CHAPTER-2

LITERATURE REVIEW

2.1 Introduction

Stability of Functionally Graded Materials (FGM) is gaining importance day by day because of its extraordinary properties required for tailor-made engineering applications. Functionally Graded Materials are those in which the volume fraction of the two or more constituent materials is varied continuously as a function of position along certain dimension(s) of the structure. Concept of FGM was first originated in 1984 by a group of materials scientists in Japan as means of preparing thermal barrier coating (TBC) materials (Loy et al.)[60]. In recent years the dynamic behavior of structural members with FGMs are of considerable importance in both research as well as industrial fields. FGMs are generally metal–matrix composites (MMCs) with continuously varying volume fractions of the constituent materials. Typically the material properties of an FGM can be designed by varying the volume fractions of its constituent phases along spatial coordinates so as to improve the strength, toughness and high temperature withstanding ability. FGMs possess a number of advantages, including an improved residual stress distribution, high temperature withstanding ability, higher fracture toughness, improved strength, and reduced stress intensity factors, which make them attractive in many engineering sector applications such as aerospace, aircraft, automobile, defense, biomedical and electronic industries. Many primary and secondary structural elements, such as helicopter rotor blades, turbine blades, robot arms and space erectable booms, can be idealized as beam type structure. As the applications of FGMs are gaining increasing importance in the aforesaid sectors, wherein these components are subjected to vibration and instability, an analysis of FGM beam type structures may be worth of an important research topic.

It was recognized by the early researchers that the bending effect is the single most important factor in a transversely vibrating beam. The Euler-Bernoulli model includes the strain energy due to the bending and the kinetic energy due to the lateral displacement. The Euler-Bernoulli model dates back to the 18th century. Jacob Bernoulli first discovered that the curvature of an elastic beam at any point is proportional to the bending moment at that point.

Many advances on the elastic curves were made by Euler. The Euler-Bernoulli beam theory, sometimes called as classical beam theory (CBT), is the most commonly used because it is simple and provides reasonable engineering approximations for many problems. However, the Euler-Bernoulli model tends to slightly overestimate the natural frequencies. This problem is exacerbated for the natural frequencies of the higher modes. Also, the prediction is better for slender beams than non-slender beams. The Rayleigh beam theory provides a marginal improvement on the Euler-Bernoulli theory by including the effect of rotation of the cross-section. As a result, it partially corrects the overestimation of natural frequencies in the Euler-Bernoulli model. However, the natural frequencies are still overestimated. The shear model adds shear distortion to the Euler-Bernoulli model. It should be noted that this is different from the pure shear model which includes the shear distortion and rotary inertia only or the simple shear beam which includes the shear distortion and lateral displacement only. Neither the pure shear nor the simple shear model is our purpose of obtaining an improved model to the Euler-Bernoulli model because both exclude the most important factor, the bending effect. By adding shear distortion to the Euler-Bernoulli beam, the estimate of the natural frequencies improves considerably. Timoshenko proposed a beam theory which adds the effect of shear as well as the effect of rotation to the Euler-Bernoulli beam.

The Timoshenko model is a major improvement for non-slender beams and for high-frequency responses where shear or rotary effects are not negligible. Following Timoshenko, several authors have obtained the frequency equations and the mode shapes for various boundary conditions. Traill-Nash and Collar (1953) gave a fairly complete theoretical treatment as well as experimental results for the case of a uniform beam. In the first part of their paper, they obtained the expressions for the frequency equation and mode shapes for six common boundary conditions: fixed-free, free-free, hinged-free, hinged-hinged, fixed-fixed and fixed-hinged. In the second part of the paper, they reported the experimental results with the numerical results obtained by the Euler-Bernoulli, shear, and Timoshenko models.

They used non-slender beams in which the shear and rotary effects were important. They reported the difference for the first and second natural frequencies predicted by each of the theoretical models and the experimental values (Wei et al.)[120]. The summary of the result is shown in Table 2.1. The frequency equations are difficult to solve except for the case of a simply supported beam. Even when the roots of the frequency equations are obtained, it is a challenge to present them in a meaningful way. A crucial parameter in Timoshenko beam theory is the shape factor. It is also called the shear coefficient or the area reduction factor. This parameter arises because the shear is not constant over the cross-section. The shape factor is a function of Poisson's ratio and the frequency of vibration as well as the shape of the cross-section.

TABLE-2.1

The percentage deviates from the experimental values obtained by Traill-Nash and Collar (1953)

Beam models	First natural frequency	Second natural frequency
Euler–Bernoulli	+14% to +26%	+78% to +133%
Shear	0% to +3%	−1% to +6%
Timoshenko	−1% to +2%	−1% to +6%

2.2 Literature Survey

Composite material are a class of advanced material, made up of one or more materials combined in solid states with distinct physical and chemical properties. Composite material offers an excellent combination of properties which are different from the individual parent materials and are also lighter in weight. Wood is a composite material from nature which consists of cellulose in a matrix of lignin (Wang)[117]. Composite materials will fail under extreme working conditions through a process called delamination. This can happen for example, in high temperature application where two metals with different coefficient of expansion are used. To solve this problem, researchers in Japan in the mid-1980s, confronted with this challenge in a hypersonic space plane project requiring a thermal barrier with outside temperature of 2000K and inside temperature of 1000K across less than 10 mm thickness. They came up with a novel material called Functionally Graded Material (FGM) (Shanmugavel et al.)[96].

Functionally Graded Material (FGM), a revolutionary material, belongs to a class of advanced materials with varying properties over a changing dimension (Atai et al.)[7]. FGMs occur in nature as bones, teeth etc. (Malinina et al.)[64]. Nature designed these materials to meet their expected service requirements. This idea is emulated from nature to solve engineering problem the same way artificial neural network is used to emulate human brain. FGM eliminates the sharp interfaces existing in composite material which is where failure is initiated (Niino et al.)[74]. It replaces this sharp interface with a gradient interface which produces smooth transition from one material to the next. One unique characteristics of FGM is the ability to tailor a material for specific.

