CHAPTER 5

FREE VIBRATION OF SIGMOID FUNCTIONALLY GRADED TIMOSHENKO BEAM ON VARIABLE ELASTIC FOUNDATION

5.1 Introduction

Engineering components do transmit forces while carrying out their intended functions. The forces acting on those components may not always be static loads. In many realistic situations, these forces are time dependent and may change their direction as well as magnitude and hence called as dynamic forces. These dynamic loadings may also go through two forms like periodic and non-deterministic. In engineering applications, loadings are quite often non-deterministic forces, for example, those from earthquakes, wind, and ocean waves, in on-shore and off-shore structures. These forces can be described satisfactorily in probabilistic terms. There are also engineering systems which are subjected to loadings that contain both periodic components and stochastic fluctuations.

Whenever a body is given an excitation either in the form of an impulse or in the form of a deformation, the body undergoes oscillatory motion upon the release of such excitation. Such motion of the body is called free or natural vibration. The frequency of the vibration is known as natural frequency of the body. Very often, the engineering components are subjected to periodic forces as expressed in Eq. (3.1). The component undergoes main or ordinary resonance when the forcing frequency is equal to natural frequency of the system. Ordinary resonance corresponds to the oscillatory response of the system in the direction of external excitation and the response amplitude increases linearly with time and can be reduced by providing damping.

5.2 Formulation

The Eq. (3.55) can be used to calculate the natural frequency of vibration which is given as;

$$\{\omega\} = \left[[M]^{-1}[K_{ef}]\right]^{1/2} \tag{5.1}$$

5.3 Results and Discussion

An SFG beam with steel-rich bottom is considered for analysis of free vibration. The length of the beam is 0.5 m, width 0.1 m with various thicknesses. The material properties are:

Steel: E = 2.1x10^{11} Pa, G = 0.8x10^{11} Pa, ρ =7.85x10^{3} kg/m^{3}.

Aluminium: E = 0.7x10^{11} Pa, G =0.2697x10^{11} Pa, ρ =2.707x10^{3} kg/m^{3}.

The shear correction factor k = 0.8667.

The reaction of foundation is divided into two parts. Those are resistance against transverse displacement of beam and interaction of the shear layer of foundation with mating surface of the beam. The respective stiffness matrices are computed from the corresponding work done by the foundation and added together to get the stiffness matrix of foundation. The effective stiffness matrix is calculated by adding the foundation stiffness matrix to the elastic stiffness matrix of the beam. The effective stiffness matrix is used in the equation of motion of the beam to reflect the effect of foundation on the free vibration of beam. When the foundation property is such that the interaction of its shear layer with the mating surface of beam is very less, the corresponding work done being very small can be neglected. Such foundation can be called as one parameter foundation or Winkler's foundation. If both the above mentioned work by foundation is taken into consideration, such foundation can be named as two-parameter foundation or Pasternak foundation. The following non-dimensional numbers are used for analysis purpose.

Non-dimensional natural frequency $\eta_1 = \omega_1 \left(\frac{\rho A L^4}{EI} \right)^{1/2}$

Non-dimensional foundation modulus, $K_1 = \frac{k_v L^4}{EI}$

Non-dimensional foundation shear modulus, $K_2 = \frac{k_p L^2}{\pi^2 EI}$

The effective stiffness matrix reduces to elastic stiffness matrix if the foundation stiffness matrix is made equal to zero and the modified equation of motion will correspond to beam not being rested on foundation. The effect of the geometry of the sigmoid beam resting on variable elastic foundations on its dynamic behavior is studied and presented in Figs. 5.1 through 5.6. The effect of interaction of shear layer of the foundation is not taken into consideration in these cases. The fundamental frequency of the beam resting on linear foundation increases with increase in the ratio (L/h). This can be justified as increase in the ratio 'L/h' causes reduction in mass of beam thereby increasing its frequency. Further, it is observed that higher the stiffness of foundation higher is the frequency of beam as shown in Fig. 5.1. This may be due to the fact that the higher foundation stiffness results in higher effectiveness of beam hence higher frequency. Figure 5.2 depicts the variation of the second mode frequency of the beam with L/h ratio. Similar result as in case of fundamental frequency is obtained. However, the effect of foundation on second mode frequency is found to be insignificant. The effect of beam-geometry on free vibration of the sigmoid beam resting on parabolic foundation for fundamental mode and second mode is shown in Figs. 5.3 and 5.4. A similar trend of results as obtained in case of beam resting on linear foundation is observed. The variation of first mode and second mode frequency of beam resting on sinusoidal foundation with L/h ratio as depicted in Figs. 5.5 and 5.6 respectively do not differ from that in case of beam resting on linear and parabolic foundations as far as the trend of variation is concerned.

