

CRITICAL BUCKLING OF SIGMOID FUNCTIONALLY GRADED TIMOSHENKO BEAM ON VARIABLE ELASTIC FOUNDATION

4.1 Introduction

Whenever, a beam is subjected to an axial compressive load, compressive stress is induced in the material of the beam. It is well known that the material will fail when the induced compressive stress exceeds the yield strength of the material of beam. But it is observed in practice that the beam may fail for such an induced stress well below the value of yield strength by bending under the applied compressive load. Such phenomenon is called critical buckling. The beam intended for transmitting transverse load subjected to axial load may be called as beam-column. The axial load under which the beam-column fails by buckling depends on the magnitude of the load and slenderness of the beam. This chapter is devoted to the analysis of buckling of sigmoid Timoshenko beam resting on variable elastic foundation.

4.2 Formulation

The Eq. (3.54) derived in chapter 3 can be used to calculate the critical buckling load as;

$$P^* = [K_g]^{-1} [K_e] \quad (4.1)$$

4.3 Results and Discussion

An SFG beam with steel-rich bottom is considered for analysis of critical buckling. The length of the beam is 0.5 m, width 0.1 m with various thicknesses. The material properties are taken as;

Steel: $E = 2.1 \times 10^{11}$ Pa, $G = 0.8 \times 10^{11}$ Pa, $\rho = 7.85 \times 10^3$ kg/m³.

Aluminium: $E = 0.7 \times 10^{11}$ Pa, $G = 0.2697 \times 10^{11}$ Pa, $\rho = 2.707 \times 10^3$ kg/m³.

The shear correction factor $k = 0.8667$.

The following non-dimensional numbers are used for analysis purpose.

The non-dimensional critical buckling load, $P^* = \frac{P^* L^2}{EI}$,

The non-dimensional foundation modulus, $K_1 = \frac{k_v L^4}{EI}$ and

The foundation shear modulus, $K_2 = \frac{k_p L^2}{\pi^2 EI}$

The effect of beam geometry on the critical buckling is investigated and shown in Figs. 4.1, 4.2 and 4.3 in the cases of beam resting on linearly, parabolic and sinusoidal elastic foundations respectively with various

stiffness values. It is observed that the buckling load decreases as the ratio (L/h) decreases. This may be due to the fact that decreasing ratio (L/h) makes the beam slender, hence more prone to instability. Besides, corresponding to any ratio, the buckling load increases as the foundation modulus increases. This happens because the foundation increases the effective stiffness of the beam.

A comparison among the various foundations as regards their role on the static stability is made and shown in Fig. 4.4. No such marked difference is found. This may be due to the length of the beam being small to reflect the effect of the stiffness values of the foundations chosen to vary along length. However the buckling load of the beam resting on parabolic foundation is found to be marginally higher as compared to the results of other foundations.

The effect of material properties on the critical buckling of the beam is studied and are shown in Figs. 4.5, 4.6 and 4.7 respectively for conditions of beam resting on linear, parabolic and sinusoidal foundations. It is learnt that variation of the power index has no effect on the static stability of the beam. This may be due to the fact that the beam remains symmetric irrespective of the power indices.

The results corresponding to various foundations are superimposed for the purpose of comparison and are presented in Fig. 4.8. The beam resting on parabolic foundation is found to have highest buckling load. This happens because the overall stiffness of the parabolic foundation becomes highest among all. The buckling loads corresponding to linear and sinusoidal foundation are very close to each other. It can similarly be concluded that the overall stiffness of both the foundations is coming out to be the same due to small length of beam.