The spectral application area of FGM opens a wide scope for scientific researches. Raju and Rao[86] obtained one term Rayleigh Ritz solutions for the fundamental frequency parameter of short beams in the presence of a concentrated axial load. Expressions for frequency parameter are presented for simply supported and clamped beams. They also included some numerical results to establish the accuracy. Lin[58] in his research used finite element method for dynamic analysis of Timoshenko beams by moving loads. He also included the analysis of short, sturdy beams and structures with higher modes being excited. Paulino and Jin[79] have presented an extension of correspondence principle to nonhomogeneous viscoelastic solids under the assumption that the relaxation (or creep) moduli be separable functions in space and time. The revisited correspondence principle extends to specific instances of thermo-viscoelasticity and fracture of functionally graded materials. Chakraborty et al.[18] have studied about the thermostatic behavior of functionally graded beam structures. They developed a beam element based on the first order shear deformation theory wherein the exact solution of static part of governing differential equation was used to derive the shape functions.

Li[57] has developed a new unified approach for analyzing the static and dynamic behaviors of functionally graded beams (FGB) with the rotary inertia and shear deformation by reducing the Euler–Bernoulli and Rayleigh beam theories from the Timoshenko beam theory. Sharma and Bhandari[98] have obtained an elasticity solution for a sandwich beam with a functionally graded core subjected to transverse loads. The sandwich is subdivided into four elements, the top and bottom face-sheets, and top and bottom halves of the sandwich core. Euler-Bernoulli beam theory is used to model the face-sheets and plane elasticity equations are used to analyze the core. The Young's modulus of the core is varied exponentially through the thickness and the Poisson's ratio is kept constant. The exponential variation of elastic stiffness coefficients allows exact elasticity solution for the problem.

Arvin et al.[6] used the higher order theory for sandwich beam with composite faces and viscoelastic core by considering independent transverse displacements on two faces and linear variations through the depth of the beam core. They observed that by increasing the fiber angles of face sheets and or core's thickness, loss factors are increased and natural frequencies are decreased. The finite element equations for a variationally consistent higher order beam theory are investigated for the static and dynamic behavior of rectangular beams. It is observed that for higher modes, the higher order theory provides a poor approximation to the true shear stress distribution which changes from parabolic to a distribution with maxima near the upper and lower surfaces and a minimum at the neutral axis (Heyliger and Reddy)[36].

Li and Kang[56] have investigated the mechanical behaviors of a non-linear functionally graded-material (FGM) cantilever beam subjected to an end force by using large and small deformation theories. They have found that different gradient indexes may change the bending stiffness of the beam so that an FGM beam may bear larger applied load than a homogeneous beam when choosing appropriate gradients. Moreover, the bending stress distribution in an FGM beam is completely different from that in a homogeneous beam. The bending stress arrives at the maximum tensile stress at an internal position rather than at the surface. Obtained results are useful in safety design of linear and non-linear beams. Mohanty et al.[69] evaluated the static and dynamic behavior of functionally graded ordinary (FGO) beam and functionally graded sandwich (FGSW) beam for pined–pined end condition using a finite element method assuming first order shear deformation theory for the analysis and studied the effect of power law index on critical buckling load and natural frequencies of FGO beam. The result shows that the critical buckling load of FGO beam with steel-rich bottom increases with power law index whereas the trend reverses for beam with Al-rich bottom.

Simsek et al.[101] have developed a micro scale functionally graded Timoshenko beam model for the static bending analysis based on the modified couple stress theory (MCST) to study the influences of the volume fraction index, the different estimation method of the material properties, length scale parameter, the aspect ratio and the Poisson effect on the static bending behavior. They found that the deflections of the micro beam by the classical beam theory are always larger than those by the modified couple stress theory. Kahrobaiyan et al.[40] have developed a new comprehensive Timoshenko beam element, on the basis of the modified couple stress theory in which the shape functions of the element are derived by solving the governing equations of modified couple stress Timoshenko beams. The results of the new .beam element are in good agreement with the experimental findings while the gap between the classical FEM and experimental results is not able. Benatta et al.[13] have presented the static response for a simply supported functionally graded hybrid beam subjected to a transverse uniform load and found that by varying the fiber volume fraction within a symmetric laminated beam and combining two fiber types to create a hybrid functionally graded material (FGM) can offer desirable increases in axial and bending stiffness. A rotating nonlinear FGM beam model accounting for arbitrary axial deformations is developed to analyze the geometric stiffening effect (Piovan and Sampaio)[80]. The developments in analysis of FGM beam type structures with diverse areas relevant to various aspects of analysis of FGM beam type structures are reviewed which include various deformation theories, effect of property distribution laws along coordinate, boundary conditions and system parameters effect (Chauhan and Khan)[19].

Murin et al.[71] proposed the homogenization techniques of spatial varying material properties and derived the effective longitudinal varying electric, thermal and elastic material properties by the extended mixture rules and the laminate theory for modal and multiphysical analyses of FGM beams and shells. The multiphysics of multilayered and functionally graded cylinders are subjected to steady-state hygrothermomagneto electromechanical loading. The results obtained are verified with those available in literature for a homogenous infinitely long cylinder and can also be applied to study the multiphysics of thin circular disks (Akbarzadeh and Pasini)[2]. Levyakov[52] has proposed a numerical method based on the finite-difference approximations and Newton–Raphson iteration technique to study deformations and stresses in tubes of circular and noncircular cross sections for a wide range of geometrical parameters. The behavior of various beam (Aluminium, Copper and Steel) under longitudinal stress with Computation Analysis (FEM package (Autodesk Inventor)) was done by setting maximum load value as 1100 N & dimensions of specimen were (500 x 50 x 12). It can also be observed that as the poison's value increases the value of stress increases when subjected to longitudinal load (Mahto et al.)[62].

