CHAPTER 6

PARAMETRIC INSTABILITY OF SIGMOID FUNCTIONALLY GRADED TIMOSHENKO BEAM ON VARIABLE ELASTIC FOUNDATION

6.1 Introduction

The behavior of a system is decided by the external loads acting on it. Those external loads may be static (time independent) or dynamic (time dependent) in nature. Besides the dynamic loads may be deterministic or probabilistic in nature. Further the external loads may be periodic (repeating with respect to time) in nature. When the presence of external excitations is such that they appear as parameters or coefficients in the governing equation of motion of the system, such systems are called parametrically excited systems. The parametric or dynamic instability of the system occurs when the frequency of external excitations happen to be integral multiple of the natural frequency of the system. The parametric resonance or instability corresponding to the value of excitation frequency two times the natural frequency is of practical importance and is called as principal mode parametric resonance.

The functional graded materials have enormous potential for application in many engineering sectors. It is obvious that such components may be subjected to parametric instability while executing their intended function. Realizing a need the present study is devoted to study of parametric instability sigmoid functionally graded Timoshenko beam resting on variable elastic foundations.

6.2 Formulation

The Eq. (3.56) can be used to calculate two sets of values of $\left(\dfrac{\Omega}{\omega_1}\right)$ from Eqs. (5.1) and (5.2) for given values of α, β_d, P^*, and ω_1.

$$\left(\frac{\Omega}{\omega_1}\right)_1 = \left[\frac{4}{\omega_1^2}\left\{[M]^{-1}[K_{ef}] - (\alpha + \beta_d/2)P^*[M]^{-1}[K_g]\right\}\right]^{1/2} \tag{5.1}$$

$$\left(\frac{\Omega}{\omega_1}\right)_2 = \left[\frac{4}{\omega_1^2}\left\{[M]^{-1}[K_{ef}] - (\alpha - \beta_d/2)P^*[M]^{-1}[K_g]\right\}\right]^{1/2} \tag{5.2}$$

The instability zones can be found by superimposing the plots between $\left(\dfrac{\Omega}{\omega_1}\right)_1$ and β_d, and $\left(\dfrac{\Omega}{\omega_1}\right)_2$ and β_d.

6.3 Results and discussion

An SFG beam with steel-rich bottom is considered for analysis of free vibration. The length of the beam is 0.5 m, width 0.1 m with various thicknesses. The material properties are given as;

Steel: $E = 2.1\text{x}10^{11}$ Pa, $G = 0.8\text{x}10^{11}$ Pa, $\rho = 7.85\text{x}10^{3}$ kg/m^3.

Aluminium: $E = 0.7\text{x}10^{11}$ Pa, $G = 0.2697\text{x}10^{11}$ Pa, $\rho = 2.707\text{x}10^{3}$ kg/m^3.

The shear correction factor $k = 0.8667$.

The following additional data are taken for analysis of dynamic stability.

$P^* = 11.33\text{x}10^7$ N is the buckling load of a steel beam of length 0.5 m, breadth 0.1 m, thickness 0.125 m and $\omega_1 = 6725 rad/s$.

The effect of the beam-geometry on the instability zones is investigated and presented in Figs. 6.1 through 6.6. The effect of interaction of shear layer of the foundation is not taken into consideration in these cases. The area of instability zone for principal mode of the beam resting on linear foundation decreases with increase in the value ratio (L/h) from 4 to 2. Also, the zone is shifted farther from the axis of dynamic load factor remarkably as shown in Fig. 6.1.

A similar trend is observed for second mode instability zone of the beam as depicted in Fig. 6.2. Hence, the stability behavior is enhanced as the beam becomes thicker. This can be justified as increase in the ratio 'L/h' causes reduction in mass of beam thereby increasing its frequency. Higher frequency of beam requires a still higher excitation frequency to cause parametric instability.

Figs. 6.3 and 6.4 present respectively the instability zones for principal mode and second mode of the beam resting on parabolic foundation. Similar trend of results is noticed as in case of beam resting on linear foundation. The effect of the geometry on stability of sigmoid beam resting on sinusoidal foundation as depicted in Figs. 6.5 and 6.6 is to improve the dynamic stability in the way as described for beam resting on linear and parabolic foundations.