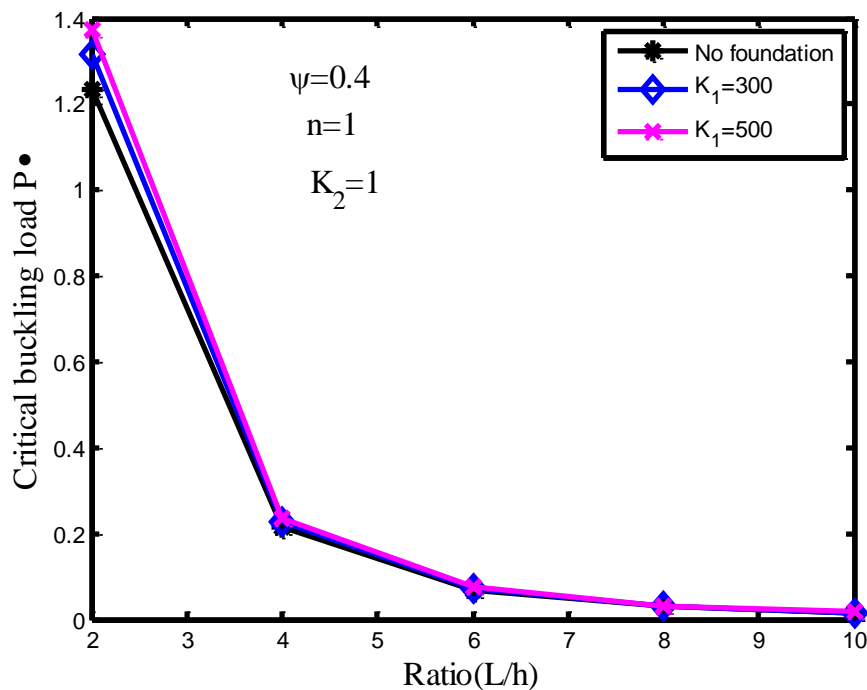


Fig. 4.1 Effect of geometry on critical buckling load of beam resting on linearly varying elastic foundation.

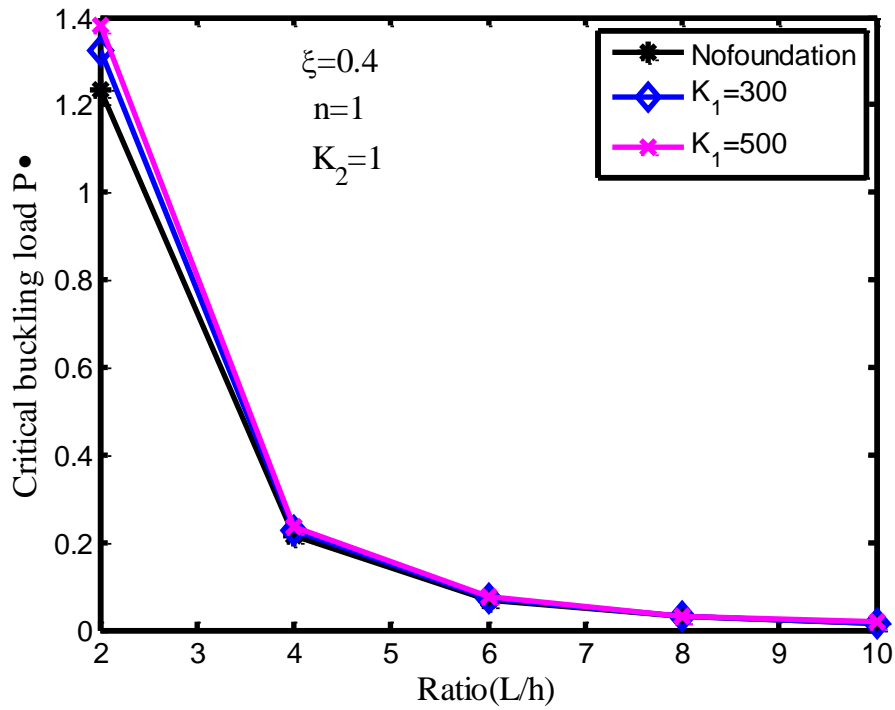


Fig. 4.2 Effect of geometry on critical buckling load of beam resting on parabolically varying elastic foundation.

