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LIST OF SYMBOLS

Although all the principal symbols used in this thesis are defined in the text as they occur, a list of them is presented below for quick reference.

(e)	Element
h	Beam thickness
k	Shear correction factor
k_1	Winkler's foundation constant per unit length of beam.
k_2	Shear foundation constant per unit length of beam
l	Element length
n	Index of power law variation
s	Slenderness parameter
t	Time
u	Axial displacement of reference plane
v	Transverse displacement in y-direction
w	Transverse displacement in z-direction
A	Cross-sectional area of beam
$A_{11}, B_{11}, D_{11}, D_{22}$	Stiffness coefficients
$E(z)$	Young's modulus
E_a	Young's modulus of alumina
E_s	Young's modulus of steel
$G(z)$	Shear modulus
L	Length of the beam
M_y, M_z	Bending moments about Y and Z axis
N	Axial force
$P(t)$	Dynamic axial load
P^*	Critical buckling load

I	Moment of inertia of cross-section
I_0, I_1, I_2, I_{22}	Mass moments
R	Hub radius
R_b	Material property at the bottommost layer
R_t	Material property at topmost layer
$R(z)$	A material property
S	Element strain energy
T	Element kinetic energy
V_y, V_z	Shear forces along Y and Z axis
W_c	Work done by centrifugal force
W_p	Work done by axial force
a_1, a_2, \dots	Coefficients of polynomials
b_1, b_2, \dots	Coefficients of polynomials
b	Beam width
c_1, d_1	Constants for Fourier expansion
e	Exponent

Matrices

$[k_c]$	Element centrifugal stiffness matrix
$[k_e]$	Element elastic stiffness matrix
$[k_{ef}]$	Element effective stiffness matrix
$[k_f]$	Element Pasternak foundation stiffness matrix
$[k_g]$	Element geometric stiffness matrix
$[k_p]$	Element foundation shear layer stiffness matrix

$[k_w]$	Element Winkler foundation matrix
$[m]$	Element mass matrix
$\{p\}$	Independent coefficient vector
$\{q\}$	Dependent coefficient vector
$\{\bar{u}\}$	Element displacement vector
$\{\hat{u}\}$	Nodal displacement vector
$\{\bar{u}_d\}, \{\bar{u}_r\}$	Deflection and Rotation vectors
$\{F\}$	Element load vector
$[\tilde{G}]$	Material constant matrix
$[K_c]$	Global centrifugal stiffness matrix
$[K_e]$	Global elastic stiffness matrix
$[K_{ef}]$	Global effective stiffness matrix
$[K_f]$	Global Pasternak foundation stiffness matrix
$[K_g]$	Global geometric stiffness matrix
$[K_p]$	Global foundation shear layer stiffness matrix
$[K_{th}]$	Global thermal stiffness matrix
$[K_w]$	Global Winkler foundation stiffness matrix
$[M]$	Global mass matrix
$\{\hat{U}\}$	Global nodal displacement vector

Greek symbols

δ	Hub radius parameter
ε_{xx}	Axial strain
η_1, η_2	First mode and second mode non-dimensional frequencies
γ_{xz}	Shear strain
θ	Rotation of cross-section plane about z- axis
λ	Shear-bending coupling parameters
$\rho(z)$	Density of material
σ_{xx}	Axial stress
τ_{xz}	Shear stress
ϕ	Rotation of cross-section plane about y- axis
φ	Increase in twist angle per unit length
ω	Natural frequency
$\bar{\omega}$	Angular velocity of beam
Ω	Frequency of axial load
$[\mathcal{N}(x)]$	Shape function matrix
α	Static load factor
β	Axial bending coupling parameter

GLOSSARY OF TERMS

FEM	Finite element method
FGM	Functionally graded material
FGO	Functionally graded ordinary
FGSW	Functionally graded sandwich
FGO-1.5 beam	FGO beam having properties along thickness as per power law with index value equal to 1.5
FGO-2 beam	FGO beam having properties along thickness as per power law with index value equal to 2
FGO-2.5 beam	FGO beam having properties along thickness as per power law with index value equal to 1.5
TBT	Timoshenko beam theory
DQM	Differential quadrature method
SSM	State space method