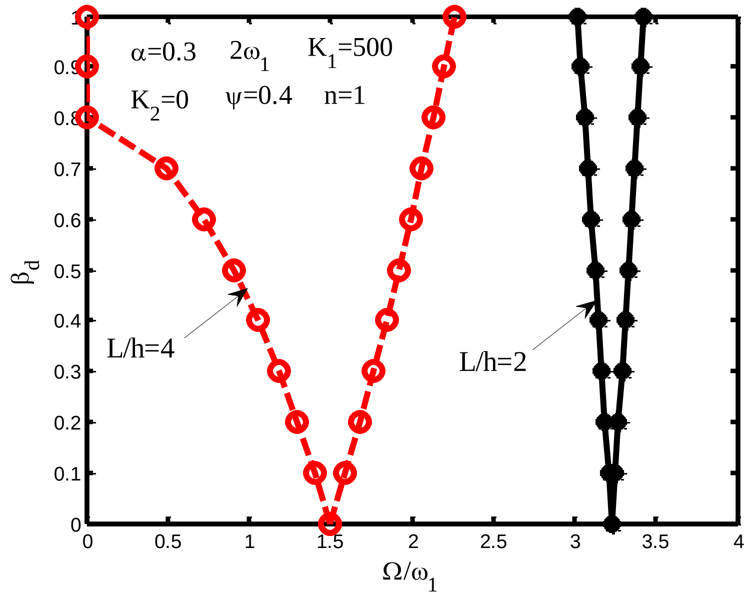

Fig. 6.1 Effect of beam geometry on instability zone for principal mode of sigmoid beam resting on linear foundation.

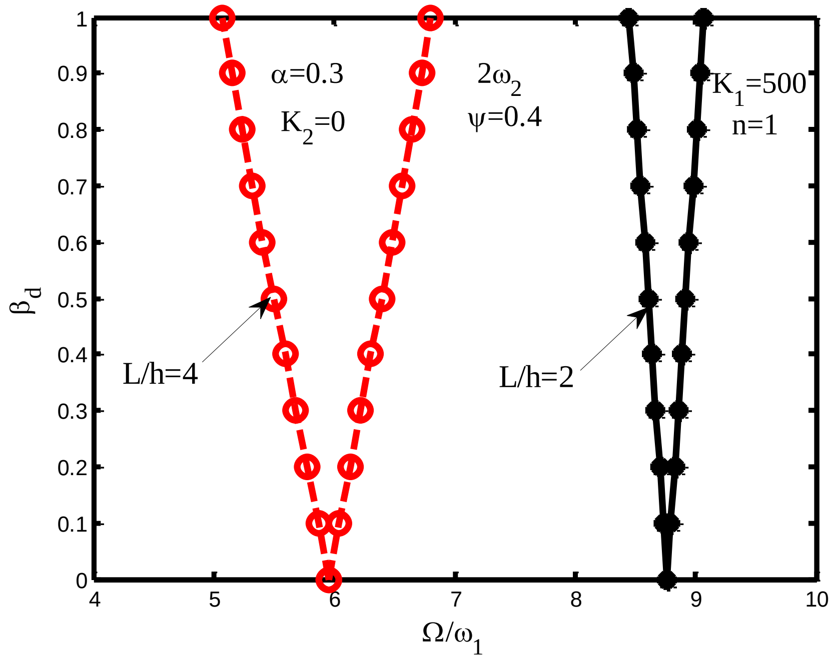

Fig. 6.2 Effect of beam geometry on instability zone for second mode of sigmoid beam resting on linear foundation.

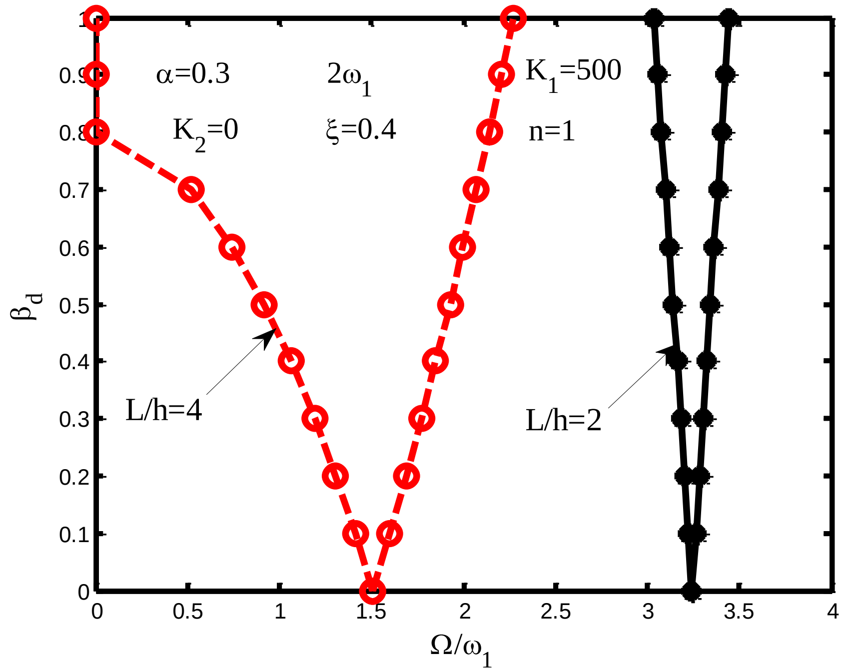

Fig. 6.3 Effect of beam geometry on instability zone for principal mode of sigmoid beam resting on parabolic foundation.