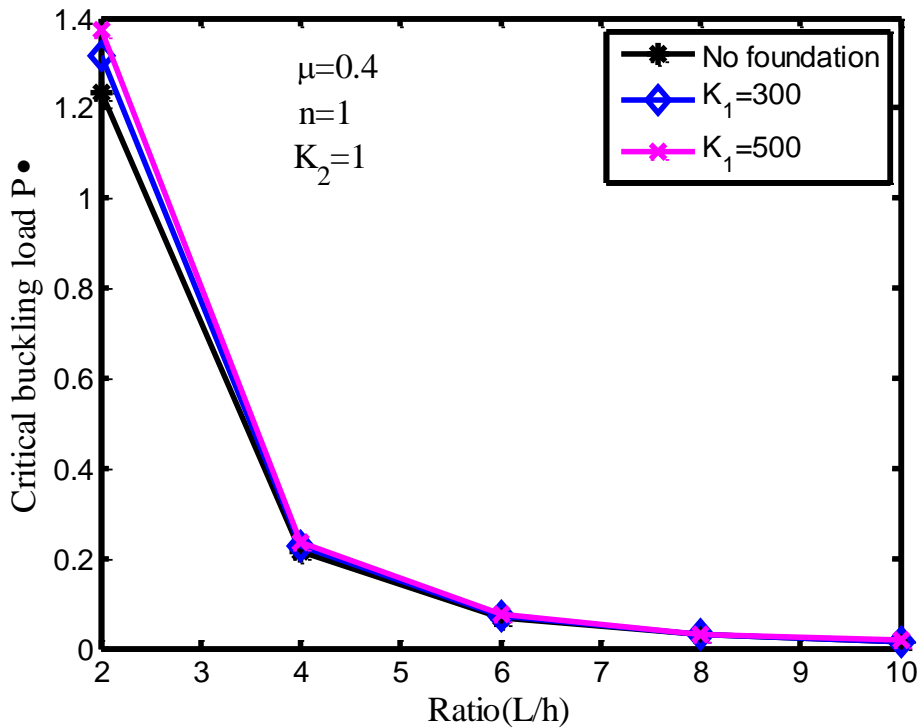


Fig. 4.3 Effect of beam geometry on critical buckling load of beam resting on sinusoidally varying elastic foundation.

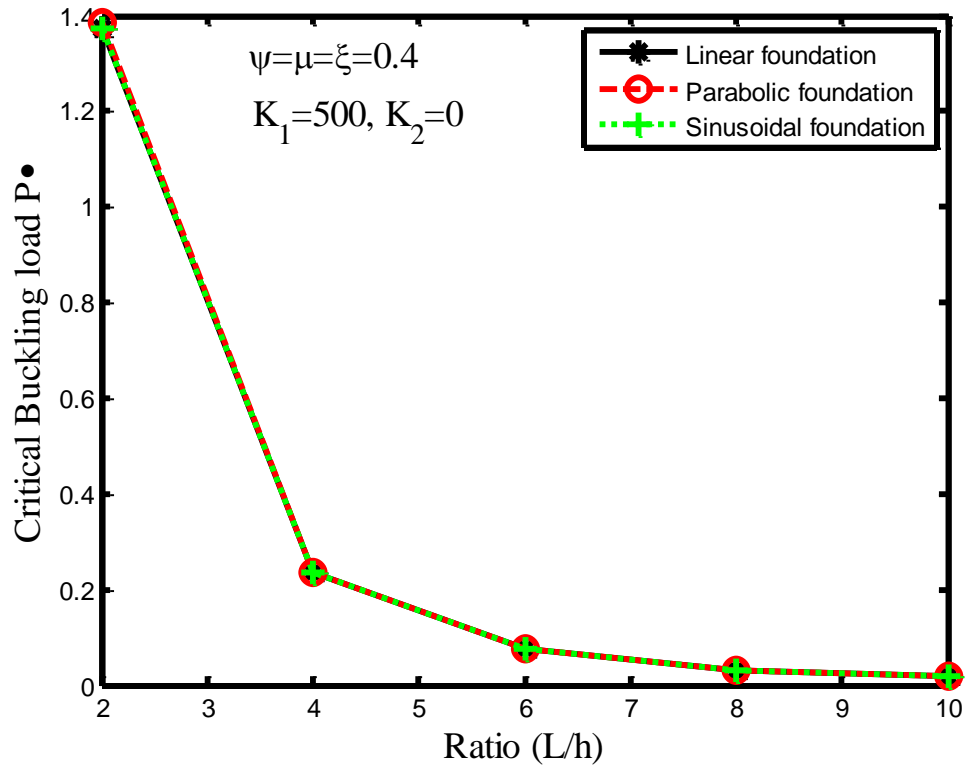


Fig. 4.4 Effect of beam geometry on critical buckling load of beam resting on variable elastic foundations.

