Chapter 2

RADIATIVE TRANSFER MODELING
Chapter 2  Radiative Transfer Modeling

2.1 Introduction

2.2 Physics of Radiative Transfer Model

2.3 Radiative Transfer model approximations
   2.3.1 Eddington Approximations
   2.3.2 Discrete Ordinate approximations

2.4 Results of Simulation and Sensitivity Studies
   2.4.1 Non-Raining (Non-Scattering) Atmospheres
   2.4.2 Raining (scattering) Atmospheres
      2.4.2.1 Eddington’s Model Simulations
      2.4.2.2 Discrete Ordinate Method Simulations

2.5 Conclusions
2.1 INTRODUCTION:

Satellite observations of the atmosphere, land, and oceans are now a major component of the environmental observing system, since they provide critical information to better understand and forecast short-term as well as climatic changes in weather. Through data assimilation techniques, the satellite observations as well as other sources of atmospheric and oceanic data, sampled at different times, intervals, and locations can be combined into a unified and consistent description of the atmospheric state. Global objective analyses are produced from these diverse observations in combination with the a priori knowledge of the evolving atmospheric state as given by numerical weather prediction (NWP) models. By far, the greatest volume of data ingested into these numerical models is from satellite instruments, whose data have contributed to a dramatic improvement of forecast accuracy over the last 20 years. The identification and proper estimation of precipitation on a global scale is of great importance in a wide range of meteorological problems. Rainfall being an important element of the global hydrological cycle, its accurate information has become important topics of increasing interest among both scientists and policy makers. This upsurge of interest has resulted from the realization that rainfall studies are directly related to ocean-atmosphere interactions, interannual variations of climate, El Nino, and other climate changes. The transfer of electromagnetic radiation represents the prime physical process that drives the circulation of the atmosphere and the ocean currents. It is apparent that an understanding of climate and its major component “precipitation” must begin with detailed understanding of radiative processes and the radiative balance of the earth and the atmosphere. In case of precipitation, the radiative transfer is applicable in microwave regime only because upwelling radiation in this band of electromagnetic spectrum over the cloud is directly responsive to precipitation microphysics (Ulaby et al., 1981). On the other hand visible (infrared) techniques are empirically based and generally suffer from an incomplete correlation between brightness (cold cloud top temperatures) and precipitation, and thus do not fall in the realm of radiative transfer. Passive microwave rainfall retrievals, although fraught with their own problems, offer a much more direct relationship between Brightness Temperatures (Tbs) and
precipitation. This is due to the fact that for all but the strongest convection, frequencies less than 35 GHz have the bulk of their signal originating from well within the rain layers itself. Microwave brightness temperatures measured from a satellite-borne radiometer results from the integrated effects of surface emission and reflection, absorption and emission by atmospheric gases, and absorption, emission and multiple scattering of cloud and precipitation particles. In studying radiative transfer in absorbing, emitting and scattering media, two very important features arise. First, in such media the absorption and emission of radiation occur not only at the boundaries of a system, but also at every point inside a medium. The same is true for scattering. To accurately describe the microwave signatures, a radiative transfer model with full inclusion of the aforementioned effects, particularly the multiple scattering by precipitation particles, is required. Understanding and simulating microwave brightness temperatures in the forward step with great care (radiative transfer, surface variability, bulk microphysics) can thus lead to more physical rainfall retrieval schemes. For complete solution of the energy transfer problem, it is necessary to know the volume field of temperature and physical properties of a medium at each point of a system. Here by a point of a system means a physically infinitesimal (unit) volume of a medium, which contains a fairly large number of particles, the interaction between which can provide the local thermal equilibrium conditions. By particles mean either a set of macroscopic particles (aerosols, water drops, snow and ice particles, volcanic ashes or particles of another nature), or a set of quantum particles (atoms and molecules of gases). With the development of more sophisticated retrieval algorithms, the need to physically interpret microwave brightness temperatures and consequently the need for fast and accurate forward radiative transfer calculations have increased for the following applications: (i) physically-based retrieval of geophysical parameters (Kummerow and Giglio, 1994; Smith et al. 1994; Evans et al. 1995) and (ii) direct data assimilation of satellite observed radiances (Eyre, 1997). A number of plane parallel radiative transfer models dealing explicitly with microwave frequencies have been attempted. Wilheit et al. (1977), made the first objective attempt to relate single-channel microwave radiances to the rain intensity. He developed a technique for quantitatively mapping precipitation over the oceans by means of satellite radiance data
at 1.55 cm (19.35 GHz). Most of the earlier radiative transfer solutions account for emission and absorption processes only, however for exact estimation of microwave radiation propagating through precipitation the consideration of multiple scattering is very important. The main differences between the solutions in this case become the ability of the radiative transfer solution to properly capture the angular distribution of the radiation by properly solving the scattering phase function. The problem of scattering phase function was solved in a simple way by Eddington approximation by considering only the second moment of angular distribution of the radiation. Therefore Eddington approximation described by Wu and Weinman (1984) is widely referenced along with the multistream double-adding scheme with a complete treatment of polarization developed by Evans and Stephens (1990). More general radiative transfer codes applicable to a wider range of frequencies are described by Shettle and Weinman (1970), Coakley and Chylek (1974), Stephens (1988), and Stamnes et al. (1988).

With the ongoing successful TRMM mission along with the continued pioneering SSM/I series of sensors, Meghatropiques sensors and the upcoming Global Precipitation Mission, there has been considerable emphasis on measuring precipitation. Therefore a great deal of confidence is required to gain insight into microwave brightness temperatures and rain rate relationship by simulating radiances through precipitation fields generated by models.