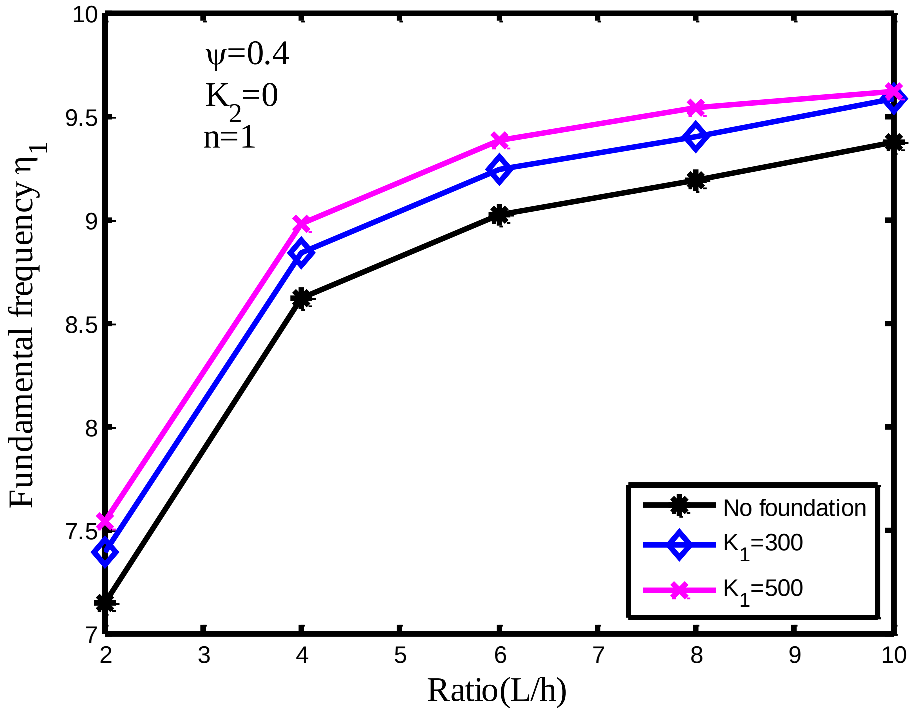

Fig. 5.1 Effect of the geometry on fundamental frequency of sigmoid beam resting on linear foundation

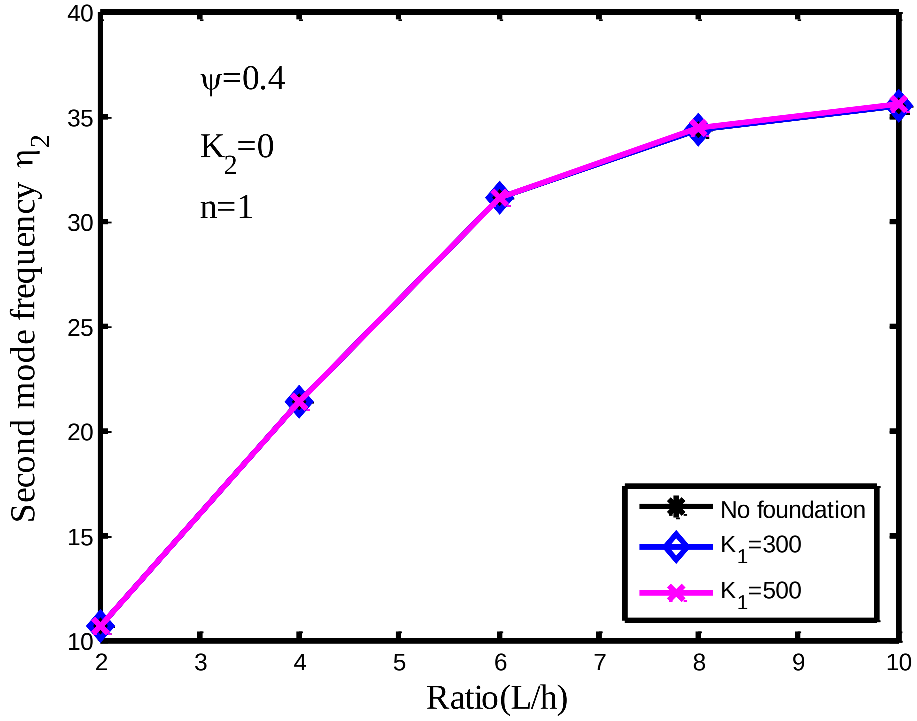

Fig. 5.2 Effect of the geometry on second mode frequency of sigmoid beam resting on linear foundation