Zohoor and Kakavand[131] have considered a two-link flexible manipulator in which Timoshenko and Euler–Bernoulli beam models are considered. The findings suggest that for two-link manipulators with relatively high slenderness ratios, there is a remarkable difference between the models, considering the foreshortening effect and un-stiffened models. It is obvious that for high precisions applications, the stiffened Timoshenko model is recommended. The shear deformation theory is used to analyze simply supported thick isotropic beams for the transverse displacement, axial bending stress and transverse shear stress. The following points are observed that the shear deformation theory is variationally consistent and requires no shear correction factor and is capable to produce excellent results for deflection and bending stress because of effect of transverse normal (Fatima-uz-Zehra and Shinde)[33]. The static analysis of a functionally graded (FG) simply-supported beam subjected to a uniformly distributed load by using Ritz method within the framework of

Timoshenko and the higher order shear deformation beam theories reveals that the stress distributions in FG beams are very different from those in isotropic beams (Simsek) [102]. Benatta et al.[12] have studied Higher-order flexural theories for short functionally graded symmetric beams under three-point bending. They concluded that by creating an FGM in the form of a symmetric composite whose fiber volume fraction varies through the thickness, the design of such beams can be improved.

The displacement field based on higher order shear deformation theory is used to study the static behavior of functionally graded metal–ceramic (FGM) beams under ambient temperature. The results obtained as depending on whether the loading is on the ceramic rich face or metal rich face of the beam, the static deflection and the static stresses in the beam do not remain the same. It was observed that the deflections for a given boundary condition are more for metal rich beam when compared to ceramic rich beam and deflection increases as the power law index increases. For power law index other than homogeneous composition the stress profile is not linear. Investigations on the static analysis of the FGM beams revealed that the deflections, stresses and the location of the neutral surface are highly dependent on power law index (Kadoli et al.)[39]. The problem of a cantilever functionally graded beam subjected to different loads is studied (Zhong and Yu)[129] and the obtained solution is valid for arbitrary graded variations of the material distribution, so it could serve as a basis for establishing simplified FGM theories. A systematic approach to solving the eigenvalue problems associated with the uniform Timoshenko beam model with pinned–pinned and cantilever beam are discussed for the properties of the natural frequencies and modes (Rensburg and Merwe)[92].

The stiffening effects of a simply supported and clamped–free symmetric piezo laminated composite type beams are investigated and found that the optimal placement for simple support boundary conditions occurs when the actuators are placed next to the supports (Waisman and Abramovich)[115]. The elastic solution is obtained for a functionally graded beam subjected to transverse loads and observed that the stress concentrations are less than that in homogeneous beams when the softer side of the functionally graded beam is loaded. The reverse is true when the stiffer side is loaded (Sankar)[94]. Rao and Gupta[88] have derived the stiffness and mass matrices of a rotating twisted and tapered beam element using shear deformation and rotary inertia in deriving the elemental matrices. The rest four natural frequencies and mode shapes in bending mode are calculated for cantilever beams. The effects of twist, speed of rotation and variation of depth and breadth taper ratios are studied.

The linear flexural stiffness, incremental stiffness, mass, and consistent force matrices for a simple two-node Timoshenko beam element are developed (Kosmatka)[46] based upon Hamilton's principle, where interdependent cubic and quadratic polynomials are used for the transverse and rotational displacements, respectively. It is observed that the Timoshenko beam element is in good agreement with the exact results for long slender isotropic beams, where there is no hint of any type of "shear-locking" effects. Iura [38] examined the effects of coordinate system on the accuracy of co-rotational formulation for planar Bernoulli-Euler's beam established that the comparison between the approximated strains in the locally convicted coordinates and the exact strains in the fixed global coordinates is enough to study the accuracy of the co-rotational formulation. The effects of material properties of the FG beam and inertia of the moving load on the dynamic behavior of the system by Rayleigh–Ritz method have been investigated and analyzed (Khalili et al.)[42] and found that the critical velocity of the beam/FG beam predicted by moving force model is smaller than the one predicted by moving mass model. Kim and Paulino[45] have used finite element method to compute fracture parameters in FGM and developed graded finite elements where the elastic moduli are smooth functions of spatial coordinates which are integrated in to the element stiffness matrix.

Coomar and Kadoli[24] have developed a functionally graded model of an air-cooled turbine blade with air foil geometry conforming to the NACA0012. The model developed by them is used in a finite element algorithm to obtain a non-linear steady state solution to the heat equation for the blade under convection and radiation boundary conditions. Marakala and Kadoli [65] have presented a finite element formulation for an Euler-Bernoulli beam subjected to thermal load in which the temperature distribution across the thickness of the beam is obtained by either dependent or independent of the heat loss caused by beam motion. Pradhan and Parida[82] have carried out non-linear finite element analyses using contact and multi point constraint (MPC) elements for evaluation of inter-laminar stresses in the adhesive layer existing between the lap and the strap adherents of lap shear joints (LSJ) made with curved laminated fiber reinforced plastic (FRP) composite panels for varied embedded delamination between the first and second plies of the strap adherents.

Belyi et al.[11] has presented their work on parametric generation of light by pumping with Bessel light beams. They developed a theory of azimuth-matched nonlinear interaction of Bessel light beams. The possibility of high-efficiency parametric conversion of a Bessel pump beam to a Gaussian signal wave beam and a Bessel idler wave beam is shown. They predicted a physical mechanism to achieve high-efficiency parametric generation of light using a pump with a multi-ring spatial Bessel structure. This mechanism is based on the compensation of energy losses in the central maximum by replenishment of energy from the lateral maxima of the Bessel beam. So, the optimum use of the specific space-frequency structure of Bessel beams offers possibilities to increase the efficiency of parametric conversion processes.

Murin[72] has studied the Cartesian stiffness matrix using methods of differential geometry and given a formulation leads to three important results: (a) The stiffness matrix does not depend on the parameterization of the manifold; (b) The stiffness matrix depends on the choice of a connection on the manifold; and (c) The standard definition of the Cartesian stiffness matrix assumes an asymmetric connection and this is the reason that the matrix is in general asymmetric.

2.3 Recent Research efforts in FGM

Lots of studies have been conducted to understand the behavior and property estimation of functionally graded materials subjected to transverse loading. A comprehensive review on performance of FGM was published in 2007 (Birman and Byrd) [15]. Other reviews on functionally graded materials available in the literature "review study on research and development" (Cherradi et al.)[22]. Tilbrook et al.[112] also conducted review study on crack propagation in functionally graded materials. A number of researches have also been conducted in the areas of analysis and modelling work on functionally graded material; some of these work can be found in the literature [48, 76, 130, 51, 47, 93, and 73]. There are still more to be done in terms of research to improve the stability under extreme operating conditions and performance of manufacturing processes of FGM.