2.2 PHYSICS OF RADIATIVE TRANSFER MODEL:

The interaction and propagation of electromagnetic radiation between the atmospheric constituents and its layers is generally described by the radiative transfer theory (RTT). Since the radiation is the most important source of energy for driving all the atmospheric processes, atmospheric dynamics is strongly influenced by how solar and the terrestrial radiations are scattered, absorbed and emitted by the earth’s surface and the atmosphere. Therefore the knowledge of radiative transfer is most fundamental in the retrieval of atmospheric and earth’s surface parameters in space-borne remote sensing. The use of RTT in relation to real media is based on some physical simplifications which allow us to advance in studying radiative transfer in composite
media, where the direct use of the Maxwell theory is troublesome. Consider a beam of radiation with intensity \( I \) propagating in the absorbing, emitting and scattering medium in a given direction. The energy of radiation will decrease owing to its absorption by substance and owing to the deviation of a part of the radiation from the initial trajectory as a result of scattering in all directions. But, at the same time, the energy will increase because of thermal radiation emission by the substance volume. The absorption, scattering and emission of radiation by a substance have effect on the energy of a radiation beam that propagates in it. In this case the total balance of change of the initial intensity can be, certainly, both positive and negative. Besides, a strong inhomogeneity of the energy balance, both over the substance volume and over the observation direction, is possible. These properties have been discussed in detail in the books by Chandrasekhar (1960), Sobolev (1963), Ozisik (1973), Siegel and Howell (1972), Buglia (1986), etc. This chapter deals with the basic formulation of radiative transfer in the atmosphere.

The fundamental property of electromagnetic radiation is that it can transport energy. Many of the units used to quantify electromagnetic radiation are based on energy. The basic unit of radiant energy is the joule. However the most fundamental radiation unit for satellite meteorology is monochromatic radiation, which is the energy per unit time per unit wavelength (frequency, wave number) per unit solid angle crossing a unit area perpendicular to the beam. The reason for radiance being the most fundamental unit is that the output voltage of the detectors/antenna on a satellite radiometer is proportional to the energy per unit time intercepted by it. Further, the sensor collects radiation from a certain solid angle and which has filters that pass radiation of only a certain narrow range of wavelengths. Normalizing the sensor output by area, solid angle, and wavelength range results in a quantity that is most closely related to monochromatic radiance.

Now, let us consider the differential amount of radiant energy \( dE_\lambda \) in a time interval \( dt \) and in a specified wavelength interval \( \lambda \) to \( \lambda + d\lambda \), which crosses an element of area \( dA \), and in directions confined to a differential solid angle, which is oriented at an angle \( \theta \) to the normal of \( dA \). This energy is expressed in terms of the specific intensity \( I_\lambda \) by
\[ \text{d}E_{\lambda} = I_{\lambda} \cos \theta \text{d}\Omega \text{d}\lambda \text{d}t. \]

The above equation defines the monochromatic intensity (or radiance) in a general way as

\[ I_{\lambda} = \frac{\text{d}E_{\lambda}}{\cos \theta \text{d}\Omega \text{d}\lambda \text{d}t} \]

Radiance also has the useful property that it is independent of distance from an object as long as the viewing angle and the amount of intervening matter are not changed.

The second most useful property describing the radiation field is monochromatic flux density or spectral flux or irradiance. It is defined as the normal component of \( I_{\lambda} \) integrated over the entire spherical solid angle. i.e.

\[ F_{\lambda} = \int_{\Omega} I_{\lambda} \cos \theta \text{d}\Omega \]

In case of isotropic radiation i.e when radiant intensity is independent of direction, the spectral flux is

\[ F_{\lambda} = \pi I_{\lambda} \]

**Extinction (Absorption and Scattering):**

The absorption and scattering are the fundamental physical process associated with the radiation and its interaction with the matters. Both absorption and scattering remove flux from an incident wave, thus jointly called extinction. The extinction is governed by a fundamental law i.e. Lambert-Beer law.

Absorption is a process that removes the radiant energy from electromagnetic field and transfers it to other forms of energy. Absorption of radiation takes place whenever a transition occurs from one internal energy state to another internal energy state, allowed by the selection rules. When the molecules are de-exited to a lower level, the difference in energy level is released and this process is called emission. Absorption is spectrally selective and the process is discrete because of quantisation. Absorption of radiation depends on the imaginary part of refractive index of matter, which in turn depends on
Chapter 2  

Radiative Transfer Modeling

the chemical composition. Basic parameter in case of absorption is absorption coefficient $K_\nu(s)$, which for particles with a non-zero imaginary index of refraction, defines the fraction of the incident radiation that is absorbed.

On the other hand scattering is the result of interaction between light and particulate matter and occurs at all wavelengths (spectrally not selective) in the electromagnetic spectrum. Any material whose refractive index is different from that of the surrounding medium (optically inhomogeneous) can scatter radiation. Scattering can be thought of as absorption of radiant energy followed by re-emission back to the electromagnetic field with negligible conversion of energy i.e. without the change in the internal energy states of the molecules. Thus, scattering can remove radiant energy of a light beam traveling in one direction, but can be a “source” of radiant energy for the light beams traveling in other directions. Scattering is basically a function of two parameters, first the wavelength of incident radiation and second the complex index of refraction of the material of which the particle is made. Scattering geometry is shown in Figure 2.1.

Basic parameters in case of scattering are the scattering coefficient, $\sigma_\nu(s)$, having the unit of per meter, which determines the fraction of the incident radiation scattered, and the particles phase function, $P_\nu(\Omega,\Omega')$ such that $\frac{1}{4\pi} P_\nu(\Omega,\Omega')$ is the probability that the incident radiation, $I_\nu(s,\Omega')$, will scatter from the solid angle $d\Omega'$ centered about $\Omega'$ into a element of solid angle $d\Omega$ centered about the direction of $\Omega$.

Phase function depends only on the scattering angle i.e.

$$P_\nu(\Omega,\Omega') = P_\nu(\cos \theta_o)$$

where, $\cos \theta_o = \Omega\Omega' = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\varphi - \varphi)$

Generally phase function is expanded in a series of Legendry polynomials.