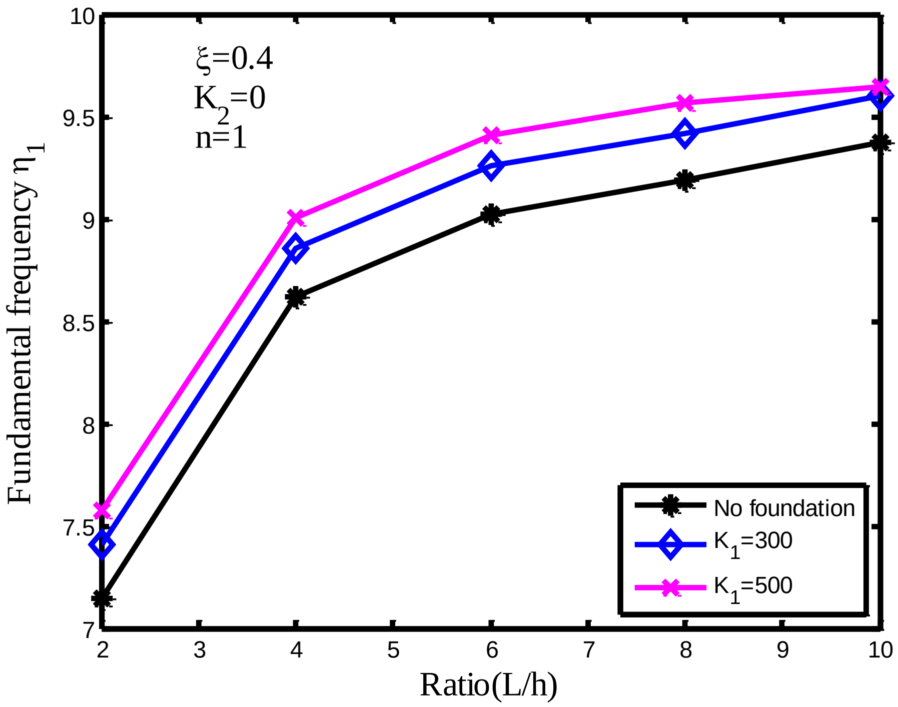

Fig. 5.3 Effect of the geometry on fundamental frequency of sigmoid beam resting on parabolic foundation

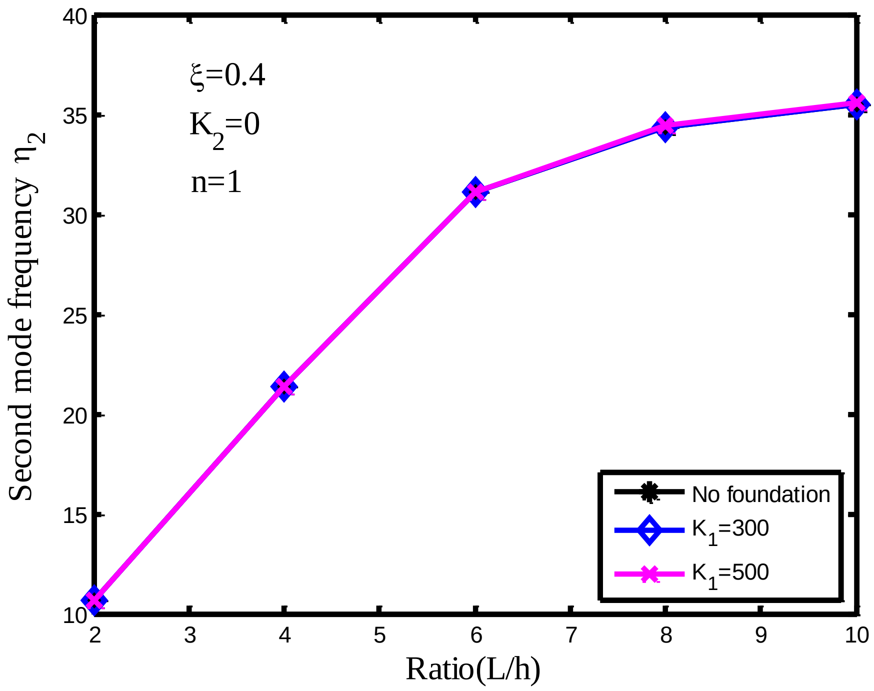

Fig. 5.4 Effect of the geometry on second mode frequency of sigmoid beam resting on parabolic foundation