2.4 Stability Analysis of FGM beam

The understanding of stability of an FGM beam subjected to environmental forces is vital before its application in engineering field. A lot of research is being conducted in this regard. A few of them are summarized below.

The static and dynamic stabilities of a laminated composite cantilever beam having linear translation spring and torsional spring as elastic supports subjected to periodic axial loading are examined (Ozturk and Sabuncu) [77] . It is found that when the stiffness of the elastic support having linear translation and linear torsional springs increases, and the position of the elastic support is moved towards the free-end of the beam, the laminated composite beam becomes more stable from the point of static stability and the dynamic unstable region without having linear torsional spring. Shahba et al.[95] have studied free vibration and stability analysis of axially functionally graded tapered Timoshenko beams through a finite element approach and investigated the effects of taper ratio, elastic constraint, attached mass and the material non-homogeneity on the natural frequencies and critical buckling load. The parametric instability behavior of a non-prismatic bar with localized zones of damage and supported on an elastic foundation is studied by using a finite element analysis (Datta and Nagraj)[25]. It is observed that the presence of damage has a marked effect on the static buckling load, natural frequency and dynamic instability zones of the bar. A flaw in the beam near the narrow end affects the static and dynamic behavior more than one near the wider end.

Mohanty et al.[68] investigated the effect of distributions of material properties of the FGO beam on its parametric instability and found that the FGO beam with steel-rich bottom is more stable as compared to that with Al-rich bottom for all the three modes and for both the types of property distributions. The parametric dynamic stability of a pinned-pinned asymmetric sandwich beam resting on a Pasternak foundation with viscoelastic core, subjected to an axial pulsating load has been investigated (Ray et al.) [91]. The vibration and stability of a non-uniform double-beam subjected to tangential follower forces distributed over the center line by use of the transfer matrix approach has been analyzed (Takahashi and Yoshioka)[109]. They observed that the eigenvalues of vibration and critical flutter loads of a free-clamped beam of varying cross-section subjected to a follower force calculated numerically by the application of the method, from which the effects of the cross-section, slenderness ratio, supported span and the stiffness of the support have been quantitatively illustrated.

There are two ways for solving the eigenvalue problems of vibrations and stability of a beam on a variable Winkler elastic foundation. The first method is based on using the exact stiffness, consistent mass, and geometric stiffness matrices for a beam on a variable Winkler elastic foundation. The second method is based on adding an element foundation stiffness matrix to the regular beam stiffness matrix, for vibrations and stability analysis. It is found that the second method is preferred, since it gives slightly better results at lower computation cost (Eisenberger and Clastornik)[30]. Swaminathan et al.[108] have made a comprehensive review of the various methods employed to study the static, dynamic and stability behavior of Functionally Graded Material (FGM) plates by both analytical and numerical methods with an emphasis to present stress, vibration and buckling characteristics of FGM plates predicted using different theories proposed by several researchers without considering the detailed mathematical implication of various methodologies. The effects of high frequency longitudinal forces on bunched beams are investigated using a computer model (Messerschmid and Month)[67] and found that the microwave instability is characterized by rapid growth rates, in time periods much less than ½ a synchrotron period & the instability has only a weak dependence on frequency, so that it can be excited over a wide frequency band in the microwave region.

2.5 Free Vibration of FGM beam

The knowledge of free vibration characteristic of FGM helps in better understanding of the stability analysis of the FGM beam. A good amount of research work has been done in this regard as follows.

Ke et al.[41] worked on non-linear free vibration of FGM micro-beam structure and determined the FGM properties by using Mori-Tanaka homogenization technique. The higher order non-linear governing equations and boundary conditions are derived by using the Hamilton principle. The result is found as, both the linear and non-linear frequencies increases significantly when the thickness of the FGM micro-beam is comparable to the material length scale parameter. Similarly Paul & Das[78] have done a theoretical study on free vibration behavior of pre-stressed functionally graded material (FGM) beam and considered Power law variation of volume fraction along the thickness direction. The governing equation for the static problem is obtained using minimum potential energy principle. The dynamic problem for the pre-stressed beam is formulated as an eigenvalue problem using Hamilton's principle. Three classical boundary conditions with immovable ends are considered for the present work, namely clamped–clamped, simply supported–simply supported and clamped–simply supported. The results are presented in non-dimensional pressure-displacement plane for the static problem and in non-dimensional frequency displacement plane for the dynamic problem. Numerical results for non-dimensional frequency parameters of unreformed beam are presented for different functionally graded materials with CC, SS and CS boundary conditions. Large amplitude free vibration problem of axially functionally graded slender non-uniform beam with various taper profiles and material gradation have been investigated (Kumar et al.)[50] with same methodology and can be applied to any type of taper pattern and axial material property gradation, as long as they are expressible in terms of mathematical functions.

An Analytical solutions to study the free vibrations of composite beams with two overlapping delaminations under axial compressive load has been developed (Della) [27] and found a linear relation between the square of the "constrained mode" and "free mode" frequencies of the simply supported beam and the axial compressive load. A non-linear relation is observed between the square of the "free mode" frequencies of a clamped-clamped beam and axial compressive load due to the opening of the delaminated layers in the "free mode" mode shape of the beam. Tang et al.[110] have analyzed free vibration of non-uniform functionally graded beams via the Timoshenko beam theory. They found that the rotational inertia of cross-section has a decreasing effect on the natural frequencies, whereas an increase in shear stiffness gives rise to an increase in the natural frequencies and the fundamental frequencies for graded shear deformation beams are closest to those for graded Timoshenko beams, while those for graded Euler-Bernoulli beams strongly deviate away from those for graded Timoshenko beams.

A numerical method for solution of the free vibration of beams governed by a set of second order ordinary differential equations of variable coefficients, with arbitrary boundary conditions, is presented (Prokic et al.)[83] and compared the calculated results with those available in the literature. Good accuracy can be obtained even with a relatively small number of nodes. Li et al.[54] worked on free vibration of axially inhomogeneous beams and found that for certain graded beams, the natural frequencies jump across its critical value and the natural frequencies lower than the critical value become pseudo-frequencies. Only when bending waves with frequencies exceeding the critical value, harmonic vibration can be excited. This is an essential difference between homogeneous and inhomogeneous beams. Results obtained apply to non-uniform beams with constant thickness and exponentially decaying width.