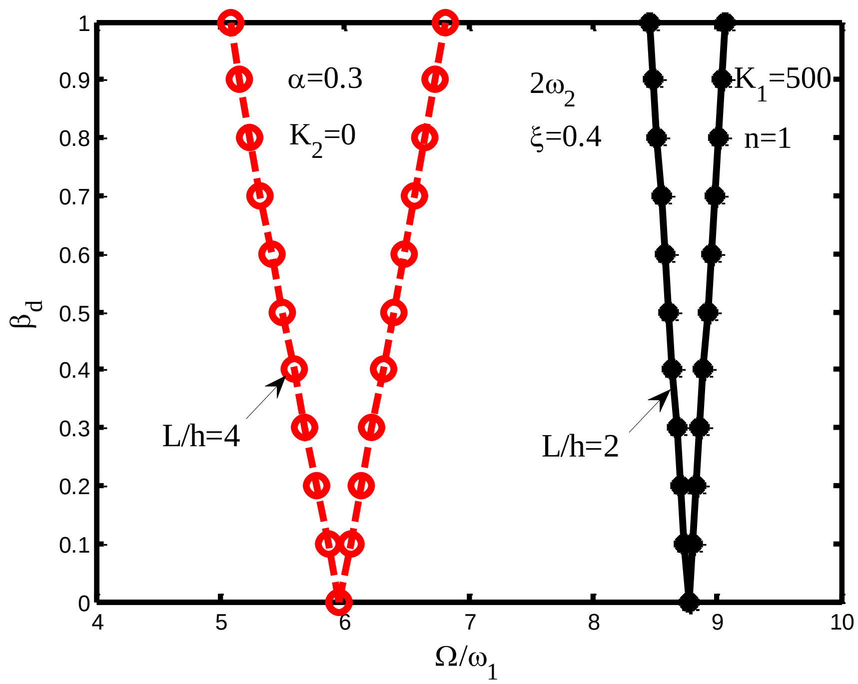

Fig. 6.4 Effect of beam geometry on instability zone for second mode of sigmoid beam resting on parabolic foundation.

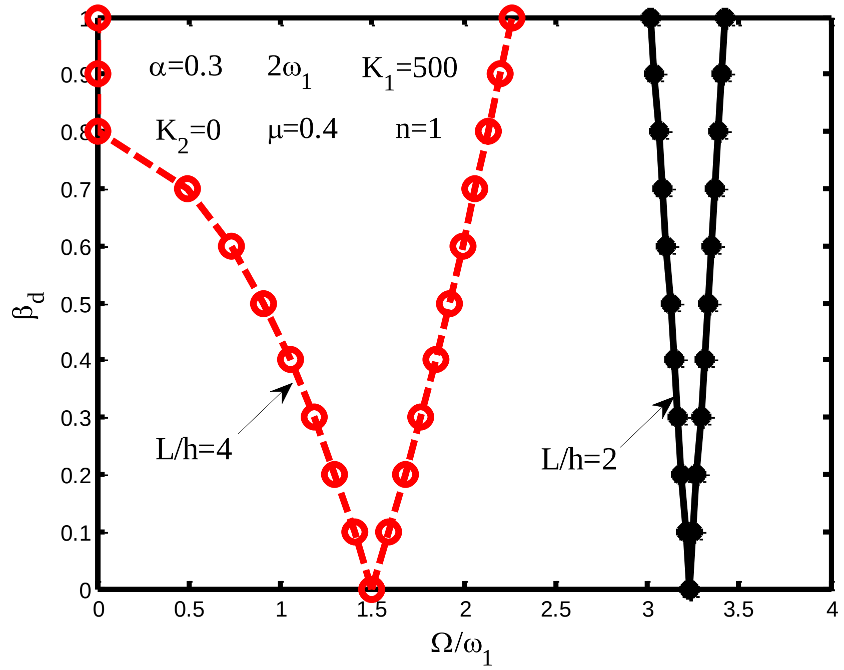

Fig. 6.5 Effect of beam geometry on instability zone for principal mode of sigmoid beam resting on sinusoidal foundation.