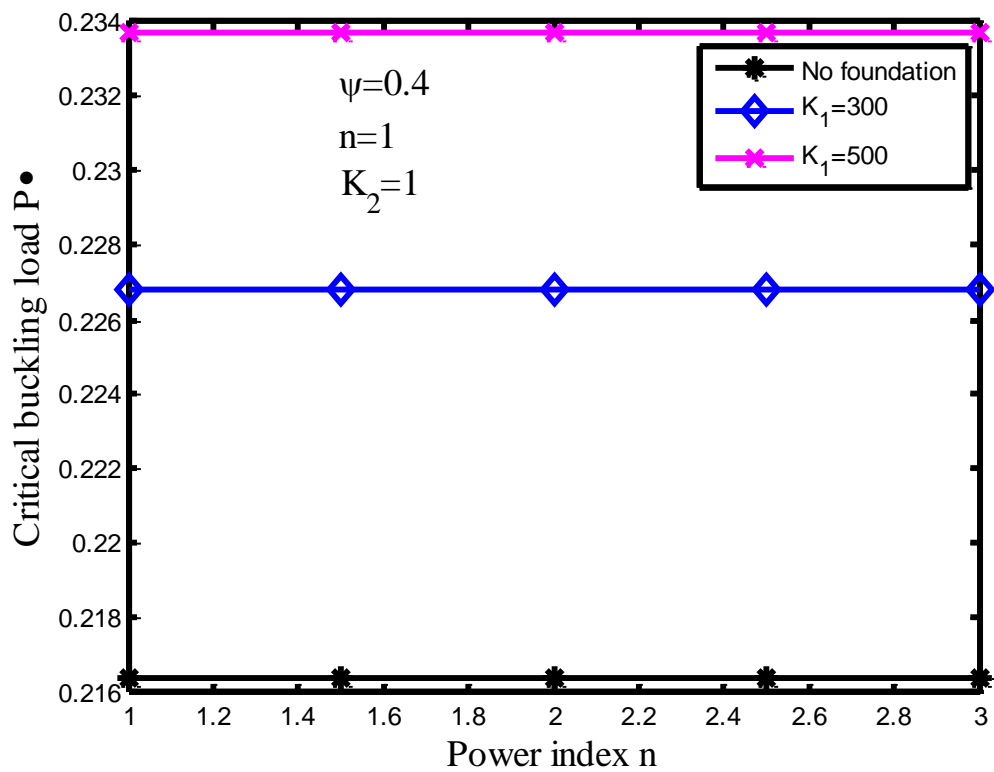


Fig. 4.5 Effect of material properties on critical buckling load of beam resting on linearly varying elastic foundations.