Radiative Transfer Equation (RTE):

The Radiative Transfer Equation (RTE) can mathematically model the transfer of energy as photons move inside a tissue. The flow of radiation energy through a small area element in the radiation field can be characterized by the radiance. The RTE is a
differential equation describing radiance and can be derived via conservation of energy. Briefly, the RTE states that a beam of light loses energy through divergence and extinction (including both absorption and scattering away from the beam) and gains energy from light sources in the medium and scattering directed towards the beam. Coherence, polarization and non-linearity are neglected. Optical properties such as refractive index $n$, absorption coefficient, scattering coefficient, and scattering anisotropy are taken as time-invariant but may vary spatially. Scattering is assumed to be elastic. It is the fundamental integro-differential equation, which governs the variation of intensity in a medium characterized by a spectral volumetric absorption coefficient, $K_\nu(s)$, and a spectral volumetric scattering coefficient, $\sigma_\nu(s)$, where $s$ is the distance along the absorbing path. Let us consider a beam of monochromatic radiation of spectral intensity $I_\nu(s,\Omega)$ traversing a small cylindrical volume element of the atmosphere in the direction $\Omega$ along the path $ds$, and is confined to the solid angle $d\Omega$ centered about the direction $\Omega$ as shown in Figure 2.2.

The general radiative transfer equation can be written as follows:

$$\frac{dI_\nu(s,\Omega)}{ds} = j_\nu - K_\nu(s)I_\nu(s,\Omega) + -\sigma_\nu(s)I_\nu(s,\Omega)$$  \hspace{1cm} (2.1)

The spectral volumetric extinction coefficient is given as

$$\beta_\nu(s) = K_\nu(s) + \sigma_\nu(s)$$

and dividing equation (2.1) by $\beta_\nu(s)$, it gives

$$\frac{1}{\beta_\nu(s)} \frac{dI_\nu(s,\Omega)}{ds} + I_\nu(s,\Omega) = J_\nu(s,\Omega)$$  \hspace{1cm} (2.2)

where

$$J_\nu = \frac{j_\nu}{\beta_\nu(s)} = (1 - \omega_\nu)B(T) + \frac{\omega_\nu}{4\pi} \int P(\cos \theta)I_\nu(s,\Omega')d\Omega'$$

is referred to as the source function and

$$\omega_\nu = \frac{\sigma_\nu(s)}{\beta_\nu(s)}$$

is called the single-scattering albedo, or particle albedo and expresses the fraction of the attenuated beam which is lost to scattering alone.
In plane-parallel atmospheres the medium is stratified in planes perpendicular to a given direction \( z \), such that the optical properties of the medium are functions of \( z \) and \( \nu \) only. Since thickness of a planetary atmosphere is generally small compared with its radius, thus this assumption is universally applied.

From Figure 2.3, \( \frac{d(\_)}{ds} = \cos \theta \frac{d(\_)}{dz} = \mu \frac{d(\_)}{dz} \)

Thus equation (2.1) in terms of \( z, \mu, \) and \( \varphi \):

\[
\frac{\mu}{\beta_\nu(z)} \frac{dI_\nu(z, \mu, \varphi)}{dz} + I_\nu(z, \mu, \varphi) = J_\nu(z, \mu, \varphi)
\]

(2.3)

here in \( J_\nu \),

\[
d\Omega' = \sin \theta' d\theta' d\varphi' = -d\mu' d\varphi'
\]

so that

\[
J_\nu(z, \mu, \varphi) = (1 - \omega_\nu)B_\nu[T(z)] - \frac{\omega_\nu}{4\pi} \int_0^{2\pi} \int P(\cos \theta_o) I_\nu(z, \mu', \varphi') d\mu' d\varphi'
\]

For convenience, introducing the concept of optical depth, \( \tau_\nu \), measured from the outer boundary downward as

\[
\tau_\nu = \int_0^z \beta_\nu(z') dz' \quad \quad d\tau_\nu = -\beta_\nu(z) dz
\]

Thus replacing height variable \( z \) by optical depth \( \tau_\nu \) in equation, (2.3)

\[
-\mu \frac{dI_\nu(\tau_\nu, \mu, \varphi)}{d\tau_\nu} + I_\nu(\tau_\nu, \mu, \varphi) = J_\nu(\tau_\nu, \mu, \varphi)
\]

or

\[
\mu \frac{dI_\nu(\tau_\nu, \mu, \varphi)}{d\tau_\nu} = I_\nu(\tau_\nu, \mu, \varphi) - J_\nu(\tau_\nu, \mu, \varphi)
\]

(2.4)

with

\[
J_\nu(\tau_\nu, \mu, \varphi) = (1 - \omega_\nu)B_\nu[T(\tau_\nu)] + \frac{\omega_\nu}{4\pi} \int_0^{2\pi} \int P(\cos \theta_o) I_\nu(\tau_\nu, \mu', \varphi') d\mu' d\varphi' \quad - (2.5)
\]

Equation (2.4) is the basic radiative transfer equation for plane-parallel atmosphere problem, which is extremely difficult to solve. Part of difficulty is due to the azimuthal
dependence of $I_\nu$ through the phase function. By expanding the phase function in a Legendre polynomial series, the azimuthally dependent terms in the function can be uncoupled. Only the azimuthally independent equation contributes to the flux calculations, which is of greatest interest in most atmospheric applications. Thus confining our solution for the azimuthally independent equation.

First of all expanding the phase function of equation (2.5), in a Legendre polynomial series of order N and applying addition theorem to separate it into the product of a function of $\mu$ and $\mu'$ only, times a function of $(\varphi'-\varphi)$ in each term. Again by expanding $I_\nu(\tau_\nu, \mu, \varphi)$ in a Fourier cosine series in $\varphi$, such that and then equating the like coefficients of $\cos m(\varphi'-\varphi)$ to separate out the azimuthally independent terms from the azimuthally dependent term.

$$
\frac{\mu}{d\tau_\nu} \frac{dI_\nu^m(\tau_\nu, \mu)}{d\mu} = I_\nu^m(\tau_\nu, \mu) - \frac{\omega_\nu}{4} (1 + 80^m_m) \sum_{l=0}^{N} \omega_l^m P_l^m(\mu) \times \int_{-1}^{1} P_l^m(\mu') I_\nu^m(\tau_\nu, \mu') d\mu'
$$

$$
- (1 - \omega_\nu) B_\nu [T(\tau_\nu)]
$$

dropping the superscripts and with $m=0$, it results

$$
\frac{\mu}{d\tau_\nu} \frac{dI_\nu(\tau_\nu, \mu)}{d\mu} = I_\nu(\tau_\nu, \mu) - \frac{\omega_\nu}{2} \int_{-1}^{1} I_\nu(\tau_\nu, \mu') P(\mu, \mu') d\mu' - (1 - \omega_\nu) B_\nu [T(\tau_\nu)]
$$

where

$$
P(\mu, \mu') = \sum_{l=0}^{N} \omega_l P_l(\mu) P_l(\mu')
$$

Thus, equation (2.6) is the most frequently used radiative transfer equation under the restrictions of plane-parallel atmosphere, phase function expandable in Legendre polynomial series, and azimuthal symmetry.