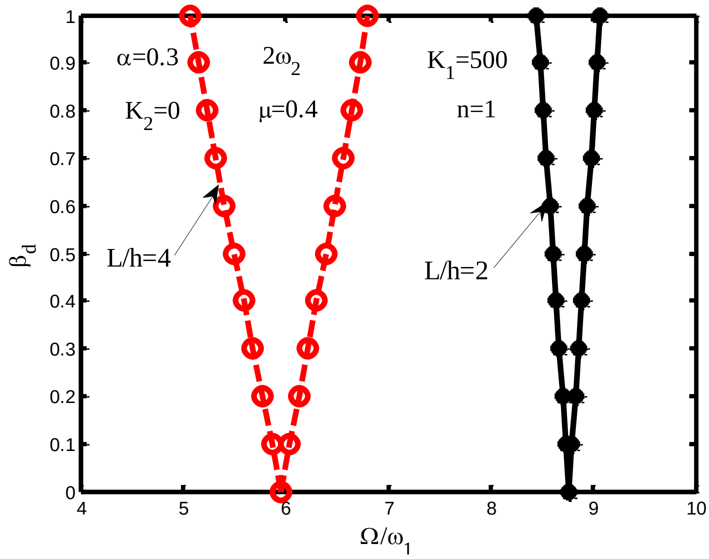

Fig. 6.6 Effect of beam geometry on instability zone for second mode of sigmoid beam resting on sinusoidal foundation.

The effect of material properties on dynamic stability of the beam resting on elastic foundations is presented in Figs. 6.7 through 6.12. The instability zones for principal mode and second mode of the beam resting on linear foundation is shown in Figs. 6.7 and 6.8 respectively. The corresponding zones for beam resting on parabolic foundation and sinusoidal foundation are shown in Figs. 6.9, 6.10 and Figs. 6.11, 6.12 respectively.

In all the above cases the instability zones are shifted nearer to the axis of dynamic load factor as the power index increases from 1 to 2. Therefore, the beam becomes more prone to parametric resonance as the power index increases. This may be due to the fact that increase of power index from 1 to 2 decreases the volume fraction of the superior constituent steel thereby decreasing the effective stiffness of the beam. The reduced frequency of the beam implies lower excitation frequency is required to cause dynamic instability.

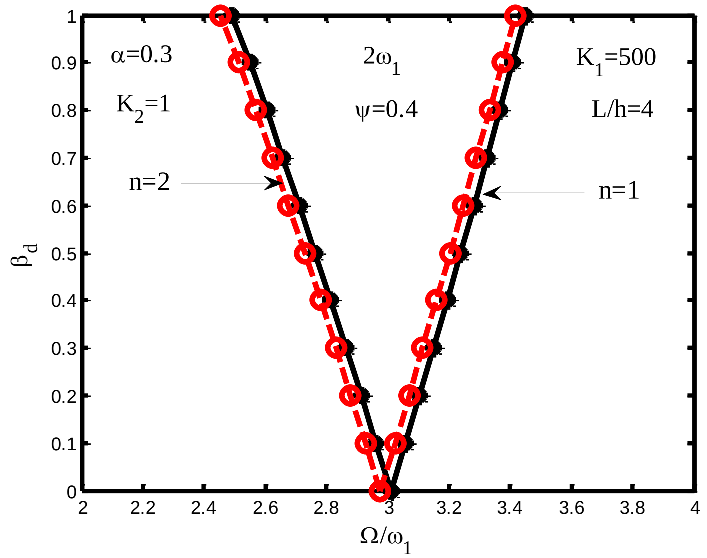

Fig. 6.7 Effect of material properties on instability zone for principal mode of sigmoid beam resting on linear foundation.

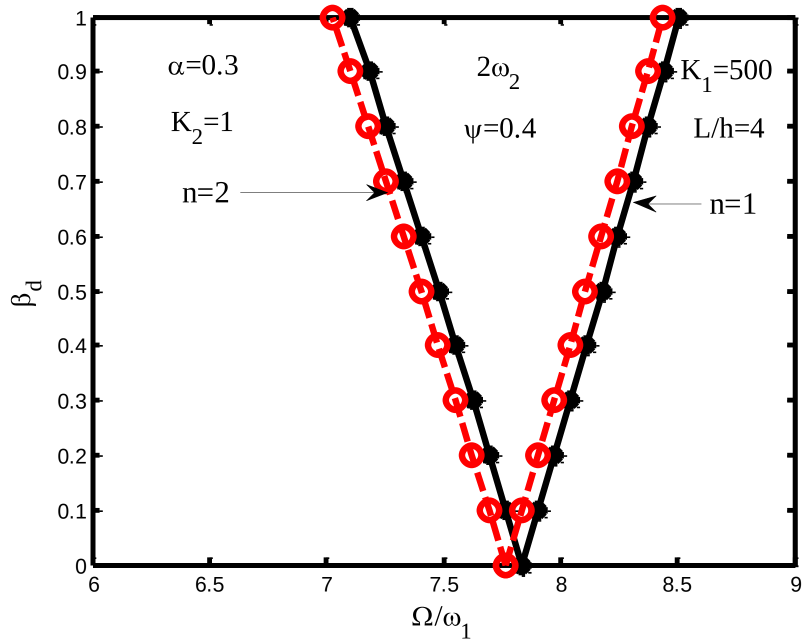

Fig. 6.8 Effect of material properties on instability zone for second mode of sigmoid beam resting on linear foundation