The differential transformation method (DTM) which is applied to investigate free vibration of functionally graded beams supported by arbitrary boundary conditions, including various types of elastically end constraints has been studied (Wattanasakulpong and Ungbhakorn)[119]. The results revealed that trend of frequency results for various modes of vibration decreases as the volume fraction indexes increase, except for the case of the E-E boundary conditions in which the trend of the first two modes is reversed owning to the effects of translational and rotational springs at both ends. It is also seen that there are considerable changes of frequencies as well as mode shapes when the stiffness of spring becomes larger. The free and forced vibration of a laminated functionally graded beam of variable thickness under thermally induced initial stresses is studied (Xiang and Yang)[121] within the framework of Timoshenko beam theory. They found that the beam thickness variation, temperature gradient, slenderness ratio, and the end support condition significantly affect both the free vibration and dynamic response as well, but in a way quite similar to what have been observed in thermally loaded homogeneous beams with variable thickness. Sina et al.[104] have used a new beam theory different from the traditional first-order shear deformation beam theory to analyze free vibration of functionally graded beams. They found that the new theory is a little different in natural frequency from the traditional first-order shear deformation beam theory and the mode shapes of the two methods are coincidental.

The free vibration of simply supported FG beam was investigated (Aydogdu and Taskin.)[8] and found that the difference between CBT and PSDBT is increasing with increasing mode number and all frequencies are decreasing with increasing k, also the results for frequency obtained as frequency parameter is decreasing with increasing k and increasing with increasing L/h ratios. An analytical solution to the free vibrations of a beam with two overlapping delaminations is presented (Della et al.)[26]. They concluded the result as the delamination lengths and locations significantly influence the fundamental frequency of a clamped–clamped beam and for two equal overlapping delaminations, the maximum influence occurs when the two delaminations are fully separated. The minimum influence occurs when the two delaminations are near fully overlapping.

The natural frequencies of honeycomb sandwich beams having debonding or delamination embedded between the face-layer laminates and the honeycomb core have been investigated (Kim and Hwang)[44]. A theoretical analysis of the effect of the extent of debonding on the flexural stiffness and on the natural frequency is compared with experimental observations and found that an increase in the extent of debonding gives a greater reduction in natural frequencies of the sandwich beams. The effect of material coupling between the bending and torsional modes of deformation together with the effects of shear deformation and rotatory inertia is taken into account by deriving an exact expressions for the frequency equation and mode shapes of composite Timoshenko beams with cantilever end conditions in explicit analytical form by using symbolic computing package REDUCE (Banerjee)[10]. Cekus[16] has used Lagrange multiplier formalism to find a solution of free vibration problem of a cantilever tapered Timoshenko beam. The sample experimental results were compared with experimental results to check the correctness. Chen et al.[20] have worked on free vibration of an arbitrarily thick orthotropic piezoelectric hollow cylinder with a functionally graded material along the thickness direction and found that frequency of the cylinder can differ significantly in the presence of a fluid medium.

A setup for the beam vibrations with different configurations and many key parameters has been made (Shinde et al.)[99] and found that the result obtained are in good agreement with the theoretical and numerical (FEA) results. Vaziri et al.[113] have developed a model to investigate lateral vibration which could be of use in building a controller for reducing vibrations in the cantilever beam and analyzed the Static and Dynamic analysis. The Multiple time scale solutions to study the nonlinear forced vibration of a beam made of symmetric functionally graded (FG) materials based on Euler–Bernoulli beam theory and von Karman geometric nonlinearity has been studied. Shooshtari and Rafiee[100] found that the nonlinear frequencies of homogenous beams and clamped–clamped graded beams are not affected by the sign of the vibration amplitude. Vela and Batra[114] have investigated a three-dimensional exact solution for free and forced vibrations of simply supported functionally graded rectangular plates Results are presented for two-constituent metal–ceramic functionally graded rectangular plates that have a power-law through-the-thickness variation of the volume fractions of the constituents. Results are also computed for a functionally graded plate that has a varying microstructure in the thickness direction using a combination of the Mori–Tanaka and the self-consistent methods.

Stafford and Giurgiutiu[107] has developed the equations of motion including shear and rotatory inertia for uncoupled lead-lag and flapping vibrations of beams rotating at constant angular velocity in a fixed plane and obtained in terms of four independent functions, each a convergent power series for exact representation of centrifugal forces, thus its accuracy is independent of the speed of rotation.

In the paper, A new approach for the free and forced vibrations of a Timoshenko beam by a single equation is proposed (Majkut)[63]. The solution to such an equation is a function of vibration amplitudes and the solution to the differential equation depends on the vibration frequency. The change of the solution form occurs when the frequency crosses a specific value $\omega = GkA/\rho I$. The correctness of proposed description was checked through the analysis of free vibration frequencies and amplitudes of forced vibrations with different boundary conditions as well as comparison with the results of finite element analysis.

The natural frequencies of a rotating functionally graded cantilever beam with concentrated mass have been studied (Ramesh and Rao)[87] and found that for a stationary beam, chord wise bending natural frequencies decrease with an increase in power law index up to a critical value after which frequencies relatively un-effected. The magnitude of the concentrated mass has been found to have an influence on the chord wise bending natural frequencies depending on the location of the mass. The relation between chord wise bending frequencies and angular speed is dependent on the location of the concentrated mass. The power law index has been found to have an influence on the relation between chord wise bending versus angular speed depending on the location of concentrated mass. Chung and Yoo[23] have proposed a finite element analysis for a rotating cantilever beam. Using the stretch deformation instead of the conventional axial deformation to derive the linear partial differential equations for a rotating cantilever beam undergoing a prescribed rotating motion and found from the time responses that the non-smooth profile of the rotating speed incurs larger vibration in the chord wise motion than the smooth one. Therefore, if the beam rotates with a smooth speed profile, a large amount of vibration can be reduced.