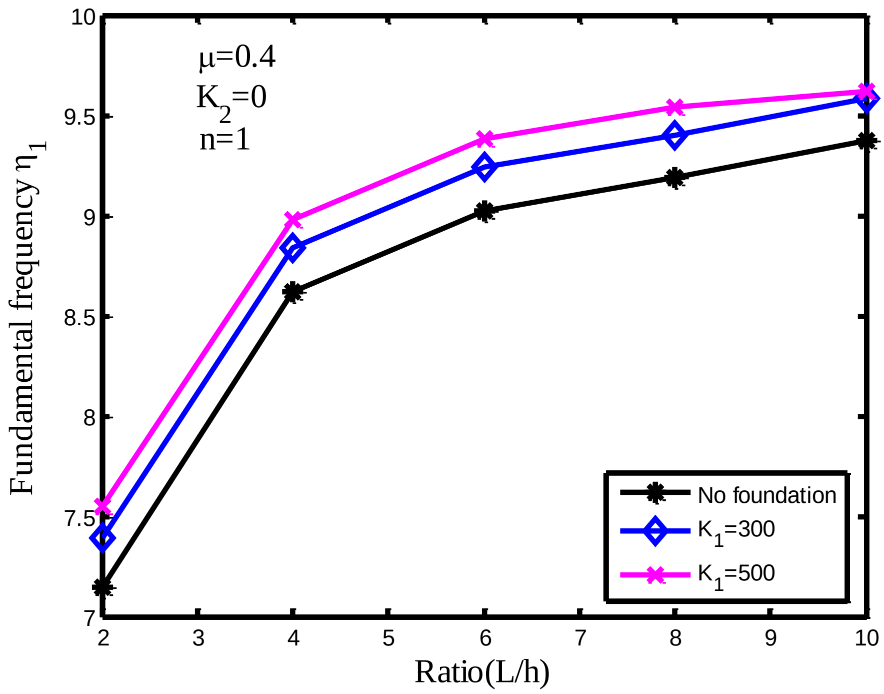

Fig. 5.5 Effect of the geometry on fundamental frequency of sigmoid beam resting on sinusoidal foundation

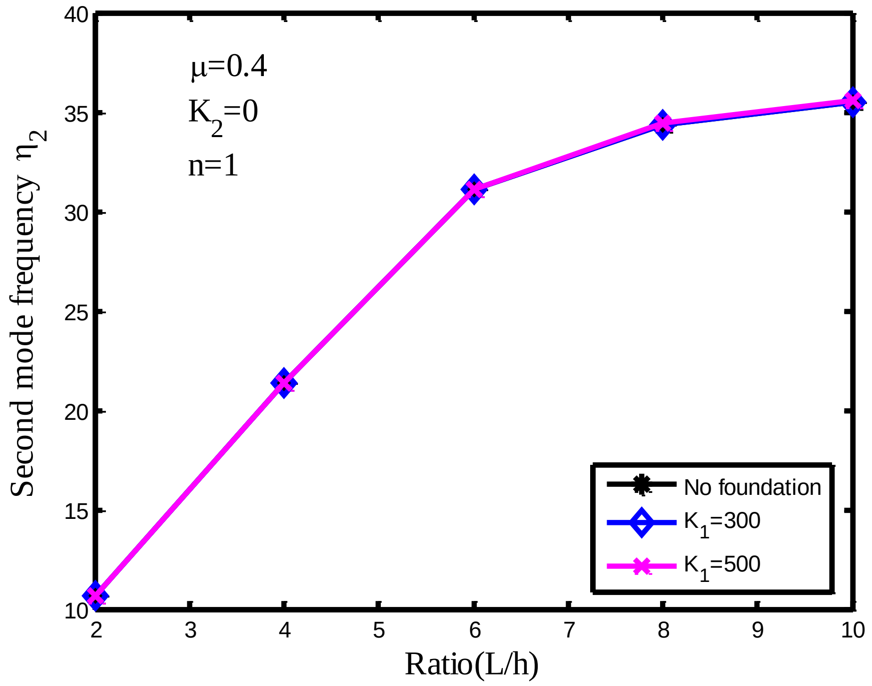

Fig. 5.6 Effect of the geometry on second mode frequency of sigmoid beam resting on sinusoidal foundation

A comparison among the chosen foundations as regards their effect on the fundamental frequency of vibration is shown in Fig. 5.7. The values of foundation stiffness at ends of beam against transverse displacement are considered the same for all the chosen foundations as shown in Fig. 3.6. The effect of interaction of shear layer of foundation is not taken into consideration in this case. It is observed that the fundamental frequency decreases with increase of foundation parameter for all the selected variable foundation model. Further, the difference in frequency values corresponding to the considered models increase with increase in foundation parameter. The frequency of sigmoid beam resting on parabolic foundation is found to be the highest among all. The frequency values in the case of beams resting on linear foundation are higher than that of beam resting on sinusoidal foundation.

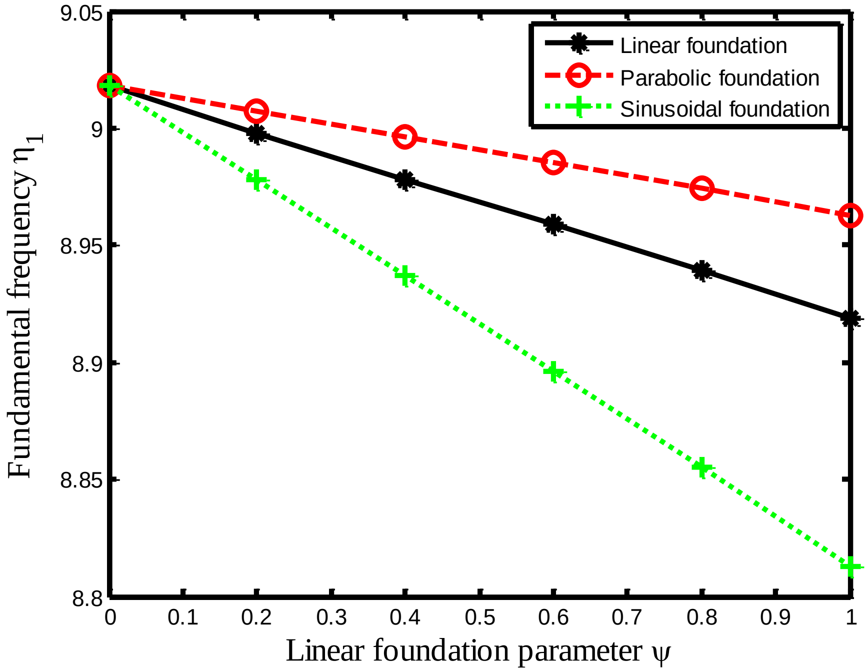

Fig. 5.7 Comparison of the effect of foundations on free vibration of the sigmoid beam.