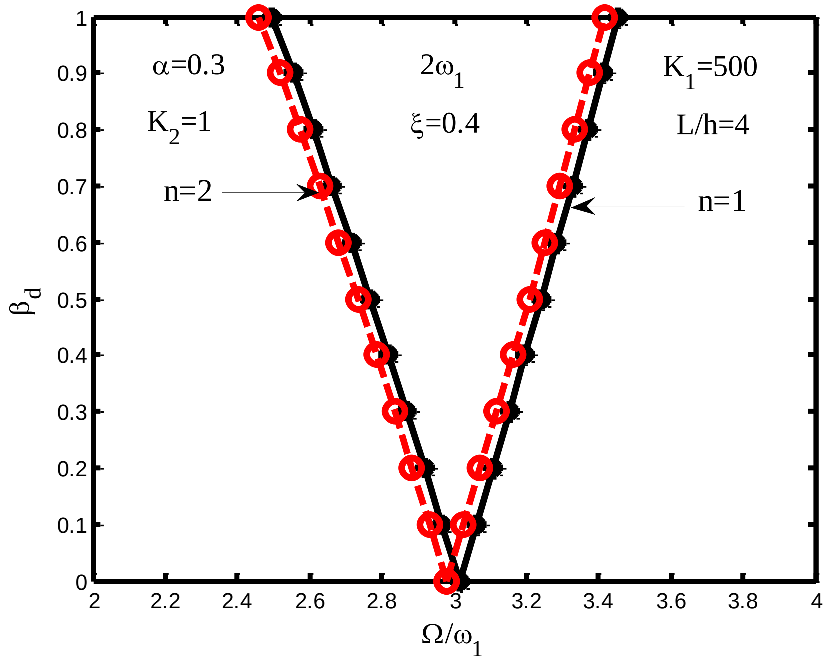

Fig. 6.9 Effect of material properties on instability zone for principal mode of sigmoid beam resting on parabolic foundation

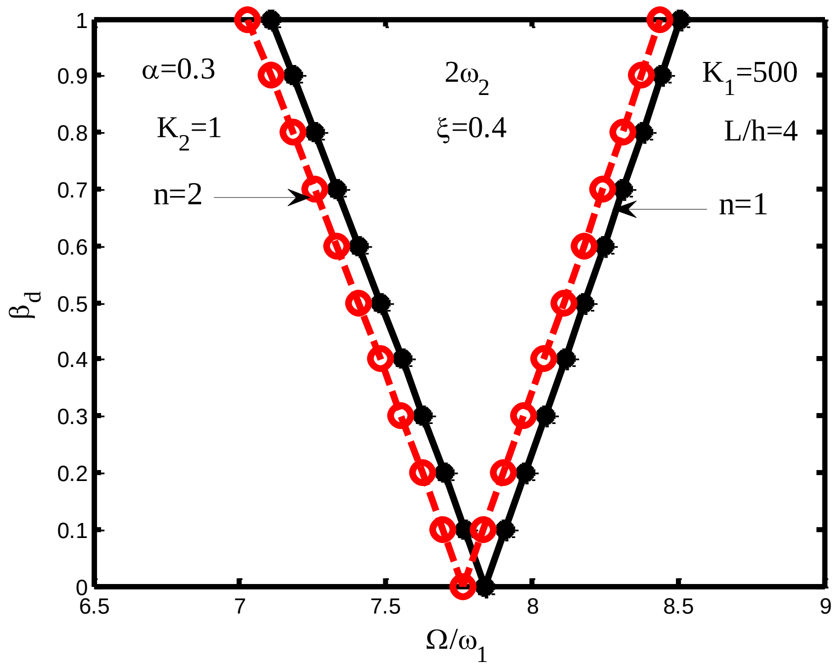

Fig. 6.10 Effect of material properties on instability zone for second mode of sigmoid beam resting on parabolic foundation