### 2.3 RadiativeTransferModelApproximations:

The mathematical difficulties that arise in solving the complete integro-differential equation (2.6) of the radiative transfer theory have resulted in the appearance of a series of approximate approaches and methods for solution of the radiative transfer equation. At present, the approximate methods of solution of the radiative transfer equation form an independent mathematical discipline. Here it is to be noted that quite different
(initially) physical prerequisites are laid down in various approaches, and, therefore, the spheres of applicability of these methods are very different from each other. As a result, the matching of solutions of various approximate methods among themselves, sometimes represents, a very complicated problem in itself. So, in the approximations of thin-optical and thick-optical layers (the latter is also called the diffusive approximation, or the Rosseland approximation) simplifications are used that follow from the corresponding limiting value of the medium's thickness. In Eddington's and Schuster-Schwarzchild's approximations the simplifications are related to the introduction of some special assumptions on the angular distribution of radiation intensity. In the method of exponential approximation of a core the integro-exponential functions in the formal solution are replaced by the exponents. The spherical harmonics method and the Gaussian quadratures method are the well-developed techniques allowing us to obtain high-order approximations using fairly simple procedures. Here two of the aforementioned approximate methods for solution of the radiative transfer equation in the schematic form shall be described. The more detailed study of the approximate methods are given by Chandrasekhar, 1960; Sobolev, 1963; Malkevich, 1973; Ozisik, 1973; Marchuk, 1976; Marchuk et al., 1986; Sabins, 1987; Thomas and Stammes, 1999; Barichello et al., 1998. The approximate methods are necessary in two ways. Firstly, they provide various simple methods for the solution of fairly complicated radiative transfer problems. In this case, however, their application is limited by the circumstance that the accuracy of the approximate method cannot be estimated without comparing it to the accurate solution or to the results obtained from accurate solutions of the Maxwell electromagnetic theory. Therefore, in using the approximate methods for studying particular natural media, some caution should be exercised, since the accuracy of any approximate method is not always clear enough. Secondly, in solving the reverse remote sensing problems, of principal significance is the possibility of describing the radiation of a studied natural medium by means of fairly simple analytical formulae.
2.3.1 Eddington’s approximations:

In Eddington’s approximation it is assumed that for an isotropic field the ratio of the second moment of the radiation field to the mean intensity is everywhere equal to 1/3.

In the plane parallel Eddington’s approximation, radiances are expanded in a series of Legendre and associated Legendre functions:

\[ I(\tau, \theta, \phi) = I(0) + I(1)\cos\theta + \ldots \]

and the phase function is similarly expanded in Legendre polynomials

\[ P(\cos\theta_0) = \sum_{n=0}^{N} \omega_n P_n(\cos\theta_0) = 1 + \omega_1 \cos\theta_0 + \ldots \]

where \( \theta_0 \) is the angle from \( \theta' \), \( \phi' \) to \( \theta \), \( \phi \), and the source function \( J(z, \theta, \phi) \) can be written as

\[ J(z, \theta, \phi) = [1 - \omega(z)]T(z) + \omega(z)[I(0) + g(z)I(1)\cos\theta] \]

where the asymmetry factor is given by \( g = \frac{\omega_1}{3} \). Using this expression the isotropic component of the diffuse radiance \( I_0 \) reduces to (Weinman and Davies, 1978)

\[
\frac{d^2 I_0(z)}{dz^2} = \lambda^2(z)[I_0(z) - T(z)] - \lambda_0^2(z)[1 - \omega(z)][1 - \omega(z)g(z)] = 0
\]

where \( \lambda^2(z) = 3\beta^2(z)[1 - \omega(z)][1 - \omega(z)g(z)] \).

The simplest solution occurs when \( \beta, \omega \) and \( g \) are independent of height, and \( T(z) \) is written as \( T_0 + \Gamma z \), where \( \Gamma \) is the lapse rate of the atmosphere. Equation (2.7) then becomes a simple second-order differential equation. It has a solution of the form

\[ I_0(z) = D_+ \exp(\lambda z) + D_- \exp(-\lambda z) + T_0 + \Gamma z, \]

where \( D_+ \) and \( D_- \) are constants to be determined from the boundary conditions. To use this simplification, clouds are generally divided into \( n \) homogeneous layers such that the above conditions are approximately satisfied in each layer. The downward flux at the top of the cloud determines the upper boundary condition, while the upward flux at the bottom of the cloud determines the lower boundary condition. Flux continuity at the layer interfaces provides the remaining boundary conditions if more than one layer is assumed.
To obtain the radiation emerging at the top of the atmosphere, \( z_0 \) is defined as the surface and \( z_n \) as the top of the atmosphere, with average layer quantities ranging from 1 to \( n \). The further details about the Eddington’s approximation are mentioned in Kummerow et al. (1993)

### 2.3.2 Discrete Ordinate Approximations:

The radiative transfer equation (2.6) in terms of polarization \( p \) (H or V) can be expressed as (Tsang and Kong, 1977):

\[
\frac{\mu}{d\tau} \begin{bmatrix} I_v (\tau, \mu) \\ I_H (\tau, \mu) \end{bmatrix} = \begin{bmatrix} I_v (\tau, \mu) \\ I_H (\tau, \mu) \end{bmatrix} - \frac{\omega}{2} \int_{-1}^{1} \begin{bmatrix} P_{VV} & P_{VH} \\ P_{HV} & P_{HH} \end{bmatrix} \begin{bmatrix} I_v (\tau, \mu) \\ I_H (\tau, \mu) \end{bmatrix} d\mu - (1 - \omega) B(\tau) \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

Where all the symbols have their usual meaning. Here Plank function \( B(\tau) \) is assumed to be linearly varying function of \( \tau \) from the top of the layer \( B_0 \) to the bottom of the layer i.e., \( B(\tau) = B_0 + B_1 \tau \). The four scattering phase functions \( P_{VV}, P_{VH}, P_{HV}, \text{ and } P_{HH} \) are the ones integrated over all azimuthal directions. Thus there exists cross-polarization scattering in the scattering source term because \( P_{VH} \) and \( P_{HV} \) are not zero.