The vibrational properties of functionally graded nanocomposite beams reinforced by randomly oriented straight single-walled carbon nanotubes (SWCNTs) under the actions of moving load have been investigated Yas et al.[127]. They found that In the forced vibration (before the passage of moving load from the beam span), the beam with symmetrical distribution of carbon nanotube (SFG) has the smallest dynamic displacement at each time and frequencies of CNTRC beam reinforced with aligned CNTs are higher than those of beams reinforced with randomly oriented CNTs. Ansari et al.[4] have investigated On the basis of the strain gradient Timoshenko beam theory, the vibrational response of FGM micro beams and made a comparison between the vibrational behavior of FGM micro beams predicted by the various beam models based on the classical theory, modified couple stress theory, and strain gradient theory.

Andrew and Vel[3] have proposed a methodology to optimize the natural frequencies of functionally graded structures by tailoring their material distribution and developed a methodology for the simulation and optimization of the vibration response of bidirectional functionally graded structures. The results show that material distribution has a significant influence on natural frequencies and that the proposed methodology can be used to design functionally graded structures with superior dynamic response. A technique of analyzing the vibration problem of reinforced concrete beam members including bond-slip of the reinforcements has been proposed by Inoue et al.[37]. They derived a finite element formulation of the problem and concludes with numerical examples with selected bond moduli and damping resulting from bond-slip, to examine the effects of bond-slip on the bending vibration of reinforced concrete beams. Yang and Shen[123] have presented Free and forced vibration analyses for initially stressed functionally graded plates in thermal environment and the results reveal that, when thermal effects are considered, functionally graded plates with material properties intermediate to those of isotropic ones do not necessarily have intermediate natural frequencies and dynamic responses. Rao and Ratnam[89] proposed a methodology for the health monitoring of structures capable of identifying the damage at the earliest possible stage using the acceleration time response data obtained from piezoelectric accelerometers. The applicability of the proposed method is tested with the welded structure model by fixing it to the multi axes electro dynamic vibration shaker.

2.6 Vibration Based Damage of Cracked FGM beam

A minute crack in the FGM beam subjected to complex environmental forces associated with vibration helps in opening the crack more and more and finally could lead to complete fracture of the beam. So a knowledge in vibration based damage is felt necessary for further analyzing the beam. Produced below are few of the similar work done by people.

The various vibration based damage diagnosis techniques presented by various researchers for a cracked composite and non-composite structures. Presently various analytical, numerical and experimental techniques are in use for damage detection in a FGM beam, laminated composites and non-composite structures for its vibration analysis. Various results infer that any damage in cracked beam structures brings a change in the physical properties like stiffness, mass and damping capacity that forces a change in modal parameters like natural frequencies and mode shapes (Agrawal et al.)[1]. Liu et al.[59] have developed an analytical solution to study the effect of edge crack on the vibration characteristics of delaminated beams adopting the rotational spring model, the ‘free mode’ and ‘constrained mode’ assumptions in delamination vibration and found that the effect of delamination length and thickness-wise location on reducing the natural frequencies is aggravated by an increasing crack depth. The location of the crack also influences the effect of delamination, but such influence is different between crack occurring inside and outside the delaminated area. The difference of natural frequencies between 'free mode' and 'constrained mode' increases then decreases as the crack moves from one side of the delaminated region to the other side, peaking at the middle. The analytical results of this study can serve as the benchmark for FEM and other numerical solutions.

Rao and Ratnam[90] studied on the use of experimental and analytical modal analysis of a welded structure used for vibration based damage identification and observed from the results that the natural frequencies obtained from the experimental modal analysis and ANSYS software version 11.0 shows a good consistency in comparison.

Gawai and Kumawat[34] has discussed as the Dynamic structures subjected to periodic loads compose a very important part of industrial machineries. One of the major problems in these machineries is the fatigue and the cracks initiated by the fatigue. Here an accurate and comprehensive investigation about vibration of cracked dynamic structures is done. On the base of these investigations the cracks can be identified well in advance and appropriate measures can be taken to prevent more damage to the system due to the high vibration level. Typical situations where it is necessary to consider more precisely the response produced by dynamic loading are vibrations due to equipment or machinery, impact load produced by traffic, snatch loading of cranes, impulsive load produced by blasts, earthquakes or explosions. So it is very important to study the dynamic nature of structures.

The free vibration analysis of a functionally graded cracked beam resting on a Winkler–Pasternak foundation has been studied (Matbuly et al.)[66] and found that the values of the natural frequencies increase with the increasing both of Young's modulus gradation ratio (E2/E1), elastic foundation modulus, $K_{f,}$ and shear foundation modulus, Ks. The values of the natural frequencies decrease with the increasing both of number and depth of cracks. The minimum values always occurs when the crack is located in the middle of the beam. The sudden changes in the mode shapes of the vibrating beam may be used to accurately determine the crack position. A theoretical investigation in free vibration and elastic buckling of beams made of FGM containing open edge cracks by using Bernoulli–Euler beam theory and the rotational spring model has been done by Yang and Chen[125] and obtained analytical solutions of the natural frequencies, critical buckling load, and the corresponding mode shapes for cracked FGM beams with clamped–free, hinged–hinged, and clamped–clamped end supports.

Yamuna and Sambasivarao[122] have investigated the natural frequency of a simply supported beam with a triangular crack, numerically by finite element method using of FE analysis software ANSYS. Different crack location effects are considered and the results are compared with that of the simply supported beam without crack. The results obtained from the vibration analysis of the beam show that the lowest fundamental frequency of the beam without crack is higher than the lowest frequency obtained for beam with cracks. When the location of the crack varies from the either end of the simply supported beam to the centre of the beam, the lowest natural frequency decreases. Thalapil and Maiti[111] has analyzed the natural vibration of monolithic beams with longitudinal cracks and developed an analytical method to address both forward problem of determination of natural frequencies knowing the beam and crack geometry details as well as inverse problem of detection of crack with the knowledge of changes in the beam natural frequencies.

2.7 FGM beam on Elastic Foundations

Since the elastic foundation on which the FGM beam rests has also significant effect on the stability of the beam. A lot of work has been done by researchers to understand the mechanics of the different types of elastic foundations and their effect on the stability of a FGM beam associated with vibration.