The effect of material properties on free vibration of sigmoid beam resting on various elastic foundations is investigated and presented in Figs. 5.8 - 5.13. The effect of interaction of foundation shear layer is not taken into consideration. The variation of first and second mode frequencies with power index is shown in Figs. 5.8 and 5.9 respectively for beam resting on linear foundation.

The corresponding results of beam resting on parabolic foundation and sinusoidal foundation are depicted in Figs. 5.10 - 5.13. It is found that the fundamental frequency decreases with increase of power index for the case of beam resting on linear foundation, parabolic foundation and sinusoidal foundation as shown in Figs. 5.8, 5.10 and 5.12 respectively. This may be due to the fact that the volume fraction of superior material steel in FGM decreases with increase of power index thereby decreasing the stiffness of beam. Further, it is observed that the fundamental frequency increases with the increase of stiffness of foundation as involvement of foundation enhances the effective stiffness of the beam.

Figs. 5.9, 5.11 and 5.13 present the effect of material properties on the second mode frequency of the beam in the cases of resting on linear, parabolic and sinusoidal foundations respectively. In all the cases the second mode frequency is found to be decreased with increase of power index. Moreover, there is no significant effect of foundation on second mode frequency as the plotted lines corresponding to no foundation, foundation with value of foundation modulus 300 and 500 are very close to each other.

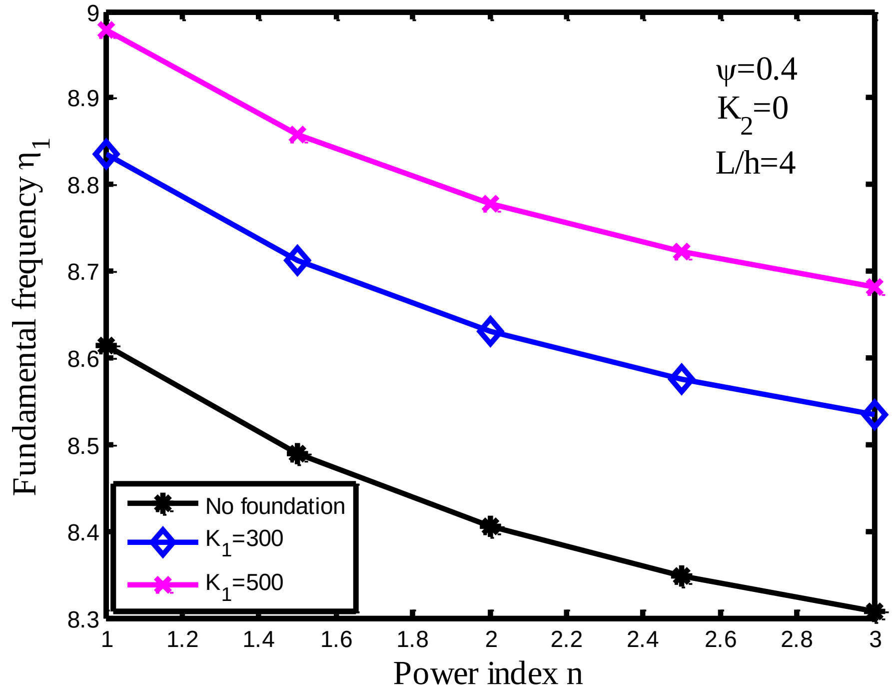

Fig. 5.8 Effect of material properties on fundamental frequency of sigmoid beam resting on linear foundation.