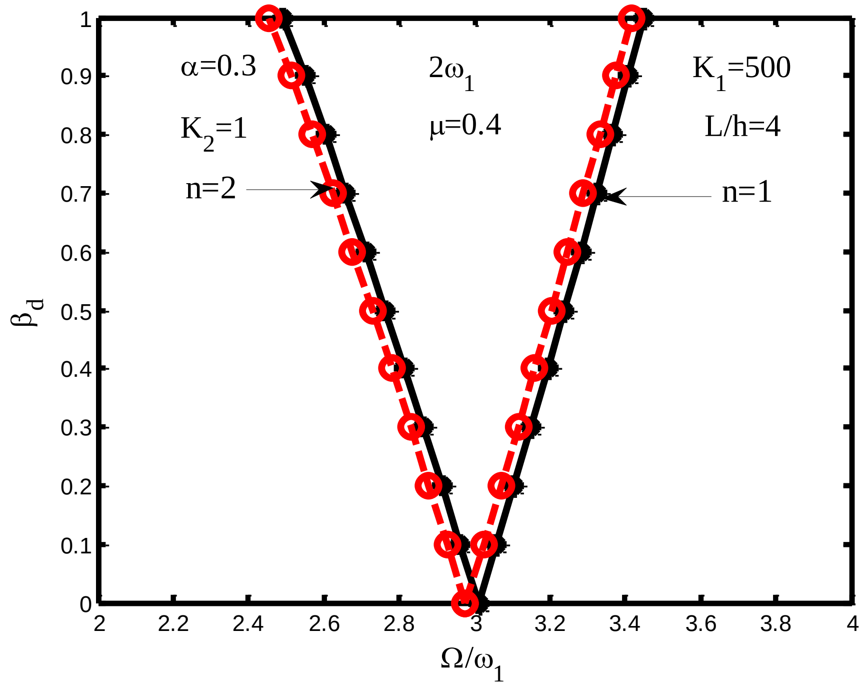

Fig. 6.11 Effect of material properties on instability zone for principal mode of sigmoid beam resting on sinusoidal foundation.

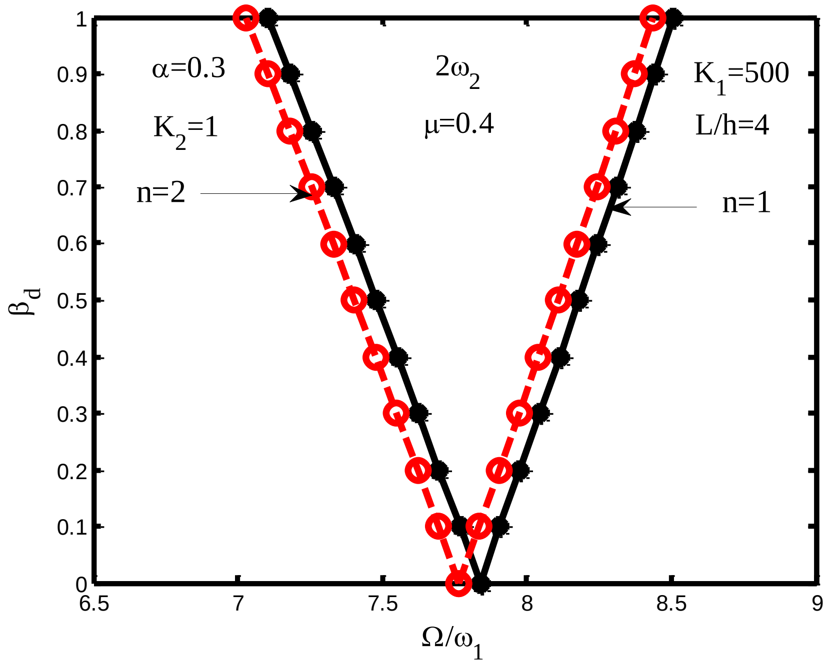

Fig. 6.12 Effect of material properties on instability zone for second mode of sigmoid beam resting on sinusoidal foundation.

The effect of the resistance of foundation on the stability behavior of the beam resting on parabolic foundation is studied and shown in Fig. 6.13. The instability zone moves farther from the axis of dynamic load factor thereby making the beam more stable. The effect of shear layer interaction on parametric resonance is depicted in Fig. 6.14. The instability region for the principal mode of sigmoid beam resting on parabolic foundation shifts farther from the axis of dynamic load factor as the value of foundation shear modulus increases from 0.2 to 0.6. Besides, the area of the instability zone decreases as the value of foundation shear modulus increases from 0.2 to 0.6. Hence, stability behavior of the beam is enhanced significantly as it is obvious from Fig. 6.14.

The effect foundation parameter on the parametric instability of the beam resting on linear foundation is studied. The instability zone shifts away from ordinate as the foundation parameter increase from 0.2 to 0.6 and it decreases its area in the process as shown in Fig. 6.15. Therefore the beam becomes more stable with increase of foundation parameter as the foundation stiffness being involved in improvement of stability behavior of beam increases with increase of foundation parameter.

