (at $\lambda_B$) because of the presence of the positive Kerr nonlinearity in the medium. If the input intensity is sufficiently high, the Bragg wavelength $\lambda_B$ no longer lies within the stop band and the wavelength $\lambda > \lambda_B$ comes in the stop band. As a result, these wavelengths get reflected. The intensity of reflected wave increases with increasing input intensity and reaches its maximum value at a particular input intensity as seen in Fig. 2.

On gradual reduction of the input intensity, the reflected intensity does not retrace the same path due to the fact that there is still enough light within the grating to keep its index high and to maintain it in a high reflection mode. Thus, switch-down to a low reflection state occurs at a lower input intensity than that at which switch up occurred, resulting in a hysteresis loop as illustrated in all the three cases in Fig. 2. This hysteresis loop shows that the FBG supports the occurrence of optical bistability.

Fig. 2 also shows the effect of the detuning on the switching characteristics of the fiber Bragg grating. It is found that for a given $\kappa L = 1$ (FBG length $L = 5 \text{ mm}$), the switch-on intensity and the width of the hysteresis loop increases with increasing detuning wavelength. For detuning wavelength $1550.5 \text{ nm}$ (Fig. 2a) near the edge of the photonic bandgap, the width of the hysteresis loop is small and the bistable device requires low switch-on intensity $I_0 \approx 150 \text{ MW/cm}^2$. When, the wavelength of the incident field is detuned more and more away from the edge of the stop band the width of the hysteresis loop and the switching intensity increases. The reason behind these features of the hysteresis loop can be explained as follows: It is well established that at low excitation intensity the FBG is highly reflective for wavelengths falling in the range $-\kappa < \delta < \kappa$ and transmissive for $-\kappa > \delta > \kappa$. The detuning parameter $\delta$ represents the difference between the frequency of the applied signal and the Bragg frequency of the grating. As the detuning parameter approaches to the edge of the stopband, the switching intensity increases. Similarly, in the reverse action the requirement of the switch-off intensity also reduces which results in an increase in the width of the hysteresis loop.

The behaviour of optical hysteresis is also studied by keeping detuning wavelength $\lambda = 1551 \text{ nm}$ constant and varying the length of the FBG. This behavior is illustrated in Figure 3 for different values of $\kappa L = 0.5$ (Fig. 3a), 1.0 (Fig. 3b) and 1.5 (Fig. 3c). It is interesting to observe that the width of the hysteresis loop decreases with increasing length of the grating if the linear coupling parameter $\kappa$ is constant. However, the switching intensity where
device is "on" is independent of the length of the FBG and it remains constant at intensity $I_0 \approx 200 \text{MW/cm}^2$. This is in marked contrast to the behavior of a nonlinear Fabry-Perot where an increase in the length of the medium leads to a decrease in the required switching intensity. The difference is due to fact that in the fiber Bragg grating an increase in length increases coupling between forward and backward propagating modes. As a result, the internal field drops off almost exponentially along the length of the grating. Hence, most of the light is reflected in the first few periods and light does not travel in remaining length of the grating. On the other hand in the Fabry-Perot device the field is uniform throughout the medium and the nonlinear interaction occurs along the whole length of the cavity. The present analytical study is in good agreement with the analysis presented by Winful et al for distributed feedback structure in III-V semiconductor materials and hence stems our formulation.

4. CONCLUSIONS

We have investigated reflective optical bistability phenomena in nonlinear fiber Bragg grating by incorporating optical Kerr effect in the coupled mode analysis. The expression for the intensity dependent reflectivity of fiber Bragg grating is obtained analytically. It is observed that the switching intensity at which device is "on" and the width of the hysteresis loop can be controlled by varying the detuning wavelength. It is also observed that the width of the hysteresis loop decreases with increasing length of the fiber Bragg grating. Current analytical study is in good agreement with the analysis presented by Winful et al and hence stems our formulation. Finally, it is envisaged that the present work will be useful in understanding the experimental demonstration of optical bistability in nonlinear fiber Bragg grating and the development of components for all-optical computing applications.

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