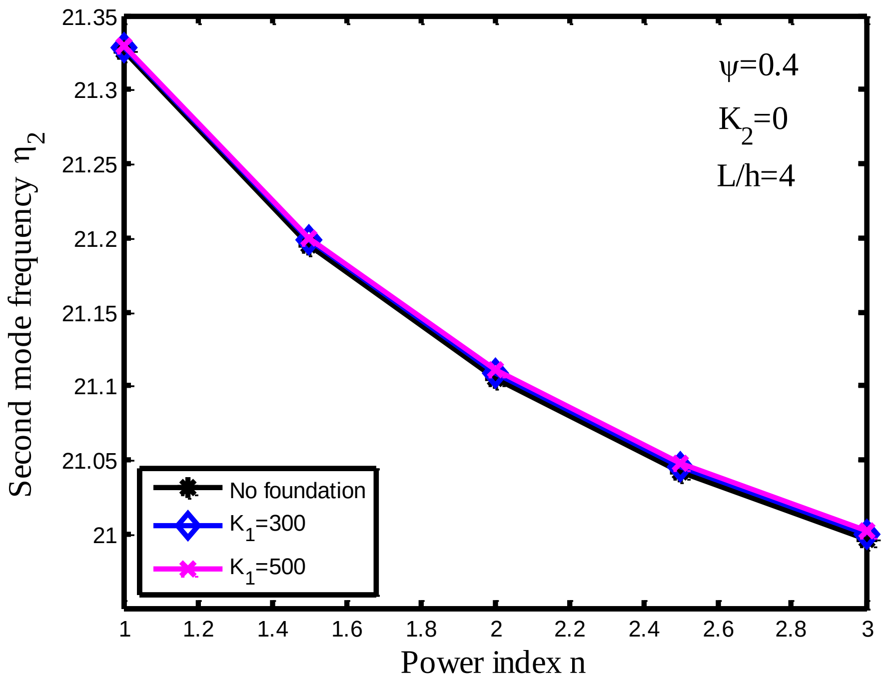

Fig. 5.9 Effect of material properties on second mode frequency of sigmoid beam resting on linear foundation.

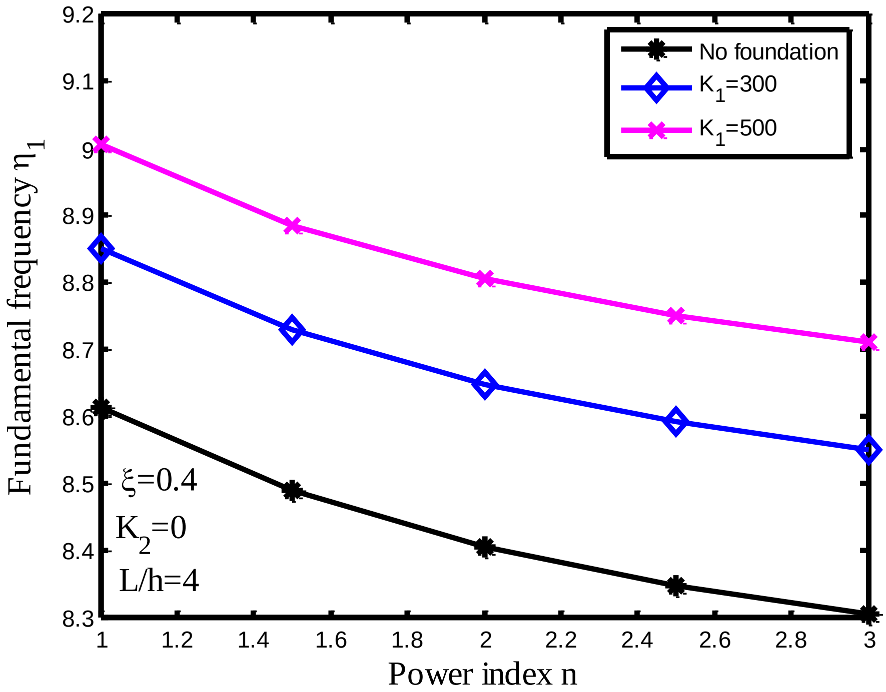

Fig. 5.10 Effect of material properties on fundamental frequency of sigmoid beam resting on parabolic foundation.

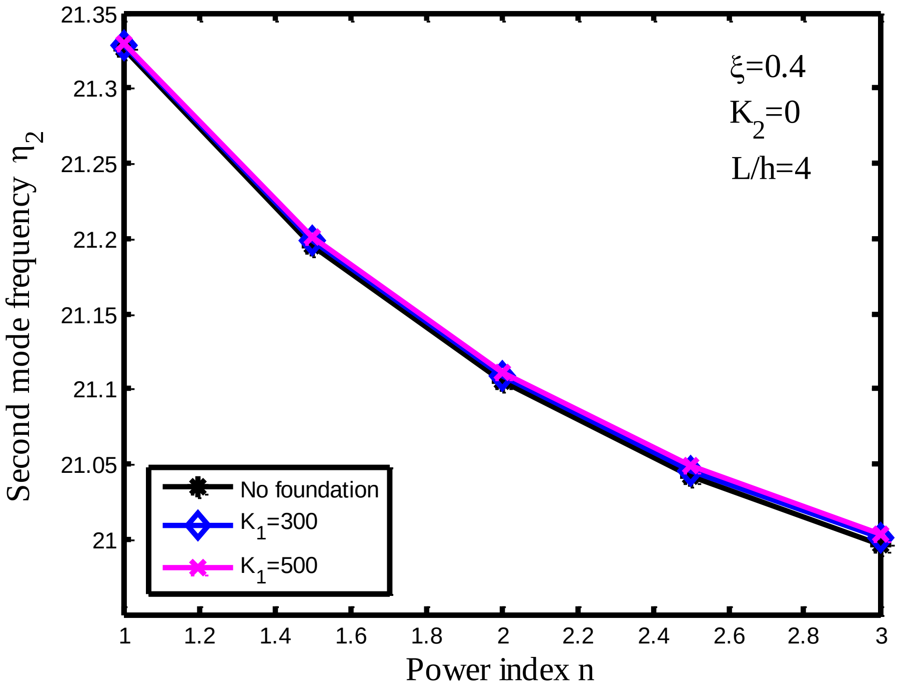

Fig. 5.11 Effect of material properties on second mode frequency of sigmoid beam resting on parabolic foundation.