The effect of steady part of the applied load on the parametric resonance of the beam resting on sinusoidal foundation is presented in Fig. 6.16. The stability of the beam improves remarkably as the value of static load factor decreases from 0.3 to 0.1 as the location of instability is shifted towards right and the area of the zone decreases as well as result of change in the value of static load factor.

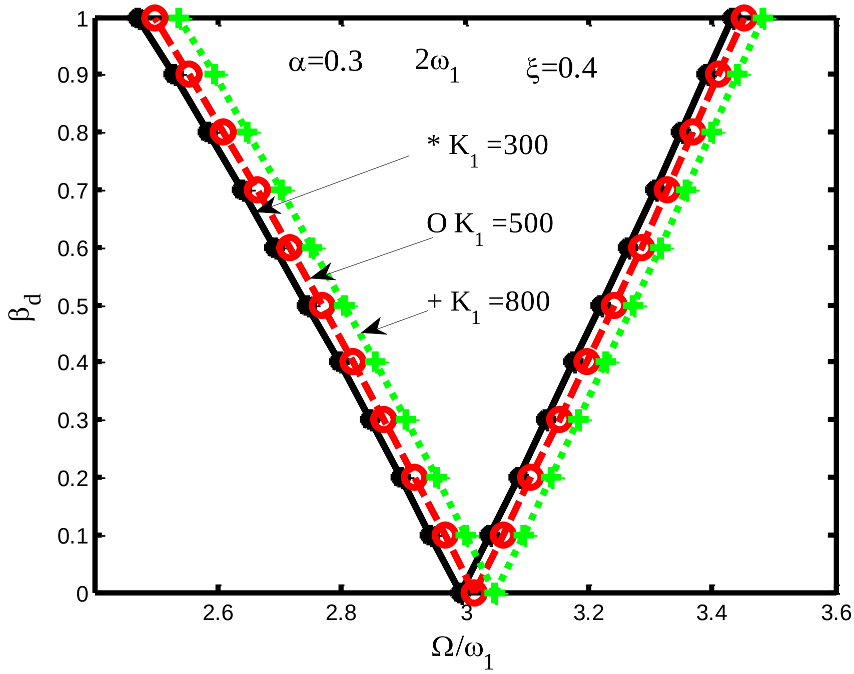

Fig. 6.13 Effect of foundation stiffness on instability zone for principal mode of sigmoid beam resting on parabolic foundation.

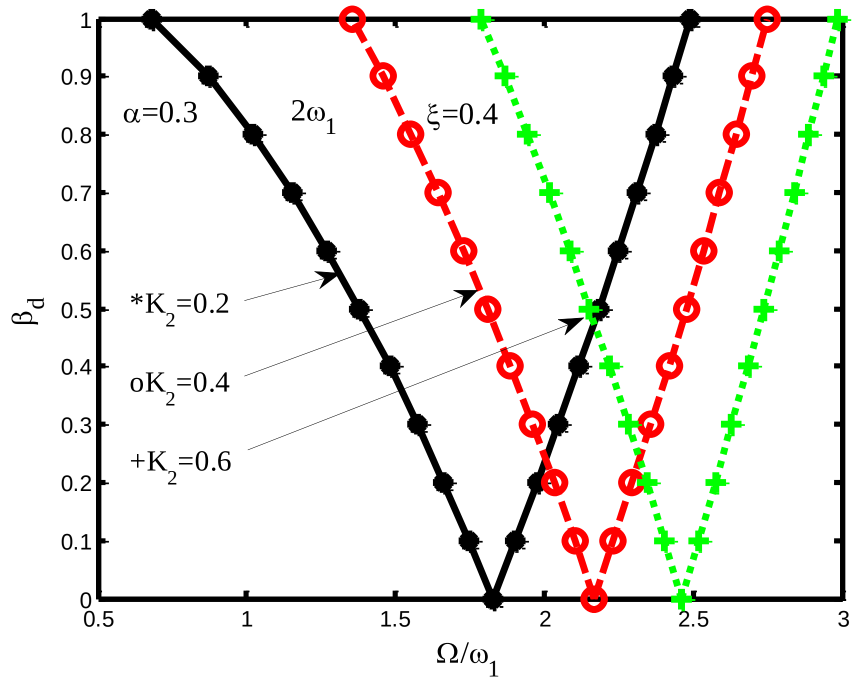

Fig. 6.14 Effect of foundation shear layer on instability zone for principal mode of sigmoid beam resting on parabolic foundation.