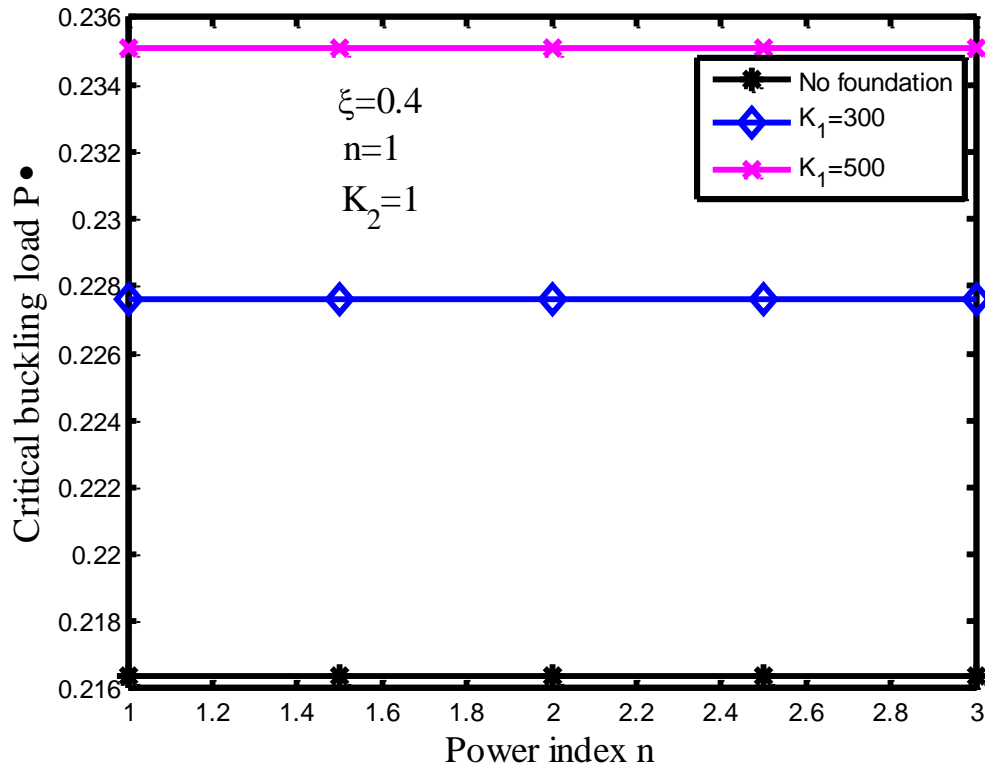


Fig. 4.6 Effect of material properties on critical buckling load of beam resting on parabolically varying elastic foundations.

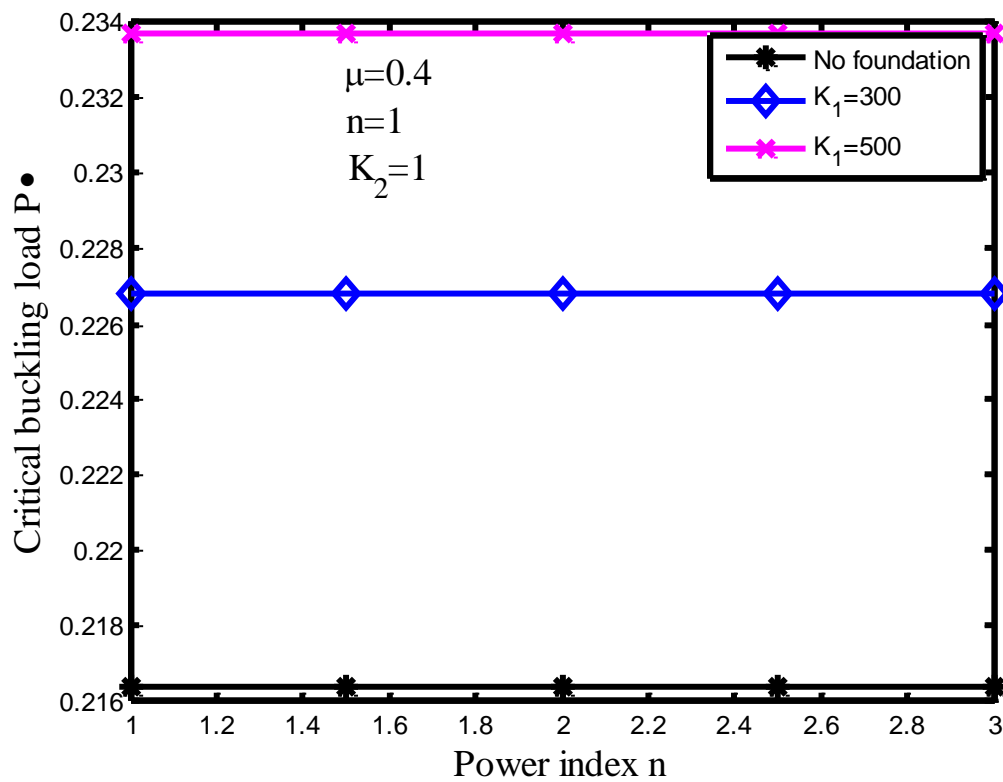


Fig. 4.7 Effect of material properties on critical buckling load of beam resting on sinusoidally varying elastic foundations.