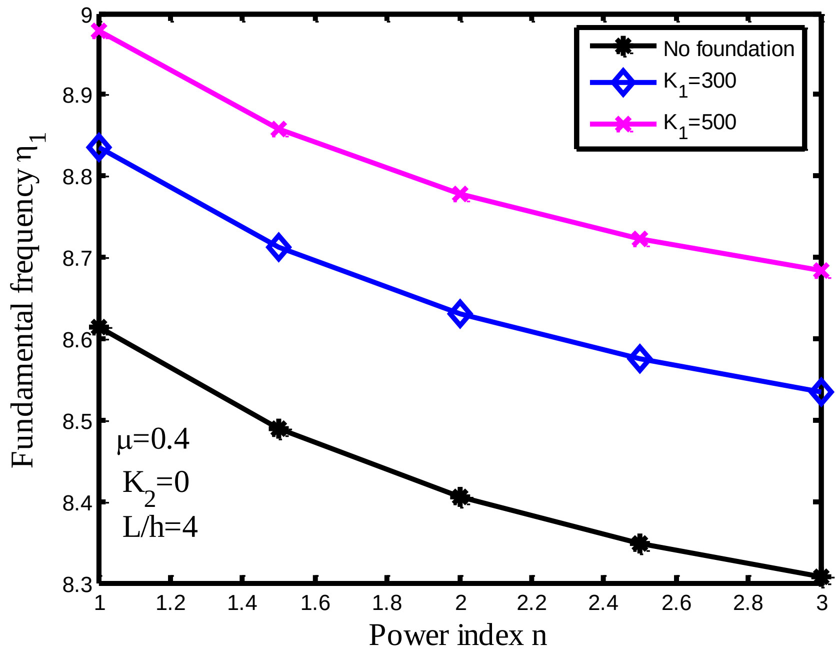

Fig. 5.12 Effect of material properties on fundamental frequency of sigmoid beam resting on sinusoidal foundation.

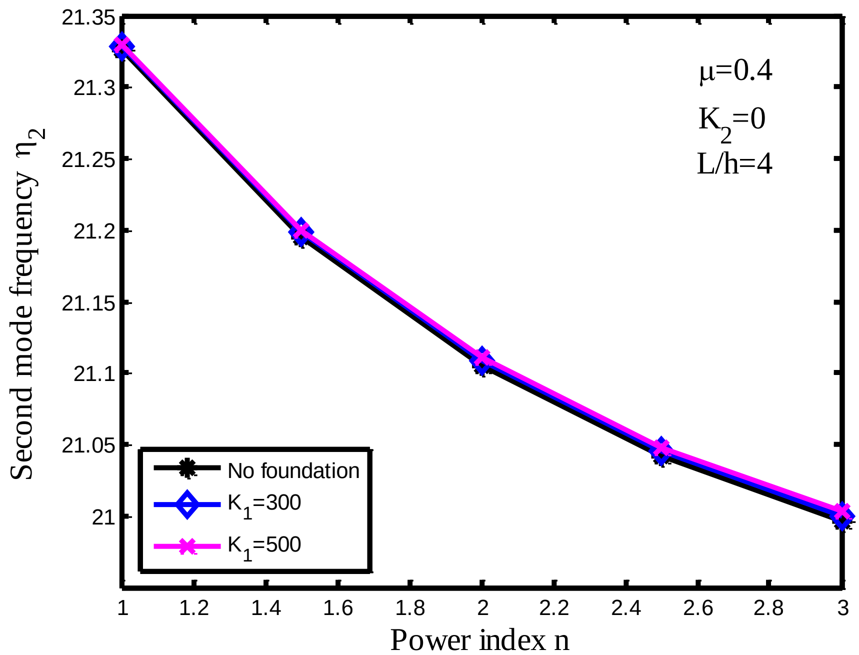

Fig. 5.13 Effect of material properties on second mode frequency of sigmoid beam resting on sinusoidal foundation.

The effect of the resistance of foundation against transverse displacement on free vibration of sigmoid beam resting on the chosen foundations is studied and presented in Fig. 5.14. The frequency increases with increase of foundation modulus as the later enhances the effective stiffness of the beam. It is also observed that the beam resting on parabolic foundation has got the highest frequency corresponding to any stiffness of foundation. The effect of two parameter foundation on free vibration is depicted in Fig. 5.15. The fundamental frequency is found to be increased non-linearly with foundation shear modulus in all the cases of chosen foundations. It is also observed that the effect of shear layer is more significant than the effect of foundation resisting the transverse deflection as it is obvious from Figs. 5.14 and 5.15.