The exact solution of \( I_p(\tau, \mu) \) is obtained by solving equation (2.8) using the discrete ordinate method with sufficient streams. In this DOM model it is assumed that cross-polarization scattering in the scattering source term is negligible and the scattering phase function is assumed to follow Henyey-Greenstein equation, and is expended with Legendre polynomial \( p_l \) as

\[
P(\cos \theta_0) = \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta_0)^{\frac{3}{2}}} = \sum_{l=0}^{N} (2\ell + 1) g^\ell P_\ell (\cos \theta_0) \equiv \sum_{l=0}^{N} A_\ell p_\ell (\cos \theta_0)
\]

For azimuth-independent case (spheres, randomly-orientated irregular particles), the cosine of the scattering angle \( \cos \theta_0 \) can be denoted as \( \mu \mu' \) (Liou, 1974), so that

\[
P(\cos \theta_0) = P(\mu, \mu') = \sum_{l=0}^{N} (2\ell + 1) g^\ell P_\ell (\mu) p_\ell (\mu') \equiv \sum_{l=0}^{N} A_\ell p_\ell (\mu) p_\ell (\mu')
\]
Where asymmetry factor $g$ is calculated following Mie theory (Bohren and Huffman, 1983), $p_{\ell}(\mu)$ is the $\ell$th order Legendre polynomial and $N$ is the number of terms to add. In DOM, $N=2n-1$ ($2n$ is the stream number). To minimize the error associated with this cutoff, a $\delta$-adjustment (scaling $g$, $\tau$, and $\omega_0$) is applied (Fu and Liou, 1992).

The formal solution of equation (2.8) without inclusion of polarization (Stamnes and Swanson, 1981) is

$$I(\tau, +\mu) = I(\tau^*, +\mu) \exp[-(\tau^* - \tau) / \mu] + \int_{\tau}^{\tau^*} J(t, +\mu) \exp[-(t - \tau) / \mu] dt / \mu$$

- (2.10)

$$I(\tau, -\mu) = I(0, -\mu) \exp(-\tau / \mu) + \int_{0}^{\tau} J(t, -\mu) \exp[-(\tau - t) / \mu] dt / \mu$$

- (2.11)

where $\mu$ and $-\mu$ denote upward and downward directions, respectively. $\tau^*$ is the optical depth of the layer. The source function $J(\tau, \mu)$ including both emission and scattering with the discrete ordinate approximation is

$$J(\tau, \mu) = \frac{1}{2} \omega_0 \sum_{i=0}^{2n-1} A_i p_i(\mu) \sum_{j=-n}^{n} a_j p_j(\mu_j) I(\tau, \mu_i) + (1 - \omega_0)(B_0 + B_\tau)$$

- (2.12)

Where $2n$ is the stream number in discrete ordinate approximation and $a_j$ is the quadrature weight for jth quadrature point.

The detail solution of the equation (2.8) is given in Pokherel (2005).

By solving the $L_j$s the radiances at quadrature angles ($\mu_i$) are obtained. To obtain radiance at any angle i.e. corresponding to the inclination angle of the radiometer equation are applied as explained in Liu et al. (1998) which is based on the state-of-art RTM using DOM that has been used here for carrying out the simulations and sensitivity studies. The satellite observed radiance at the top of the atmosphere is then obtained. Brightness temperature can be calculated from the radiance using Planck’s function. Horizontally and Vertically polarized radiances are calculated separately because of the difference of their surface emissivity.
2.4 RESULTS OF SIMULATION AND SENSITIVITY STUDIES:

The simulations and sensitivity studies have been carried out for various frequencies (10.65, 19.35, 22, 24, 37, 85.5, 89 and 157 GHz), corresponding to different weather conditions including raining and non-raining atmosphere. Radiative transfer simulations that include scattering are complex, because the scattering due to hydrometeors, liquid water and rainfall are difficult to separate at higher frequencies (e.g., 85.5 and 157 GHz). Here separately two simulations, with and without inclusion of scattering, were performed.

2.4.1 Non-Raining Atmospheres:

The main objective of the present simulations is to create a data base that could reasonably ascertain the accuracy of a radiative transfer model under the clear sky conditions over the global tropics covering 300 south to 300 north for the general passive microwave frequencies that would make the basis for interpretation of satellite microwave radiometric data explaining the emission part of it. This would to help in the retrieval of oceanic and atmospheric parameters like wind speed, water vapor and liquid water, from existing TRMM-TMI channels (for confirmation) and recently launched Indo-French Megha-Tropiques satellite (for testing and validation) to a desired accuracy, through assessment of the simulated and observed brightness temperatures fields. The input parameters used for this RT-model are GDAS model reanalyzed 6-hourly surface wind speed, near surface air temperature, sea surface temperature and vertical profiles (61 layers; upto 30 km height from surface at a separation of 500m) of cloud liquid water, temperature and relative humidity. The simulated brightness temperatures (Tbs) (Figure 2.5 to 2.8) in connection with the oceanic and atmospheric parameters (wind speed, water vapor, liquid water, and rain.) from Wentz et al. (taken from www.ssmi.com) (Figure 2.9 a, b, c & d) will be discussed. Though the simulations are performed only for the clear and cloudy sky conditions without rain occurrences, the larger dynamic ranges of the Tbs in 37 GHz show, some impact of raining conditions represented as high cloud liquid water contents of the clouds. Figure 2.4 shows the scatter plot of simulated Tbs between Vertical and horizontal polarizations.
for various TRMM radiometric channels 10.65, 19.35, 37 and 85.5 GHz using Kummerow model under clear-sky condition on 13\textsuperscript{th} June 2010. At lower frequencies upto 37GHz, the Tb of V and H polarizations vary constantly because of large emission and very low scattering from the raindrops that causes increase in the Tb values. On the other hand at higher frequency e.g. 85GHz, the emission and scattering both are dominant and thus the observations in Figure 2.4 shows some mismatch with rainfall at the lower Tbs and the higher Tbs where the scattering depresses the Tbs. This is also clear from the Figure 2.8 and 2.9. Thus the overall scatter plot of the simulated Tbs (V vs H) does represent the strong physical basis for the interpretation of microwave radiometer’s data and to apply suitable inversion techniques for the retrieval of the ocean and atmospheric geophysical parameters.