Arefi[5] evaluated the nonlinear responses of a functionally graded (FG) beam resting on a nonlinear foundation and found that increasing the non-homogenous index, increases both linear and nonlinear deflections. Furthermore, it can be concluded that increasing the non-homogenous index, increases the percentage of difference between linear and nonlinear responses. Thermo-mechanical buckling and nonlinear free vibration analysis of functionally graded (FG) beams on nonlinear elastic foundation has been investigated (Fallah and Aghdam)[32] and found that the effect of the vibration amplitude on the nonlinear natural frequency in the higher temperature is more than lower temperature. The thermo-mechanical vibration analysis of functionally graded (FG) beams and functionally graded sandwich (FGSW) beams are presented (Pradhan and Murmu)[81]. The functionally graded material (FGM) beams are considered to be resting on variable (i) Winkler foundation and (ii) two-parameter elastic foundation. It is observed that for two-parameter elastic foundation non-dimensional frequency increases with increase in second parameter elastic modulus, power-law index and decreases with applied temperature. If furthermore, increase in the temperature of the core material increases the frequency parameter.

The deterministic and random vibration analysis of a Rayleigh-Timoshenko beam on an elastic foundation, which may stimulate bridges, runways, railways, pipelines etc. the Reyleigh-Timoshenko beam is assumed to be uniform, damped with generalized boundary conditions and subjected to axial load. The mechanical and thermal post-buckling analysis of FGM rectangular plates resting on nonlinear elastic foundations have been studied (Zhang and Zhou)[128] using the concept of physical neutral surface and high-order shear deformation theory, and investigated the post-buckling behavior of FGM rectangular plates with two opposite simply supported edges and other two opposite clamped edges using multi-term Ritz method.

Sofiyev[105] studied the buckling analysis of functionally graded material (FGM) circular truncated conical and cylindrical shells subjected to combined axial extension loads and hydrostatic pressure and resting on a Pasternak type elastic foundation. They found analytically the critical combined loads of FGM truncated conical shells with or without elastic foundations.

Duc and Thang[28] presented an analytical approach to investigate the nonlinear static buckling and post-buckling for imperfect eccentrically stiffened functionally graded thin circular cylindrical shells surrounded on elastic foundation with ceramic-metal-ceramic layers (S-FGM) and subjected to axial compression. The static and dynamic buckling of an FGM beam subjected to uniform temperature rise loading and uniform compression have been studied (Ghiasian et al.)[35] and found that for sufficiently stiff softening elastic foundation, post-buckling equilibrium path becomes unstable. Furthermore, when the thermal post-buckling equilibrium path is stable, no dynamic buckling occurs according to this criterion.

Sofiyev[106] focused on the thermal buckling analysis of FGM shells resting on the two-parameter elastic foundation and graded the material properties of the constituents in the thickness direction according to the power-law distribution with the surrounding elastic medium is modeled as an elastic foundation of the Pasternak-type.

Bagherizadeh et al.[9] have investigated the mechanical buckling of functionally graded material cylindrical shell that is embedded in an outer elastic medium and subjected to combined axial and radial compressive loads assuming the material properties to vary smoothly through the shell thickness according to a power law distribution of the volume fraction of constituent materials and modelled the elastic foundation by two parameters Pasternak model obtained by adding a shear layer to the Winkler model considering the simply-supported boundary condition. The effect of randomness in properties on the elastic buckling of FGM rectangular plates which are resting on an elastic foundation has been investigated and subjected to uniform in-plane edge compressions including the interaction between the plate and foundation in the formulation with a two-parameter Pasternak model (Yang et al.)[126].

Duc and Qua[29] have investigated the nonlinear response of eccentrically stiffened FGM cylindrical panels on elastic foundation subjected to mechanical loads and determined the explicit relations of load-deflection curves for simply supported eccentrically stiffened FGM panels by applying Bubnov-Galerkin method, the Lekhnitsky smeared stiffeners technique with Pasternak type elastic foundation and stress function.

2.8 Buckling analysis of FGM beam

Most of the researchers have dealt with buckling analysis of FGM beams associated with free, forced vibration by using different theories. Due to the increased applicability of functionally graded materials in the diversified field, it is important to understand environmental causes for buckling of FGM beams from bellowed literature.

Shariyat and Asemi[97] have studied the shear buckling analysis of the orthotropic heterogeneous FGM plates for the first time and considered the influence of the Winkler-type elastic foundation. Li and Batra[53] have derived Analytical relations between the critical buckling load of a functionally graded material (FGM) Timoshenko beam and that of the corresponding homogeneous Euler–Bernoulli beam subjected to axial compressive load for clamped–clamped (C–C), simply supported–simply supported (S–S) and clamped–free (C–F) edges. For C–S beams, the transcendental equation has been derived to find the critical buckling load for the FGM Timoshenko beam which is similar to that for a homogeneous Euler–Bernoulli beam. For the FGM beams Young's modulus, E, and Poisson's ratio, m, are assumed to vary through the thickness. The significance of this work is that for the C–C, S–S and C–F FGM Timoshenko beams, the critical buckling load can be easily found from that of the corresponding homogeneous Euler–Bernoulli beam and two constants whose values depend upon the through-the-thickness variations of E and m.

Rahimi et al.[85] have investigated the post buckling behavior of functionally graded beams by means of an exact solution method and achieved a closed-form solution for the post buckling deformation as a function of the exerted axial load which is beyond the critical buckling load. Bhangale and Ganesan[14] have studied the buckling and vibration behavior of a functionally graded material (FGM) sandwich beam having constrained viscoelastic layer (VEL) is studied in thermal environment by using finite element formulation and found that the critical buckling temperature for a FGM sandwich beam increases as the power law index η increases. By increasing the ratio of the core to stiff layer, thermal buckling temperature increases.

Buckling of beams made of functionally graded material under various types of thermal loading has been investigated (Kiani and Eslami)[43] and found that the critical bucking temperatures for the C–R and S–S boundary conditions are identical in each case of thermal loading and Based to the Euler–Bernoulli beam theory, the critical buckling temperature is proportional to the square of h/L ratio for both FGM and pure isotropic beams. Again in each case of thermal loading and boundary conditions, the critical bucking temperature for the FGM beams is lower than fully ceramic beam but greater than fully metallic beam.