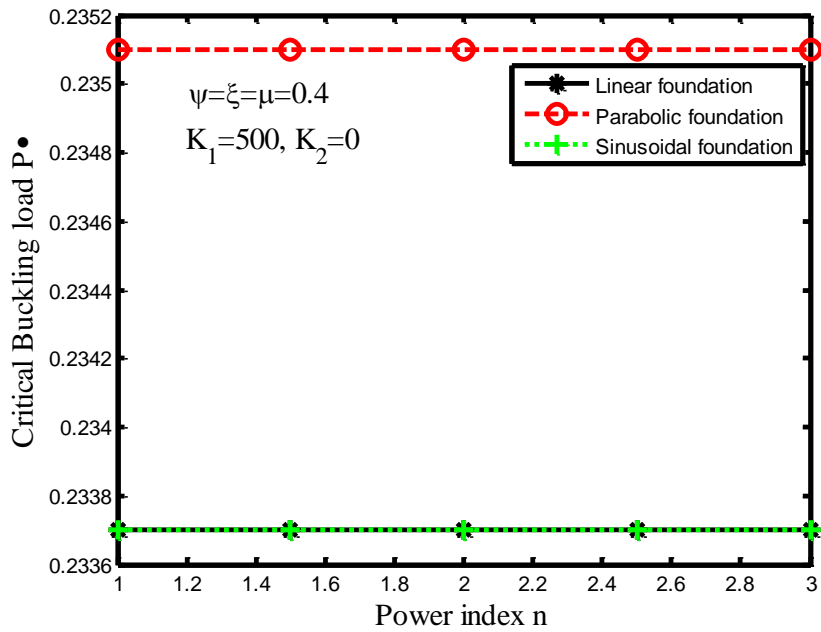


Fig. 4.8 Effect of power index on critical buckling load of beam resting on variable elastic foundations.

The effect of foundation on critical buckling load is investigated and shown in Figs. 4.9 and 4.10. Figure 4.9 presents the contribution of foundation by resisting the deflection of beam and the effect of interaction of shear layer of foundation is shown in Fig. 4.10. The static stability increases with increase of foundation modulus and foundation shear modulus as well which is obvious from Figs. 4.9 and 4.10. It is observed from Figs. 4.9 and 4.10 that the contribution of shear layer of beam in improving the stability behavior of beam is more significant. Figure 4.11 presents the comparison among various foundations as regards their role on stability behavior of beam. The parabolic foundation renders highest stability to the beam while the sinusoidal foundation causing the lowest buckling load of beam.

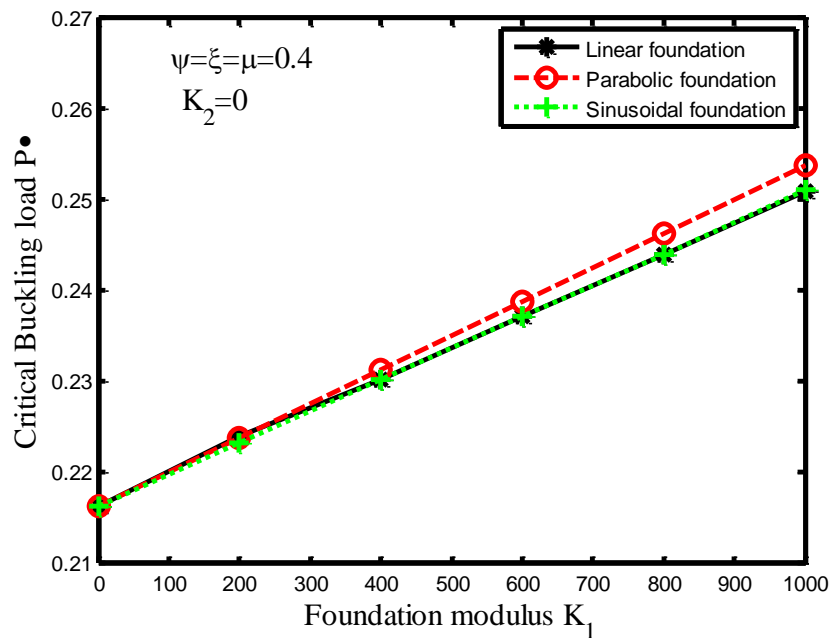


Fig. 4.9 Effect of resistance of various foundations against deflection of sigmoid beam on its buckling.