It is clear from the Figure 2.5 to 2.9 that most of the signatures related to the variations of wind, water vapor, CLW and high CLW as rain are very well picked up by the lower resolution channels in the clear sky conditions. This is due to the sensitivity of lower resolution channels to cloud liquid water which is one of the representatives of rainfall (Greenwald et al., 1993). Since, the lower frequencies are mainly emission based, so it detects the rainfall signature via cloud liquid water. The sensitivity of microwave radiances to variations in Cloud Liquid Water (CLW) is well understood but still there is a large source of error in microwave retrievals of CLW. The role of CLW in characterizing rainfall events more realistically over the oceans has been pointed out by Pokhrel et al. (2003) and Varma et al. (2003). The variations of Tbs for the CLW from 10 to 85 GHz channels indicating an order of sensitivity from 37, 19, 10 and 85 GHz more pronouncedly with the horizontal polarization than vertical for all the channels. The lower channel 10 GHz (V and H) have a very less sensitivity, 19 GHz (V and H) channel shows a significant sensitivity while 37 GHz (V and H) channels shows a mild dependency to the water vapor in the atmosphere. It is interesting to note further that the 85 GHz (H) shows some correlation with 19 and 37 GHz channels due to its sensitivity to Total Precipitable Water (TPW) as well. In case of Sea Surface Temperature (SST), the lower frequency channels like 10 V, 19 V and H show sensitivity to the surface winds and sea surface temperatures and in TRMM these
frequency channels are utilized for the retrieval of these parameters. SST and winds has potential in characterizing cyclogenesis based on their threshold limits as preconditions for the deep convection (Gadgil et al., 1984).

2.4.2 Raining Atmospheres:

Kummerow (1993) has reported that the simplicity and computational efficiency of this radiative transfer model lends it primarily to satellite remote sensing applications and can reproduce brightness temperatures in the MW regime (6.6-183 GHz) and in good compliance with the realistic measurements. The error in simulated Tbs is within 0.2 K in the absence of scattering which is far smaller than those introduced by usual uncertainties in the input parameters. However, at frequencies higher than 80 GHz, the scattering by cloud ice and snow particles becomes strong enough to be detected by space-borne microwave radiometers and led to more error in the simulated Tbs and the problem of physical retrieval of precipitation becomes complex (Liu, 2004). Even though, the problem of deducing the Tbs from atmospheric quantities is rigorously solved by means of RT equation, a new global variable as a function of temperature profile can greatly improve the modelization (Bosisio and Mallet, 1998).

In the case of raining atmosphere the size and type of hydrometeors will affect the transfer of radiation with emission as well as scattering. In this approach, the emission coefficient is assumed to be the extinction coefficient; i.e., the sum of both absorption and scattering coefficients which are calculated using the Mie scattering theorem. This approach is a better approximation when the medium is optically thick, highly scattering and almost isotropic. In this case, the scattering effect is similar to emitted radiation except that the emission coefficient describes both emission and scattering processes. This is used for retrieving rain attenuation and liquid water content (Westwater et al. 1980; Zhang et al. 1985). For liquid cloud, it is usually assumed that absorption dominates over scattering. The Rayleigh approximation is applicable for computing the absorption coefficients of cloud droplets and the absorption approach is used for calculating the atmospheric radiation or brightness temperature. Recently, however, it has been found that a small number of large cloud droplets can contribute significantly to the scattering even when the mode size of the particle size distribution
is in the order of 10 to 100 micrometers (Kobayashi and Adachi, 2008). In this case, the Rayleigh scattering approximation is not valid and an exact solution must be used. The emission part has been studied in details in the previous section, now the combination of both scattering and emission is presented here as follows:

2.4.2.1 Eddington Model – Simulations:

In this section theoretical simulation of brightness temperatures for Meghatropiques-MADRAS channels as a function of rain rate using 1-D Eddington approximation (Kummerow, 1993) has been carried out. Figure 2.10 (a-e) shows the scatter plot of simulated Tbs as a function of rain rate with ice content for 19, 24, 37, 89 & 157Ghz channels at both polarizations H and V. From the Figure 2.10 (a) it is clear that the brightness temperature increases from 150 to 260 and 200 to 260 K for 19 GHz H and V polarizations respectively up to rainrates of 10mm/hr and thereafter it starts decreasing from rainrates of 10mm/hr to 30mm/hr. Corresponding to low rainrates upto 10mm/hr, the signals are highly polarized due to secularly reflecting ocean surface viewed at an oblique angle. While at the high rainrates upto 30mm/hr, the signal becomes unpolarized because of randomly polarized emissions produced by raindrops. In addition to these, there seems to be the influence of some degree of scattering even in these channels, which are expected to be emission/absorption dominant in their radiative contributions.

Figure 2.10 (b) shows the variation of Tbs with rainrates for 24GHz channel at H and V polarizations. In this case also the Tb increases from 225 to 260K and 245 to 260K for H and V polarizations respectively upto the rairates of 5mm/hr and after then it starts decreasing for rainrates 5mm/hr to 30mm/hr. In case 24 GHz channels the decrease is sharper than the 19 GHz channel. The 24 GHz Tbs for H and V have a marked difference below 5 mm/h rain rate. Thus 24 GHz can have minimum impact in rainfall retrieval.

At frequency 37GHz, the Tb shows the reverse trend as shown in Figure 2.10(c). From the figure it is clear that the scattering is dominant above the rainrates 5mm/hr, the Tb
decreases exponentially because of scattering and thus this channel is highly affected by the rain.