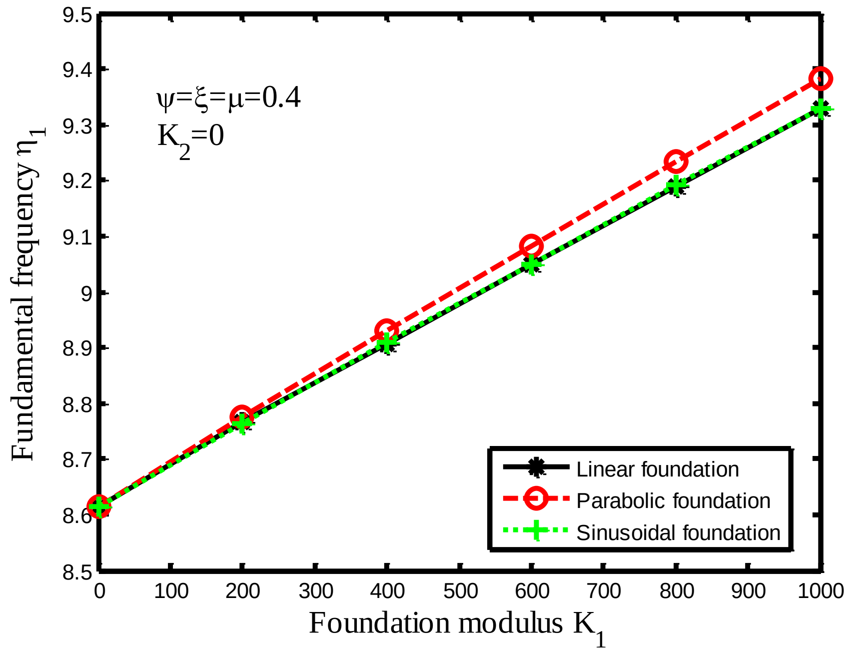

Fig. 5.14 Effect of foundation modulus on fundamental frequency of sigmoid beam resting on variable elastic foundations.

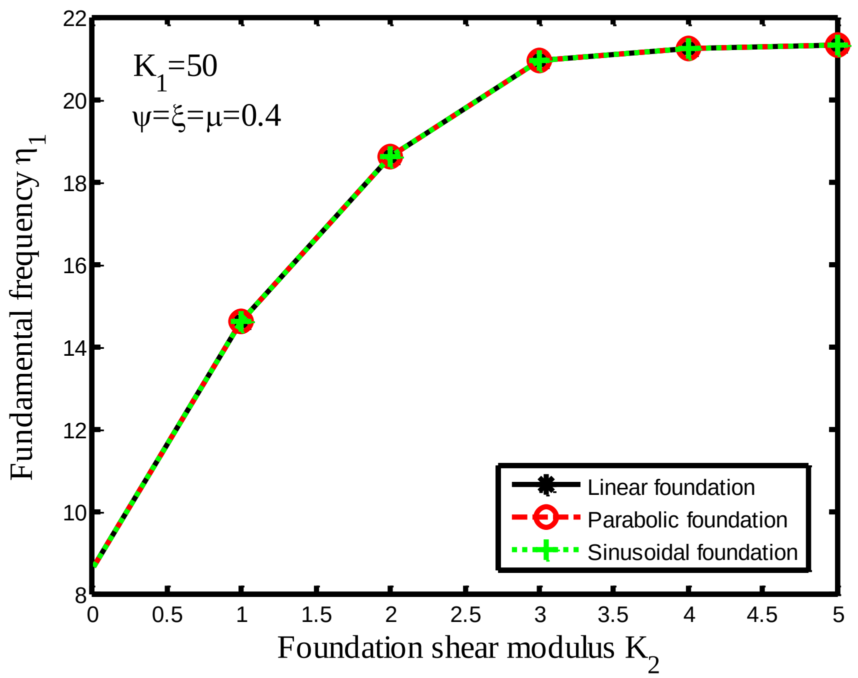

Fig. 5.15 Effect of 2-parameter foundation on fundamental frequency of sigmoid beam resting on variable elastic foundations.

5.4 Closure

A study on the free vibration of functionally graded Timoshenko beam with sigmoid distribution of material properties along the thickness and resting on variable elastic foundations is carried out using finite element method. Foundation stiffness varying linearly, parabolically and sinusoidally along the length of beam is considered for analysis. The findings of the study is summarized as follows:

- More the beam becoming slender higher is the frequencies of the beam.
- Increasing the value power index decreases the frequencies of beam thereby making it more prone to resonance.
- The presence of foundation increases the fundamental frequency of the beam.
- There is a marked difference between the effect of interaction of foundation shear layer and the effect of foundation against transverse displacement on free vibration of beam obtained from the investigation.
- The interaction of foundation shear layer ensures remarkable improvement in the fundamental frequency of the beam.

The next chapter is devoted to the study of parametric instability of the sigmoid beam resting on variable elastic foundations.