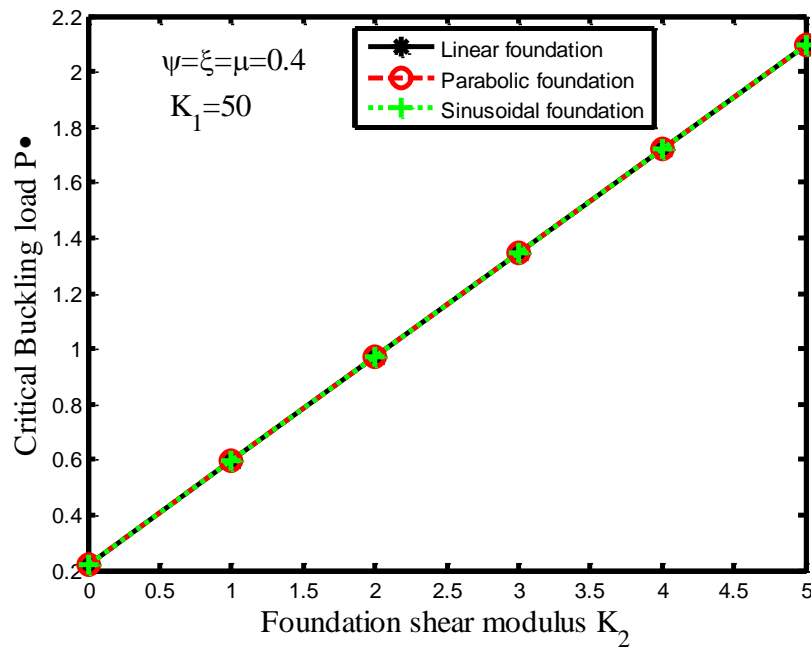


Fig. 4.10 Effect of interaction of shear layer of variable foundations on buckling of sigmoid beam.

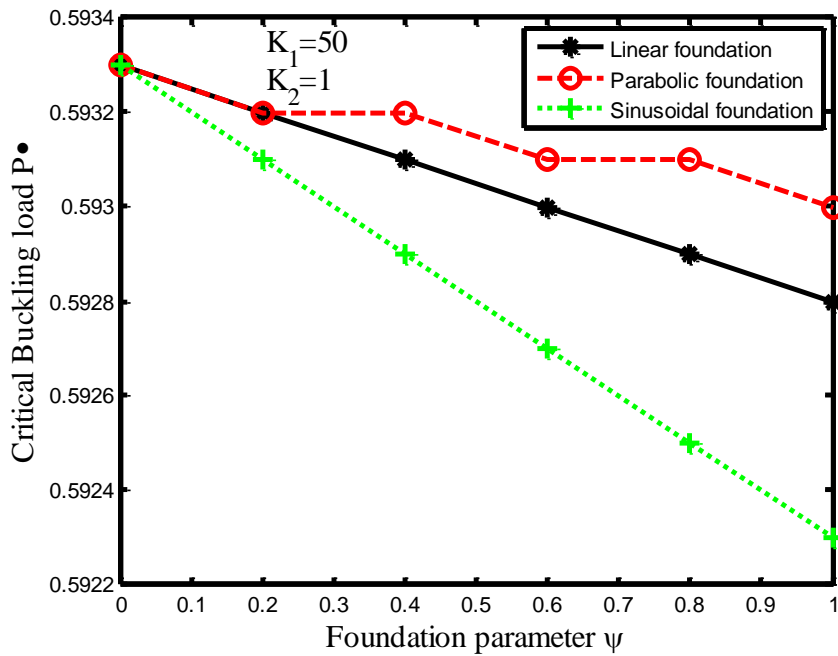


Fig. 4.11 Comparison among the variable elastic foundations regarding their effect on the buckling of sigmoid beam.

4.4 Closure

A study on the critical buckling of functionally graded Timoshenko beam resting on variable elastic foundation is carried out using finite element method. The variation of material properties is considered as per sigmoid law along the thickness of the beam. Foundation stiffness varying linearly, parabolically and

sinusoidally along the length of beam is considered for analysis. The findings of the study is collected as follows:

- More the beam becoming slender lower is the buckling load of beam thereby justifying the Euler's theory of critical buckling.
- Sigmoid distribution of properties along the thickness of beam makes no effect on the static stability of the beam.
- The presence of foundation improves the static stability of the beam.
- There is a marked difference between the effect of interaction of foundation shear layer and the effect of foundation against transverse displacement of beam obtained from the investigation.
- The interaction of foundation shear layer ensures remarkable improvement in the static stability of the beam.

The next chapter is devoted to the study of dynamic behavior of the sigmoid beam resting on variable elastic foundations.