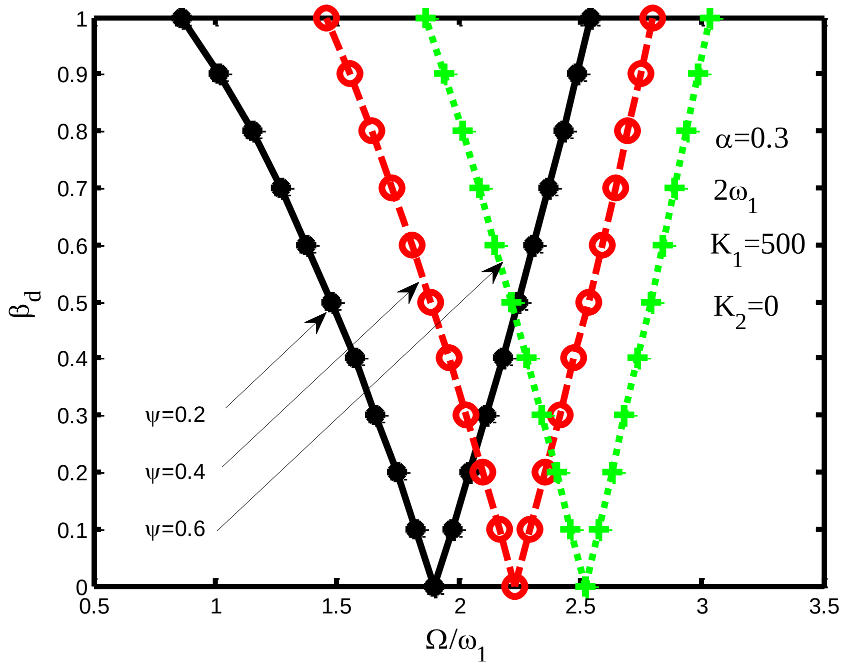

Fig. 6.15 Effect of foundation parameter on instability zone for principal mode of sigmoid beam resting on linear foundation

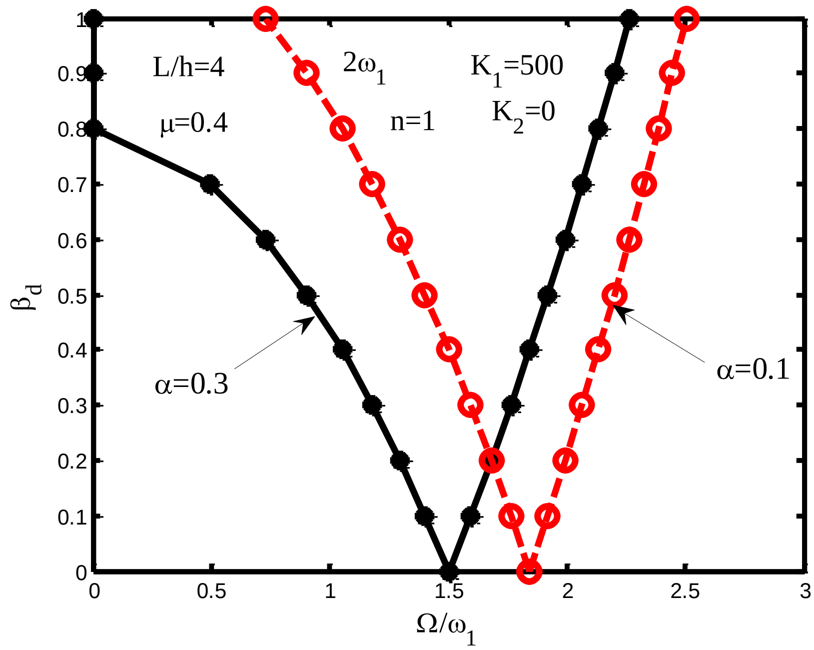

Fig. 6.16 Effect of steady part of load on instability zone for principal mode of sigmoid beam resting on sinusoidal foundation.

6.4 Closure

A study on the dynamic stability of functionally graded Timoshenko beam with sigmoid distribution of material properties along the thickness and resting on variable elastic foundations is carried out using finite element method. Foundation stiffness varying linearly, parabolically and sinusoidally along the length of beam is considered for analysis. The findings of the study is summarized as;

- Increase in slenderness of beam makes it more prone to parametric instability.
- Increasing the value power index enhances the chance of parametric resonance.
- The presence of foundation improves the dynamic stability of the beam.
- The interaction of shear layer of the foundation with mating surface of the beam improves its dynamic stability more significantly as compared to interaction of the foundation with its transverse displacement.
- Increase in the value of foundation parameter reduces the chances of parametric resonance.
- The increase in steady part of the load remarkably enhances chances of parametric instability.

The next chapter is devoted to the summary of findings of the studies carried out for the beam resting on variable elastic foundations.