Li and Song[55] have characterized Geometrically non-linear deformation of axially extensional Timoshenko beams subjected mechanical as well thermal loadings by a system of coupled and highly non-linear ordinary differential equations, which results in a complicated two-point boundary-value problem and investigated the effects of shear deformation on the bending and buckling response quantitatively. The numerical results show that shear deformation effects become significant with the decrease of the slenderness and with the increase of the shear flexibility. Numerical methods for solving the elastic and the buckling load of simply supported tapered columns subjected to compressive end load have been developed and found that by varying the section ratio, the strongest columns are identified for each taper and cross-sectional shape (Oh and Lee)[75] .

Cheng et al.[21] has presented a method of continuous analysis for predicting the local delamination buckling load of the face sheet of sandwich beams and the result obtained as the buckling load increases as the shear stiffness increases and the effect would be more pronounced for higher values of the shear stiffness. Murin et al.[70] have presented a new 3D-beam finite element of double symmetric cross- sectional area, made of a Functionally Graded Material (FGM) which can be used in modal, elastostatic and buckling analysis of single beams and beam structures.

2.9 Frequency response of an FGM beam

The frequency response of FGM beams associated with vibration under various boundary conditions plays a major role in stability analysis and has been studied by people. Few of them are presented below.

Failla[31] has studied on the frequency response analysis of beams and plane frames with an arbitrary number of Kelvin–Voigt viscoelastic dampers. Using the theory of generalized functions within a 1D formulation of equations of motion, exact closed-form expressions are derived for beam dynamic Green's functions and frequency response functions under arbitrary polynomial load, for any number of dampers. From the nodal displacement solution, the exact frequency response in all frame members is also obtained in closed analytical form.

Simsek[103] analyzed about fundamental frequency of FG beam having different boundary conditions within the framework of the classical, the first-order and different higher order shear deformation beam theories. The frequency equation was obtained by using Lagrange's equations and the boundary conditions of beams were satisfied with Lagrange multipliers. The effects of delamination due to concentrated mass loading with respect to modal frequency variations in composite beams numerically and experimentally have been investigated (Yang and Oyadiji)[124] and found that the frequency curve deviations have a local or global characteristic depending on whether the delamination occurs at the near surface or the mid-plane of the composite beam, respectively. Consequently, the frequency curves can be employed as NDT tool for delamination identification and localization.

Kulkarni[49] has estimated damping in Cantilever beams of different materials (steel, brass, aluminum) and concluded that material damping is higher for steel in comparison with brass and aluminum. The increase in material damping could be correlated to the stiffness of materials. The damping ratio increases and natural frequency decreases with decrease in thickness for each material. But it is vice versa in case of length. The damping of specimen made up of aluminum was found to be lowest than either steel or brass. The dynamic response of Timoshenko beams made of functionally graded materials (FGMs) has been investigated (Wattanasakulpong and Mao)[118] and found that the proposed modelling and analysis method can provide accurate frequency results of the beams as compared to some cases in the literature. Qureshi and Bertocchi[84] have worked on Crash performance of notch triggers and variable frequency progressive-triggers on patterned box beams during axial impacts. They concluded that the value of the peak force spike could be lowered and controlled effectively by changing the maximum amplitude of the third wavelength .The loss of energy absorption could be minimized by controlling the progression exponent.

The minimum stiffness of a simple (or point) support that raises a natural frequency of a beam to its upper limit has been investigated for different boundary conditions (Wang et al.)[116] and concluded that a stiffer intermediate support is needed to raise a higher order frequency to its maximum.

Maharathi et al.[61] have developed a modified general transfer matrix method for the steady state response analysis of linear flexible rotor-bearing systems in the frequency domain with fixed matrix size. Chakraborty and Gopalakrishnan[17] have employed the spectral finite element to analyze the wave propagation behavior in a functionally graded (FG) beam subjected to high frequency impulse loading, which can be either thermal or mechanical.

2.10 Problem Definition

The problem undertaken here is to evaluate the static and dynamic behavior of simply supported sigmoid functionally graded material beam on different elastic foundations. A finite element method is used considering first order shear deformation for the analysis. The element chosen in the present case is different from the conventional elements in the sense that the shape functions of the element is obtained from the exact solution of the static part of the governing differential equation derived from Hamilton's principle. Along with this the shape functions are dependent on length, material and cross-sectional properties ensuring better accuracy of the solution. The material properties are assumed to follow sigmoid distribution of power law.

2.11 Closure

Present chapter helps in understanding into various previous developments in the area of structural dynamics of functionally graded material beams. They are broadly categorized into ten sections. In Section 2.1, introduction on FGM and its characteristics are presented. In Section 2.2, literature on different materials i.e., alloys, composite materials and advantages of FGM over them is reviewed. Section 2.3 describes about recent efforts taken by people on analysis of FGM. In Section 2.4, literature on static and dynamic analysis of FGM and factors effecting is reviewed. Section 2.5 depicts the findings regarding the effect of various system parameters on the vibration and stability of FGM beams. A review on vibration based damage due to crack initiation and expansion is done in Section 2.6. Section 2.7 presents an exhaustive review of the literature on vibration of isotropic and FGM beams on elastic foundation. In Section 2.8, the effect of spatial variation of properties on the static, free vibration, forced vibration and buckling behavior of FGM plates is discussed. Section 2.9 presents a review on frequency response of an FGM beam associated with vibration is carried out. Having reviewed all possible areas, the present problem is defined in Section 2.10.

It is understood from the reported literature that a considerable work have been done on the dynamic stability of structural components made of metals, alloys and composites. A review of the literature also reveals that a lot of work have been done on the free, forced vibration and buckling of FGM beams. Some works also have been done on the dynamic stability of isotropic and composite beams. But the amount of work done on stability of FGM beams is found to be very less as realized from the reviewed literature. Therefore, it may be concluded in this section that stability study of FGM beams is worth of a research topic to be taken up.