Figure 2.10(d) shows the trend of Tb with rainrate for 89 GHz channel at both polarizations H and V. The 89 GHz channel shows the same trend as that of 37GHz. Generally 89 GHz is greatly affected by the presence of ice in the precipitating clouds which is clear from the figure by the decreasing slope with increasing rainfall from 5mm/hr to 30mm/hr. The Tb decreases sharply from 260 to 200K and 275 to 200K for H and V polarizations respectively in the presence of ice content in the precipitation clouds as rainrate increases from 5mm/r to 30mm/hr.

The behavior of Tb with rainrate for 157GHz channel is shown in Figure 2.10 (e). This also shows the same trend as that of 89GHz. Only difference is that this channel is more affected by the presence of ice than the 89 GHz and it is clear from the figure as it has a sharper slope than the 89GHz. From the figure, it can be seen that the 157 GHz reveals a further dip in the brightness temperatures suggestive of its increased ability to pick up ice/rain signature. In view of this, in the context of rainfall retrievals with multi-channel sensors, this high frequency channel is of great significance in delineating ice/rain structures. The 157 GHz channel shows the good correlation between the precipitating hydrometeors and the brightness temperature. Thus this relation can be used to retrieve the instantaneous surface rain rate parameter, which is one of the useful rainfall products to be retrieved from passive microwave observations.

Figure 2.11(a-e) shows the variation of simulated Tbs as a function of rain rate without ice content for 19, 24, 37, 89 & 157 GHz channels at both polarizations H and V. Figure 2.11 shows the similar trend as that in Figure 2.10. At lower frequency channels 19GHz, 24 GHz & 37GHz, the variation of Tb is same as that in case of with ice simulations because these channels are less affected by the ice content in the precipitation clouds. But there is significant difference at higher frequency channels viz. 89 GHz and 157 GHz. In case of no ice content in the precipitation clouds, the Tb decreases from 265 to 225K and 275 to 225K for 89H and 89V channels respectively while in case of ice content the decrease is more prominent as it falls from 260 to 200K
and 275 to 200K for 89H and 89V respectively. Similar trend is followed in 157 GHz channel.

An overall assessment through these simulation studies are able to be made, the degree of importance of the variables affecting brightness temperature measurements at any given frequency is precipitation through large variation in the brightness temperature vis-à-vis other parameters. In particular the higher the scattering parameters (like hydrometeor size, phase number density etc.) are associated with the higher the precipitation rate. These studies are consistent with the results obtained by Spencer et al. (1989) while analyzing the brightness temperature and rain rate relationship for SSM/I channels at 18, 37 and 85 GHz using RT model developed by Wu and Weinman (1984). Due to a different input data base in their simulations having the ice precipitation present above the freezing levels, they found a large slope in the dynamic ranges of brightness temperatures, particularly at 85 and 37 GHz. The consistency of the simulations and observations provides strong theoretical basis in the use of both forward radiative transfer and TMI/MADRAS observations for the retrieval of useful oceanic and atmospheric parameters from these channels. Similar data base of input and output field vectors has been created and studied during the detailed simulations carried out by Gairola et al. (2004).

2.4.2.2 Discrete Ordinate Model – Simulations:

It has been proved by various authors that the discrete ordinate method (DOM) with sufficient stream number is an accurate and stable way to solve radiative transfer problems (Liou, 1973; Stamnes and Swanson, 1981). The detailed information about the model used here has been elaborately discussed in the previous section 2.3.2. Figure 2.12 (a-e) and Figure 2.13 (a-e) show the theoretical upwelling brightness temperatures for TMI and MADRAS frequencies channels respectively as a function of rain rates ranging from 0 to 45 mm/hour as calculated from this model using four streams. The ice and cloud profiles have been taken from Smith et al. (1994) and similar to that taken by Pokherel et al. (2003) but for MSMR channels. The input profile of the ice has a density of 0.50 gm/cm$^3$. From the figure it is clear that the form of the curves represents
contributions from both emission and scattering associated with liquid and frozen hydrometeors. Similar to the nonraining and raining cases of Eddington’s approximation, here also the Tbs are primarily dependent on the surface emission, atmospheric emission and scattering. In the case of DOM the strong dependence of ice and hence associated scattering is clearly evident. Figure 2.12(a) shows the variation of brightness temperature with rainrate for 10 GHz channel (V and H polarization). It is clear from the figure that the Tb first increases from 110 to 240K and 180 to 250K for H and V polarizations respectively up to the rainrates of 20 mm/hr and afterward it starts decreasing and saturates after 35 mm/hr of rainrate. Although 10 GHz channel is not so sensitive to ice scattering but a little amount of scattering is still persisting. The response of Tb with rainrate for 19 GHz and 21 GHz channels show the similar trend as that of 10 GHz channel in presence of ice scattering but with a small difference in their dynamic range of Tbs as shown in Figure 2.12(b) and 2.12(c) respectively. In both frequencies, first the Tb increases up to the rainrates of 5 mm/hr and then there is a decreasing slope, which immediately merges after the rain rate of 10 mm/hr. Figure 2.12(d) and 2.12(e) shows the Tb variation with rainrate for 37 GHz and 85 GHz channel respectively. The large slope in the dynamic ranges of brightness temperatures is apparent in both 37 and 85 GHz channels. The 85 GHz channel shows the sharp decreasing trend in Tbs after 1 mm/hr of rainrate than the 37 GHz channel as the Tb falls from 240 to 150K and 245 to 145K for 85Hz and 37GHz channel respectively. This sharp decrease in slope is seen due to the ice precipitation present above the freezing levels. Therefore this potential of 85 GHz can be utilized for the rain retrieval over the land areas.

Figure 2.13 (a-e) shows the similar behavior as that of 2.12 (a-e) except that it has an extra higher frequency channel of 157 GHz and a little variation in the frequency of 21 GHz, and 85 GHz channels. 157 GHz channel will be used for the retrieval of ice at cloud tops as it is more sensitive to ice content. In place of 21 GHz, here it has 24 GHz channel which is more suitable for integrated water vapor and 89 GHz channel is used in place of 85 GHz because of its suitability to retrieve convective rain over land and sea areas. From the Figure 2.13 (d) and 2.13 (e), it is clear that the slope of 157 GHz
channel is more sharp compare to 89 GHz. That shows the dependence of this higher frequency channel on ice content scattering.

It is thus evident from the simulations for 10 to 85 GHz, that any of the channels are no longer good indicators of rain through their emission signatures alone in the absence of ice aloft. The ice scattering signatures are well picked up at higher frequencies by present model which corroborates with the actual measurements, e.g. the Tbs from TMI and rainfall from PR observations. Therefore a combination of emission and scattering channels would be possible for rainfall retrieval using any inversion techniques like multiple regression or neural network approach (Gairola, et. al., 2001, Kumar, et. al., 2007).

Since the DOM is an advanced RT model solution, it is also highly pertinent to have better characterization of input fields through cloud resolving models than the input data used herein. The cloud resolving models are an important tool and have been used to study many processes in atmospheric sciences, including cloud-ocean surface interactions (Tao, et. al., 1991). The TRMM relies on Goddard Cloud Ensemble Model (GCE) to validate remotely sensed estimates of precipitation and diabatic heating. The attempt with DOM mentioned in this thesis will help in achieving Meghatropique and coming GPM project goals.

2.5 CONCLUSIONS:

In this chapter the brightness temperatures at different frequencies corresponding to the TMI and MADRAS sensors as a function of rainfall rate using well-established radiative transfer models have been simulated. The simulations are done under clear sky conditions as well as in raining conditions. The two important methods namely: Eddington’s approximation and Discrete Ordinate method with four streams have been used for simulations from these radiative transfer models.. In first attempt, the simulation of brightness temperatures for various TMI frequencies 10 to 85 GHz channels in non-raining (non-scattering) conditions have been carried out using Kummerow RT-model. The database of the geophysical parameters for forward radiative transfer modeling is constituted by a set of analysis of the GDAS and
ECMWF forecast. These simulations have been formulated with the specifications of the TRMM-TMI, like frequencies, incidence angle and polarization over the frequency range of 10 to 85 GHz, but the results are general and thus can be applied in the case of MADRAS and other future coming radiometers. The simulated Tb results are in good relation with the accumulated 3-days averaged Wentz TMI Precipitation for non-raining conditions characterizing all the possible dynamic ranges of input and output field vectors. The rainfall signatures are very well picked up by the lower channels under the clear sky conditions due to impact of cloud liquid water, while the higher channels show some discrepancy because of suppressed scattering.

Secondly, the simulations have been carried out in case of raining atmospheres using Eddington’s and Descrete Ordinate methods. In raining atmosphere the contribution from emission and scattering associated with the presence and absence of liquid hydrometeors (clouds and rainfall) has been characterized. Eddington’s approximation is used to calculate the scattering effects by large liquid hydrometeors, whereas Discrete Ordinate Method is applied to calculate the scattering from ice as well. The main objective of the RT-simulations is to delineate the more appropriate frequency channels that are most sensitive to the rainfall especially in case of brightness temperatures. Therefore the study carried out in this chapter will contribute to select the suitable predictor variables for the combined use of different satellite sensors to retrieve rainfall as well as other geophysical parameters with greatest possible accuracy. This type of sensitivity study based on the simulation of brightness temperatures for non-raining, raining or cloudy conditions is useful in interpretation of radiometric observations. The channels of lower frequencies are more sensitive to the rain as well as surface roughness whereas the higher frequency channels show greater sensitivity to the rainrate. Furthermore, the higher frequency channel, viz. 157 GHz has a great potential to sense precipitation due to ice contents in the clouds. Therefore the proper combination of both type of channels are very much essential for the development of the advanced rainfall retrieval algorithm. This type of study will be important for the geophysical parameters retrieval from the currently launched Indo-French MEGHA-TROPIQUE satellite and future coming GPM group of satellites.
Chapter 2  
Radiative Transfer Modeling

Figure 2.1: Scattering Geometry

Figure 2.2: Radiation passing through a small volume element

Figure 2.3: Geometry for a plane-parallel atmosphere
Figure 2.4: Scatter plots of the simulated Tbs from the TRMM-TMI radiometric channels.

(a) Simulated TMI Tb (K) at 10.65 GHz (H)

(b) Simulated TMI Tb (K) at 10.65 GHz (V)

Figure 2.5 (a-b): Simulated Tbs for the TRMM-TMI channel (10.65 GHz)
Chapter 2
Radiative Transfer Modeling

Figure 2.6 (a-b): Simulated Tbs for the TRMM-TMI channel (19.35 GHz)

Figure 2.7(a-b): Simulated Tbs for the TRMM-TMI channel (37 GHz)
Figure 2.8(a-b): Simulated TBs for the TRMM-TMI channel (85.5 GHz)

Figure 2.9(a): 3-days averaged Wentz TMI Wind speed ending at 13th Jun, 2010 downloaded from www.ssmi.com/tmi
Figure 2.9(b): 3-days averaged Wentz TMI Water Vapor ending at 13th Jun, 2010 downloaded from www.ssmi.com/tmi

Figure 2.9(c): 3-days averaged Wentz TMI Cloud Liquid Water ending at 13th Jun, 2010 downloaded from www.ssmi.com/tmi

Figure 2.9(d): 3-days averaged Wentz TMI Precipitation ending at 13th Jun, 2010 downloaded from www.ssmi.com/tmi
Figure 2.10 (a-e): Simulated brightness temperatures for cloudy/raining atmosphere with Ice Content for frequencies from 19 to 157 GHz
Figure 2.11 (a-e): Simulated brightness temperatures for cloudy/raining atmosphere without Ice Content for frequencies from 19 to 157 GHz
Figure 2.12 (a-e): Theoretical Brightness temperatures as a function of rain rate for TRMM-TMI frequencies as calculated by DOM (Liu, 1998) and from 10.0 to 85 GHz.
Figure 2.13(a-e): Theoretical Brightness temperatures as a function of rain rate for MEGHATROPIQUES-MADRAS frequencies as calculated by DOM (Liu, 1998) and from 19.0 to 157 GHz.